Convective Heat Transfer
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Lecture – 41
Partially divided enclosures

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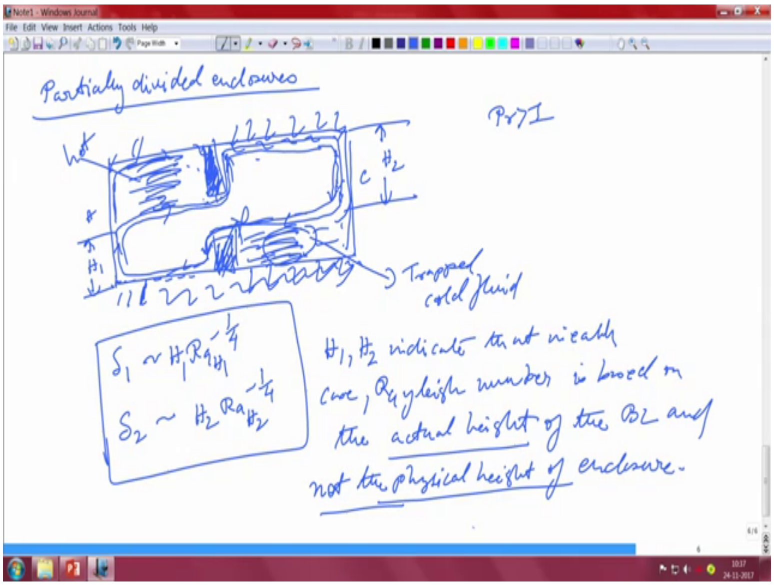

Partialy divided enclosures

$Pr > 1$

$$\delta_1 \sim H_1 Ra_{H_1}^{-1/4}$$

$$\delta_2 \sim H_2 Ra_{H_2}^{-1/4}$$

H_1, H_2 indicate that in each case, Rayleigh number is based on the actual height of the BL and not the physical height of enclosure.

Last class we started looking at this multiple enclosure business. So, partially divided enclosures. So, this partially divided enclosures is has is very realistic as I told you in the last class. So, because we can have this if you do not design your room say for example, properly if it has got lots of partitions, you might end up in a situation that you might get a very wrong profile.

So, let us see how to analyze a very typical problem, take this we will analyze this and then we will try to heuristically draw a few. So, there is a prop which is comes out like this, the prop which goes like that. So, this is like that. So, this is insulated, this is insulated this prop is insulated this is also insulated got it.

So, this is the heated wall H is the cold wall C, now what kind of a recirculation pattern would you expect right? So, naturally what will happen to the fluid is that it encounters a hot wall right? So, it will start to rise right? Now if it rises along this wall it can it go to till the top these are insulated walls, remember this props are all insulated walls. So, will it actually come down and then again go up and then kind of form this entire recirculation pattern. So, that is one interesting question.

So, for example, it can go will it go something like this, this is what I am trying to imply and come down draw like this, that would be one pattern that can immediately come to your mind that why not this, but the idea is it is not that thing at all, that is because these walls are actually adiabatic in nature. So, why should it actually come down, if it goes up it is facing an adiabatic wall throughout right? So, why should it go up that much, instead if I kind of you know I will redraw this whole thing.

How about if I redraw this whole thing over here, this is still the 2 walls these are all adiabatic in nature this is the hot this is the cold. So, this is still the thing now it goes up. So, why cannot it turn here right? It can turn here because of the heat it will go up anyways right? Then it will come down right? But it would not come down the full way because it encounters an adiabatic wall over here. So, it will come down like this and it will go like that.

So, this will be kind of the path of least resistance that the flow will follow, because it has got no obligation to go up in both sides. So, what will happen here you will get trapped a hot fluid, where the fluid that has gone up would not recirculate at all, because you have an antiemetic partition over here and here you will have trapped cold fluid got it.

