Convective Heat Transfer
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Lecture – 42
Inclined enclosures

So, we intend to wrap up a natural convection in this particular class, but before we do that let us look at one more problem set which is basically, what we call inclined enclosures.

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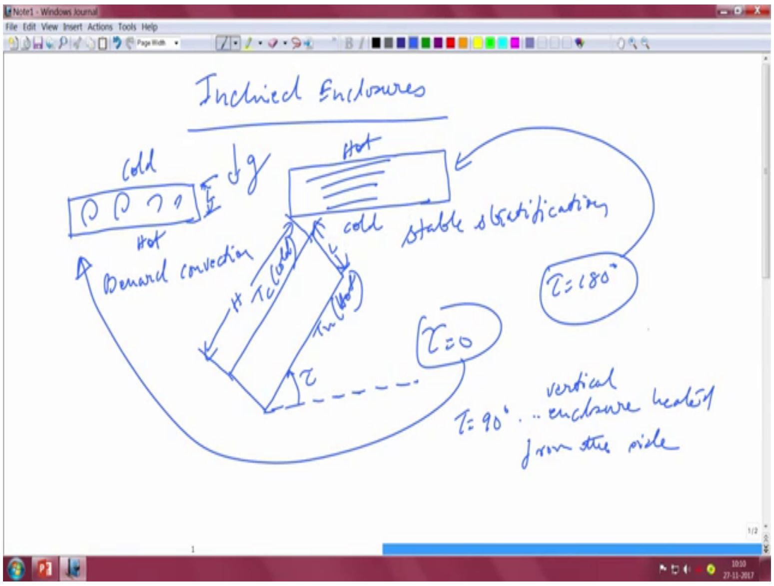

So, we saw the 2 limits right we saw that when you actually have let us take 3 different types of enclosures here and this is hot, this is cold. So, this is one type of enclosure which we saw gave rise to this relevant type of convection. So, this is hot at the bottom cold at the top.

So, let us take another type of enclosure which is the same enclosure if I just turn it around right turn it completely, then what will happen is that it will be cold at the bottom and hot at the top. So, this gives rise to what we call stable stratification this is the Bernard convection, which we talked about in the last class.

So, this is stable stratification. So, in between these 2 limits you can rotate and have this particular enclosure at any angle.

So, let that angle be something like this. So, this is the same enclosure this is a hot Th, this is a cold, there this is the angle which we call this tau, this is the length which we call as H. So, as you can see when this tau actually becomes equal to 0 all right, you have hot at the you have hot and cold depending on what is the orientation?

So, if your tau is 180 degree so, if you just rotate and go on increasing this tau. So, that it becomes it just rotates the other way. So, it becomes cold at the bottom and hot at the top so, you will get one angle. And similarly if you just make it 0 on the other side you will get another angle. So, it will be hot at the bottom cold at the top all right.

So, when tau is equal to 0, when tau is actually equal to 0 you have the configuration which is represented by this, which is there in a Bernard, when tau is equal to 180 degrees you have the configuration which is given like that all right.

So, I mean depending on whether. So, this is your hot. So, just in case this is your cold. So, the 2 configurations that we have over here is that when tau equal to 180 degree, it gives rise to stable stratification and when tau is equal to 0 it gives rise to Rayleigh number assuming that the Rayleigh number is greater than 1708 as we saw in the last class. And in between these angles any angle of tau between 0 and 180 degree you will have inclined enclosures.

Inclined enclosures when tau is equal to 90 degree you will have hot and cold in enclosure. So, it is like a vertical enclosure, which we did in the last class. So, and gravity is; obviously, acting in this direction for all these cases all right. So, 90 degree corresponds to tau is equal to 90 degree corresponds to the enclosure which is heated from the side that we did rigorously in during the earlier lectures heated from the side and it is a vertical enclosure.

So, vertical enclosure heated from the sides so; obviously, this inclination of tau has got a dramatic effect, as we can see that when tau is equal to actually 180 degree you have purely a conduction driven problem, because it is a stratified it is a stable is stratified, while on the other hand we saw that Bernard convection can give rise to a very high Nusselt number value or a heat transfer just, because of this convection cells that you create all right.

So, in other words if I have to represent this whole thing in the form of our graph.