So, you will have trapped cold fluid here and trapped hot fluid there, only recirculation will take place in this particular fashion. So, whoever is sitting if you are sitting in that side of the room, you will feel cold if you are sitting on the upper side of the room you will feel hot right? So, because you understand the fluid if it does go this particular way, that mean if it starts to make that kind of a trajectory that is not the path of least resistance for the fluid because, it has got an antiemetic wall it has to come down and then again go up. Why should it come down when the fluid is heated, right? It needs a mechanism of a cold wall to reject the heat and come down correct, but on the other hand why does it rise over here, here it rises because the fluid is still hot.

So, it can still rise and it is an antiemetic wall. So, that adiabatic wall is not sucking any heat out of the fluid. So, it will continue to rise and it will go laterally till it encounters the cold wall, as soon as it encounters the cold wall it cools down right? And it starts to return. Why does not it go all the way down and then recirculate? Because here after it is cold what makes it go up, a cold fluid cannot go up this obstacle there is no buoyancy all right? Like in the same reason why should the fluid come down in this upper part, because there is no buoyancy. There is notice you have not changed the density of the fluid at all..

So, what will happen is that whatever fluid is there is trapped. So, you create a hot and a cold air pocket just by the virtue of this design that you have implemented over here. So, once again let me just narrate one more time, normally as we have seen in other applications the fluid goes up then takes a right of a turn and then it comes down, here this is an adiabatic wall, it which cannot extract any heat out of you right?

So, the hot fluid that has gone over there has got no mechanism of coming down right? So, that fluid would not recirculate at all it will remain trapped whereas, the fluid over here if you look at this particular line, the fluid goes up it gets heated some of the fluid actually still goes up, because there is no mechanism to extract any heat out of the system. It goes up and then it takes a right-hand turn, then it encounters this cold wall and it starts to sink from the cold wall, because the cold wall is cold that it can reject the heat and it can come down.

Now, the fluid which is here actually, cannot has got a very high density the fluid that is here has got a high density. So, it cannot climb up anymore that is not possible. So, the fluid turns from here, because this is the sideways and then it can sink, because it is a cold fluid. So, it can sink. So, it sinks and then it completes the loop. So, this is the recirculation loop that you would actually have right?

If you term this height as H1 and this height as H2. So, these are the 2 heights H1 and H2 that if you define these are the 2 heights from the geometry of the internal partition, now for prandtl number greater than 1. So, we can do a purely heuristic analysis over here. So, delta 1; that means, the thermal boundary layer thickness is H Ra H to the power of minus 1 4th.

In this case it will be H1 and this will be R a H1. So, this is the height this is the effective height of the enclosure the effective height of the enclosure is not that whole height, as you can see the whole height is H, but it is only H1 which is participating in the natural convection process similarly, there is delta 2 which is on the other side. So, that has H2 into Ra H2 to the power of minus 1 4th.

So, these are the 2 terms that we have. So, H1 comma H2 indicate that in each case the Rayleigh number is based on the actual height of the boundary there, rather than the physical height of the enclosure. So, this part should be quite evident from the diagram and whatever I did over here.

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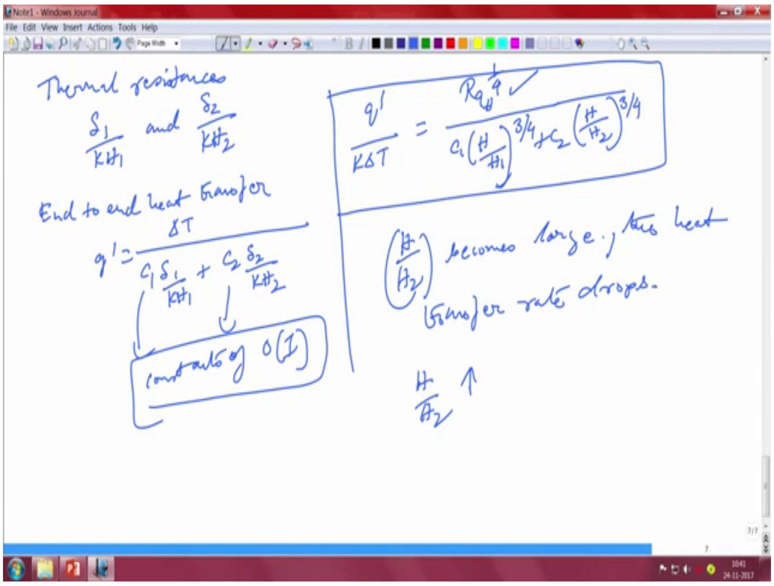


Thermal resistances

$\frac{\delta_1}{KH_1}$ and $\frac{\delta_2}{KH_2}$

End to end heat transfer

$$q' = \frac{\delta T}{C_1 \frac{\delta_1}{KH_1} + C_2 \frac{\delta_2}{KH_2}}$$

constants of O(1)

$$\boxed{\frac{q'}{K \Delta T} = \frac{Ra_H^{1/4}}{C_1 \left(\frac{H}{H_1}\right)^{3/4} + C_2 \left(\frac{H}{H_2}\right)^{3/4}}}$$

$\left(\frac{H}{H_2}\right)$ becomes large., the heat transfer rate drops.

$\frac{H}{H_2} \uparrow$

So, the let us calculate the 2 thermal resistances. So, the thermal resistances are delta 1 KH1 and delta 2 KH2 these are the resistances. So, if I calculate the end to end heat transfer; that means, the heat transfer from the warm to the cold. So, that will be q 1 delta T divided by C 1 delta 1 by K H1 plus C 2 delta 2 by K H2.

So, these are constants of order 1 by definition. So, you can cast it in a slightly non-dimensional type of form, K delta T this would be given as R H to the power of 1 4th divided by C 1 H by H1 to the power of 3 4th plus C 2 H by H2 to the power of 3 4th.

So, C 1 and C 2 are by definitions constants of the order one, this heat transfer is cast in this particular non-dimensional form, in this non-dimensional form where it is given mass q prime by K delta T is given by this particular term over here. Now, looking at the figure the previous figure. So, this is the overall heat transfer, right? Which is basically you can see it is like a kind of a superposition of the 2. So, not an exact superposition.

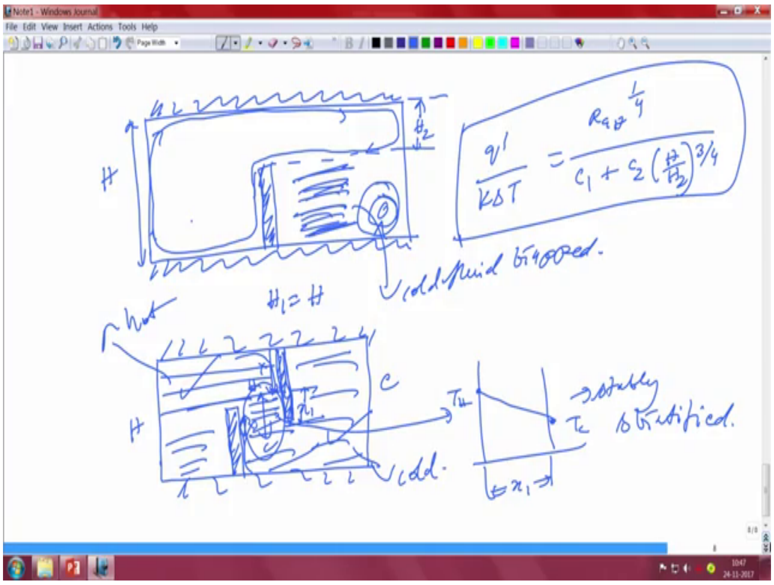
But as you can see that this particular figure if you once again watch it carefully, we see that if we play with the height; that means, H by H2 and H by H1, we can get something interesting things that can come out of it. So, H by H1 and H by H2 are the 2 parameters, that comes directly in this particular equation, right? Because Rayleigh number we have now cast it in terms of H.