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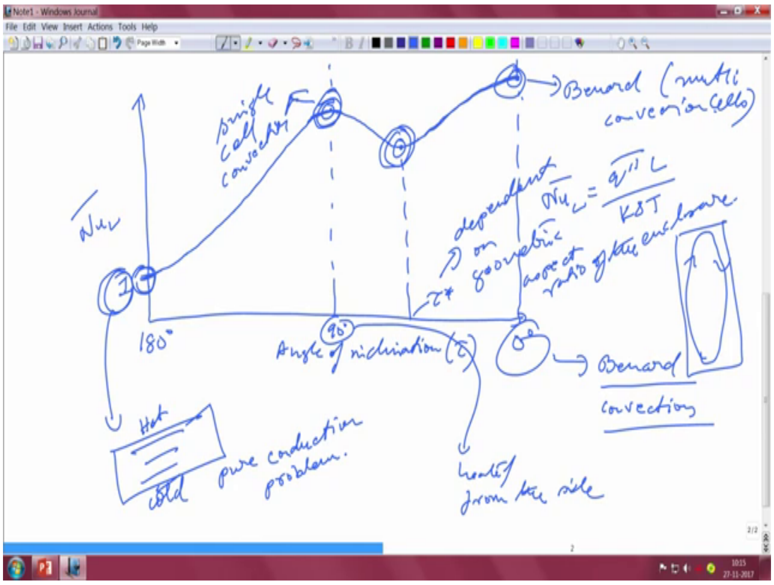

So, let us look at let us first look at what it can be and then. So, this is Nusselt number based on l where l is basically this dimension, if you go back to the previous slide this is your l. So, this is your l. So, in this particular case one Nusselt number equal to one implies the pure conduction limit right and this is your angle of inclination, the inclination which is given as tau all right.

So, this part let us represent it as degree and this part let us put a line here, this is called 0 degree. So, at 180 degree we know it is going to be 1 Nusselt number is going to be 1, because at 180 degree it corresponds to the stable stratification. So, it is hot at the top cold at the bottom it is like stable stratification that is what that one is right.

So, stable stratification is basically a pure conduction problem it is just a conduction gradient that you create. And there is somewhere in the line there is 90 degree. So, what we will what can tentatively happen is that you will go up towards the stable configuration towards the vertical enclosure, we will reach some kind of a peak Peakish over there, because this is basically what we call heated from the side right heated from the side.

So, what do you expect after this after this is supposed to fall a little bit, that is because once again you are twisting the enclosure to the other direction all right. So, it will fall a little bit this is not drawn according to scale the remember. So, it falls a little bit. So, this particular point is given some critical value which is tau star. So, there is a it starts from one it goes up reaches a maximum then it kind of droops a little bit kind of droops a little bit at this particular location.

Again as we go on to the 0 th part which is basically the Bernard convection region, you are supposed to get a kick up once again correct. So, you are supposed to get it rise to a certain extent. So, as you can see over here this is the Bernard convection. So, the Bernard convection as you can see over here. So, this is the generalized curve. So, you start from one you reach a particular maxima and then you it droops a little bit and then it rises once again because you go to the Bernard convection region.

So, this is the whole trajectory. So, this is heated from the side and this is the critical number at which it should fall right. So, this is in general. So, the Nusselt number value that you have is given by q double prime into l divided by K delta T. So, the idea is that as 185 as we start from 180 degree to 0 the heat transfer mechanism first switches' from pure convection to a single cell convection.

So, this is basically or what is called the single cell convection, because this is what we saw right because in these kind of enclosures what you see is basically you get a single cell convection correct that is what we saw right in tall enclosures, shallow enclosures you get the single cell convictions.

So, it rises from a pure conduction to a single cell convection to a maxima which is around 90 degrees as the or tau or your angle of inclination decreases below 90 degree, the Nusselt number decreases and passes through what we call a local minima? So, this local minima is given by tau star. So, this tau star is a function of the geometric aspect ratio of the channel of the enclosure. So, this is dependent on the geometric aspect ratio of the enclosure.

It is dependent on the geometric aspect ratio of the enclosures and after this the after this tau star it starts to rise once again, because of the multiple convection cells this is multi convection cells right multi cells kind of a thing that is what you are really Bernard convection is all about.