So, H is a enclosure overall height this partition heights we are playing with. So, 1 is H1 is H2 and let us see. What happens when you play with those partition heights. So, we can see that as the opening left above the partition above the partition decreases, or in other words H by H2 becomes large H by H2 becomes large the heat transfer rate drops correct.

So, as H by H2 becomes large; that means, the denominator becomes large all right? So; that means, overall heat transfer rate should drop down. So, H by H2 becoming large meaning that H by H2 increases meaning that, H becomes H2 becomes a smaller part of H, right? Or in other words the gap at the top is kind of decreasing; that means, this gap that you are creating between the 2 that upper partition that gap is actually decreasing all right?

So, that gap as it decreases your heat transfer rate decreases, because your total area that is available for the natural circulation actually goes down right? So, that is the reason why this actually happens and the heat transfer rate actually decreases because of that. So, as you can see. So, this kind of agrees with our intuition and as you decrease the overall gap between the 2, that way we make the partition kind of sit down a little bit this particular gap actually decreases, as a result of that the overall heat transfer rate should actually come down all right?

So, this is exactly what is seen over here same happens when you actually have H by H1, the same thing actually happens let me see you raise the whole thing you lowered the whole thing down, and you decrease the gap towards the bottom this also kind of shrinks. So, this correlates the heat transfer with a single internal partition, but you can have. (Refer Slide Time: 14:17).



So, in the case for example, let us take a look at a specialized case of this. So, again we draw the same box. Now, instead of 2 partition we have a single partition. So, this is the total height H. So, this is the height that we are concerned about it is H2. So, the total fluid comes down like this. So, that is H2 that is a single partition here H1, 1 is equal to H. So, the lower part actually becomes equal to 1 all right the equation that we did that one part becomes equal to 1.

So, in other words q prime by K delta T becomes R a H to the power of 1 4th then it becomes C 1 just C 1, plus C 2 into H by H2 to the power of 3 4th. So, that will be the term. So, here also you can see you only have the cold fluid trapped all right, the hot fluid actually circulates. So, people who are sitting in this part of the room, will actually face you know a continuous recirculation pattern whereas, people who are sitting if you are a guy who is sitting over here, you will find that you will feel it is it is very cold.

Actually, I mean not very cold, but relatively cold, because any this air is basically stagnant and stratified or stagnant and trapped here. So, this is very unhealthy if you are if you think about air conditioning or think about the natural circulation in a loop natural circulation in a room, then this particular guy is going to experience a lot of stale air, because the actual circulation is happening from the top. So, that is one thing. So, this acts like a fluid trap, here there is only one fluid trap when you have another partition coming at the top, you get a second fluid trap after happening on the top corner all right?

So, there you can create different types of fluid traps and these are all given by this kind of equations, which is very simple comes from simple scaling arguments right? And from existing correlations that we already found out through the duration of this course all right? Now, let us take a look at this also say for example, this is a another type of enclosure.

So, now what you do is that you make it a little longer this you draw down a little bit. So, these are both insulated right? So, if you design a room like this then you are in big problem that is because, let us see this is hot this is cold once again. So, the hot fluid as it rises as say the hot fluid starts to rise, it encounters this then it has to go up like this then it has to come down, then has to go like this and then there is no mechanism of bringing the fluid down, from this part of the enclosure right no mechanism cannot come down like this. Because hot fluid, why should it come down all right not under the action of a recirculation recirculating flow.

And cold fluid why should it go up it also cannot go up right? So, what you create in essence is that you create a region which is hot here, you create a region which is cold here, because there is no research and in this region, you get a temperature gradient from hot to cold. So, I understand. So, if you look at this region zoom in what you will find is that, from hot to cold you will have a temperature gradient. So, this is TH and this is TC and this will happen over that region which is given as say x 1.

But other than this the fluid is trapped here the fluid is trapped here, but; obviously, they cannot come down all right suddenly. So, there is an interfacial region across which the hot and the cold fluid will kind of migrate from one state to the other. So, this is for example, as a typical example that this is stable is stratified. In fact, when you actually do thermo cline-based energy storage, this actually operates on such principle. So, and this region is purely hot this region is purely cold.

So, you get no recirculation at all because as I said, hot fluid cannot come down and cold fluid cannot go up. So, you basically have taken away everything, actually can sustain a circulation. So, initially there can be some circulation, but after that it kind of stops..

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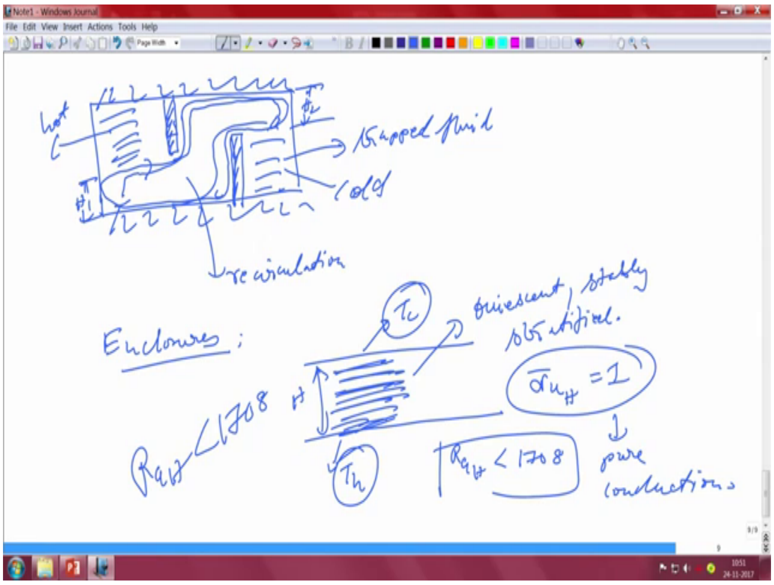

So, similarly the same design if you just take it say like this, extend this out a little bit. Now, in this particular case in the moment you are going to get a recirculation, because now the hot fluid you are giving it a chance to go up, it will go up like this and it will come down because this has encountered the wall.

So, it will maintain a recirculation of this kind, here it will be once again cold this will be hot when you get basically this is H1 and this is basically H2. So, this is the recirculation that you create, and this is basically the trapped that the trapped fluid. So, this is like a fluid trapped phenomena that you have. So, this is natural convection is possible in this particular case because look at it, you are enabling the fluid to go up and then you can it can go further up, because it is heated here you are offering a heat sink. So, that it can come down, it would not sink because then it cannot come up. So, it will just follow this particular trajectory.

So, initially after some time this is the steady state pattern that will actually evolve in this kind of things. Let us look at enclosures which are heated from below. So, you can get different types of enclosures and we can see that you can design and you can kind of from your common sense from your intuition you can actually plot, what will be the stream line? What is the heat transfer from end to end? We already showed that. So, what about enclosures which are heated from below?

So, let us take this as a as the enclosure. So, in this case this one is say hot, this one is say cold and this is the height H. So, hot to cold here is a fundamental temperature difference that has been created. So now, what will happen is that when the enclosure is sufficiently long and sufficiently wide, we are going to define something called a critical Rayleigh number.

So, if the Rayleigh number based on height is less than 1708, this is basically stable stratified stable is stratified; that means, it is questioned stably stratified and of course, the Nusselt number H is of the order 1, because it is the pure conduction limit, right? So, it is pure conduction essentially all right it will be pure conduction this is kind of stably stratified. So, this is TC this is TH all right?