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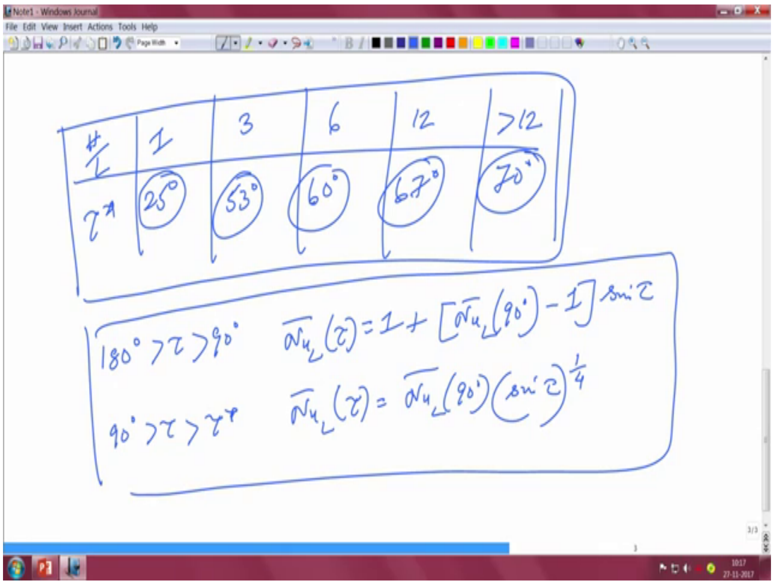

So, you can represent all these things using this kind of a H by L kind of a ratio this is your tau star. So, H by L ratio one is about 25 degrees, when it is 3 it is about 53 degrees, when it is 6 about 60 degrees, when it is 12, which is about 67 degrees, when it is greater than 12 it is about 70 degrees?

So, this is the value of tau star that you would normally get depending on the different geometric aspect ratios. So, it P is. So, it droops at around 20 degrees. So, that tau star will be somewhere beyond 90 it will be 50. So, it is all on the right hand side of 90. So, from 0 to 90 it will peak somewhere. So, 53 60 63 and 70.

So, you can write all these things in terms of a generalized correlation that is tau greater than 90 degrees Nusselt number l tau equal to 1 plus Nusselt number l at 90 degrees minus 1 into sin tau. So, this is a generalized expression that for a single cell that is for a stall enclosure or for a single enclosure kind of a thing. So, that is the correlation.

So, these are all correlations remember does not have much of a physical meaning, but for example, this one is Nusselt number l at 90 degrees and to sin tau raised to the power of 1 4th. So, these are the 2 basic correlations that we have. So, this is how they this been represented very well by different people.

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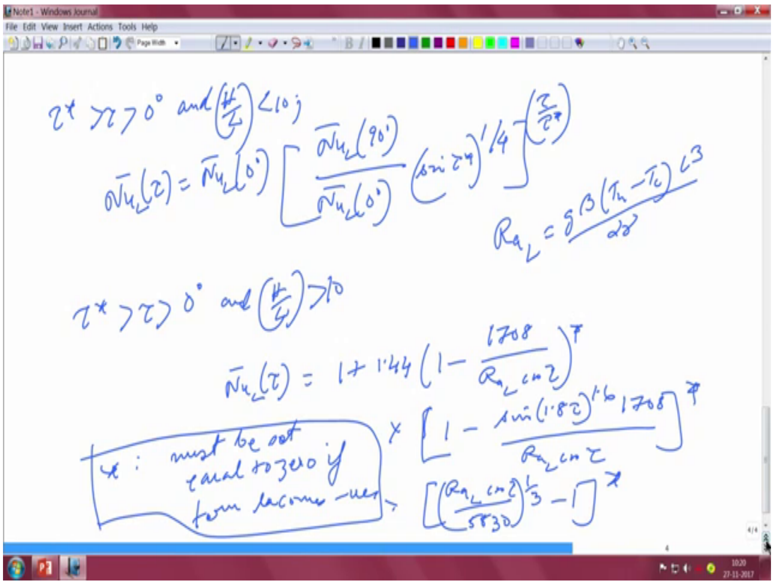

There are also other types as we will see because this is up to tau only. So, when tau star is greater than tau greater than 0 degrees and your H by L ratio is less than 10, you have your Nusselt number L tau is equal to given as Nusselt number at 0 degrees, into Nusselt number L at 90 degrees, and Nusselt number L at 0 degrees, sin tau star raised to the power of 3 by 4 sorry raised to the power of 1 4th and tau by tau star power.

Similarly, your tau star greater than tau greater than 0 degree and H by L ratio is greater than 10 you have a really complicated expression, which is given by 1.4 4, 1 minus 1 7 0 8 these are all the empirical relationships. So, should not attach much importance to them except that they are useful and handbooks kind of arguments, basically multiplied by 1 minus sin 1.8 tau 1 by 6 sorry 1 7 0 8 divided by the full 1 Rayleigh number l cos tau into a Rayleigh number l cos tau divided by 5 8 3 0 raised to the power of 1 third minus 1.