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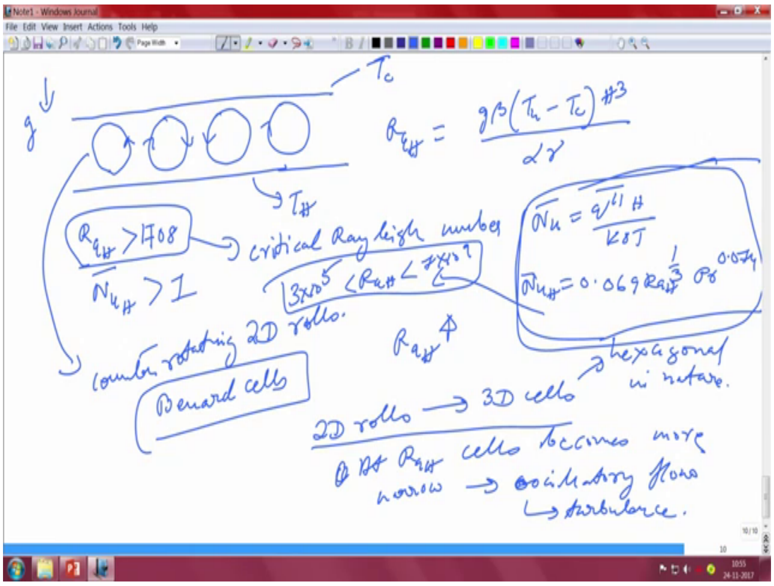

So, this is only valid for Rayleigh number less than 1708. So, as and when we increase the Rayleigh number we take the same configuration, so the Rayleigh number H is greater than 1708 and Nusselt number H is actually, now greater than 1 because we have now the limit. So, this is heated this is TH this is TC cold.

What you start to get is called the cellular patterns, look at the rotation aspect of this. So, as soon as a Rayleigh number crosses this 1708 which is called the critical Rayleigh number you start getting this recirculating cells, now between the top and the bottom, let us write the definition of Rayleigh number also just in case TH minus TC.

So, immediately you would see that the flow consists of this counter rotating 2 dimensional rolls. So, these are counter rotating counter rotating 2 dimensional rolls the cross sections of which are almost squared the flow pattern is called basically the bernard cells, as in call as in Rayleigh bernard convection. So, the cellular flow becomes it becomes more and more complicated, as we go on increasing the Rayleigh number.

In fact, if you search on the web you will find that you get spectacular patterns that actually form as we go on increasing the Rayleigh number. So, the 2-dimensional rolls 2 D rolls first starts to break up into 3 D, 3 D cells all right? Which are hexagonal in nature and even at higher Rayleigh numbers, this the cells becomes at high Ra H cells become more narrow and far more complicated and it leads to oscillatory flows after that and for at the end you get to turbulence, as you go on increasing the Rayleigh number.

So, the bernard convection has been a topic of active research for many many years around the Rayleigh bernard convection, and the there has been varieties of measures that has been proposed for example, the Nusselt number will be one of them. It will give us the q prime bar divided by H by K delta T this would be the standard definition there has been lot of correlations, which we are offering here without any semblance of a proof 69 line or Rayleigh number to the power of 1 3rd prandtone number to the power of 0.74.

So, the x this is this has got a range. So, this correlation is valid for Rayleigh number greater than 3 into 10 to the power of 5 and less than 7 into 10 to the power of 9. So, that is where it is kind of going to be valid. So, as we can see that rolls can be complicated the there this is just a global national number, which is useful for engineering definition.

But we all below a certain Rayleigh number there is basically no flow it is just stable stratification. So, there are different types of experiments historically people have done a lot of analysis on this really Bernard. We are not going to go into the details you can find some pretty pictures over the internet, where it will show the flow transitions and also a lot of Rayleigh numbers regimes will be there. So, we are not going to go into those kind of details.

But at the same time this is the canvas has been laid that. This is what happens when you actually have enclosures just from simple engineering sense, you can design enclosures and you can also see that when the enclosure is heated from the bottom what happens, now if you just invert the whole thing; that means, it is hot at the top cold at the bottom it will always remain like that, because there is no mechanism to induce any instabilities into the system.

So, in the next class what we will do is that, we will go we will do a little bit on inclined enclosures; that means, as the enclosure is inclined. So, you go from hot core all the way up to hot cold in other way and in the in this way you cover a whole gamut of inclinations, again we are not going to do a proper analysis just to is going to give you some scaling arguments over there, and then we are going to move on to the turbulent flow regimes.

Thank you.