The star essentially means that must be set equal to 0 must be set equal to 0 if the term becomes negative. So, that is the catch and the Rayleigh number in all these cases are based on g beta. So, that is that that is that.

So, similarly if you can have you can have similar really Bernard and this I will just pose the things and would not delve too much into it.

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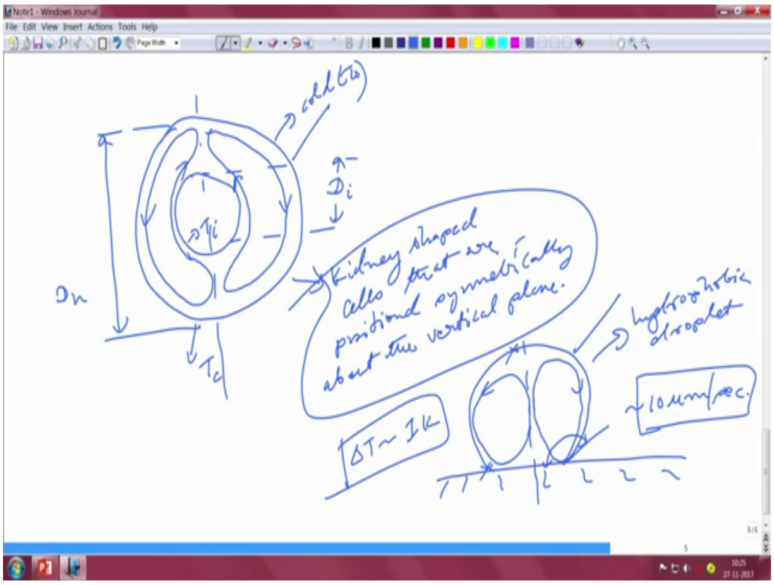

So, this is for example, a warmth cylinder which is heated as Th then you have a cold cylinder which is heated at T c. So, the smaller cylinder has got a diameter of say di and the larger cylinder has got a diameter of say D h. So, this is cold t 0 and this is warm that is very it is T i or T i or T h whatever it is a T i to keep. So, what kind of a flow do you expect?

You expect the flow to basically rise around the cylinder, but at some point of time it would reverse and come back exactly at the symmetry line. Now this is cold, because there will been another cell of equivalent type because this is symmetric. So, this cell will also go up like that and come back. So, you basically get a 2 cylinder kind of a rotation 2 rotational cells just created, because of the vertical symmetry you have a symmetry line which passes vertically through the whole thing.

So, if it is in the laminar regime. So, basically these are called kidney shaped cells that are positioned are positioned symmetrically, about the vertical plane about the vertical plane. So, these cells are basically essentially heated and cooled from the side the overall heat transfer correlation has got the features of whatever we learned earlier.

So, if you saw all the equations that we did earlier it is basically very similar to that except that this is done in a cylinder, but you can kind of you know kind of see that this kind of a cells are very common and we did all these analysis in all the other chapters earlier.

So, there are a lot of correlations which we are not going to go into the details please read the junk for the details of the correlation, but I wanted to show that this pattern that you create evolves basically because of the symmetry, if there was no symmetry this pattern would not have existed. And this is kind of intuitive also that you get heating, then it rises, and then it gets cold and it comes back from the other side.

So, even in some of the other cases say for example, if you take the case of a societal droplet here this comes from our own research work. So, this is what we call a hydrophobic droplet well, the droplet is not hydrophobic it is sitting on a hydrophobic substrate basically. So, that; that means, this angle that it makes it is basically an obtuse angle all right there is an obtuse angle that it makes.

So, what happens over here is that here also will you get this kind of natural convection flows. So, this here the natural convection flow we would not talk about the origin of the flow as such, but you create cells like this. Once again because the symmetry of the center line you get flows from both sides, this is also a natural convection driven flow just because you know you have difference in temperature between the top and the bottom. So, that is essentially one drives the flow.

So, this is very minute differences this temperature differential can be of the order of delta t can be of the order of one kelvin even then you develop a very strong recirculating flow of course; the velocity the flow velocity that you create over here is of the order of tens of microns per second. So, it is not that high conventionally, but from a droplet perspective this kind of flow is actually quite crucial and this kind of flows are the ones which actually leads to a lot of transport related issues.

Very important flow field if you talk about the talk about different types of applications like surface printing then medical biomedical and other such related applications. So, it is not just here this is between 2 cylinders, it can happen within a droplet as well this kind of flow this also can be analyzed by a simple taking into account this this enclosure models. At least as a first cut estimate you can get some numbers and later on you can use those numbers to do a lot of things.

So, it is similarly one should also read about which we are not going to cover, but stuff that is left as a reading material.

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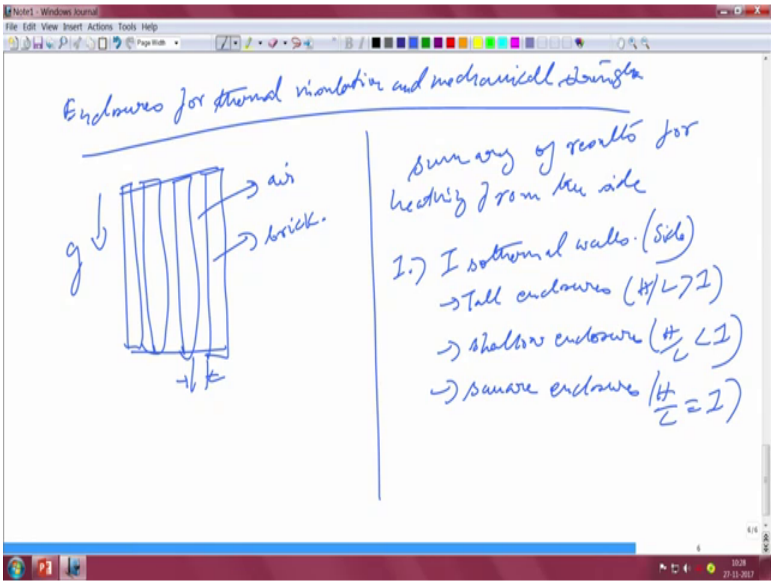

You can read enclosures for thermal insulation and mechanical strength this section from bajan. We are not going to go into the details of this do this be basically if you look at figure 5.3 0 in bay in bajan.

So, this will have all these enclosures it is like this. So, electronic chip cooling and other things will also have that. So, these are the kind of enclosures that you have as g given this way. So, there is air saying inside and this may be say brick. So, you can generate and there will be this length scales. So, you can analyze these problems and see that. So, it is an insulating wall with alternate layers the air is in between 2 insulating walls essentially.

So, you can look into these kind of problems there is quite a few. So, this chapter can be actually this part you can kind of read because there is nothing no physics as such except that this is a more like a conceptual kind of a stuff that you can read.

Also one more reading material that I am going to assign is basically you can look into the there are lots of correlations, which you can see there is also a section called summary of results for heating from the side or heating from the side.

So, out of this you can have isothermal walls isothermal walls side walls basically. So, there you have the tall enclosures tall enclosures just read through them H by L and the graphs that are associated with them then you have the shallow enclosures, obviously.

So, shallow enclosures will be H by L should be less than 1 much less than 1 actually then you can have the square enclosures we have covered most of them in details, but these are there are several correlations and things like that which will give you a nice idea of what these enclosures are all right.

So, similarly there will be there are graphs which basically figure 5.5 5.1 5 5.1 6 and equations 5.63 and 5.6 4, which basically tells you what are the correlations that are available for these kind of enclosures.

But these are nothing I mean these are not technically very new materials, but these are just basically existing correlations how the streamlines and the path lines actually look like. So, there are chapters on partially divided enclosures also which we did in a in the previous class and we also looked at triangular enclosures is something that we never looked at, but this can be something that you can look at in your spare time.

You can also look at pictures of the Rayleigh Bernard flow as I said as you go on increasing the Rayleigh number the flow becomes increasingly more complicated and as a result, the heat transfer limitations heat transfer enhancement is quite significant in those kind of cases as well. So, basically this at this particular point we wrap up the natural convection section we have covered most of the interesting concepts in details.

There are certain reading materials that you can read which is nothing to tell you can just read some graphs and correlations handbooks, but the basic concepts we have given that how frequency how friction balances buoyancy or inertia balances buoyancy and things like that.

So, in the next class what we are going to do is that we are going to look into some of the practice problems or some sample problems from the natural convection, natural convection literature. Before we move on to what we call the turbulent flow turbulent flow is important, because for both all types of whether it is natural or forced turbulent flow see we always saw that as we move on to turbulence something else happens.

So, you are just going to look at from a bird's eye view that what is turbulence because turbulence itself is a full-fledged course, which we cannot do in such a short period of time. So, we will try to do turbulence try to give you an idea of what turbulence is what are the relevant time scale length scales etcetera in this turbulence.

So, in the next class what we are going to do now we are going to look at some of the sample problems before we move on to turbulence. So, here the natural convection literature basically ends.

Thank you.