Convective Heat Transfer

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Lecture – 39
Regime IV- Shallow Enclosure Limit I

So, we formulated the problem. Now, what Gill did not do at that particular point, he did not go up to the energy equation that mean they do not solve for the Nusselt number. So, we will look at the Nusselt number solution and then we will look at some of the data that Gill actually predicted.

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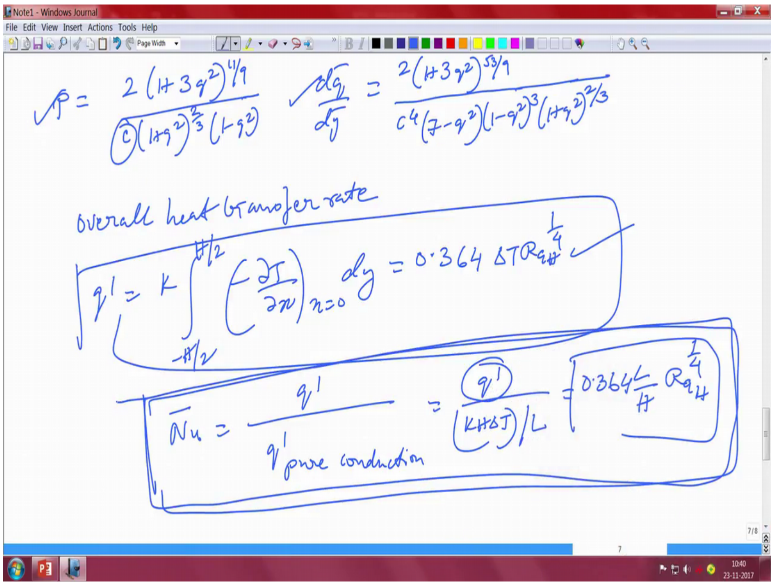

So, the overall heat transfer rate if you look at it, it is q prime K minus H by 2; 2 plus H by 2 minus dt by dx x equal to 0 dy. So, basically that is the integration that was performed and it comes out to be 0.364 delta T and Rayleigh number to the power of one- fourth. So, that is the expression that we get.

Now, if we define the Nusselt number case in this particular way noting that majority of the relationships basically depends on Nusselt number. So, the Nusselt number average, average Nusselt number is nothing but the q prime divided by the q prime in the pure conduction limit. So, that is the Nusselt number. So, this will roughly translate to the actual q prime divided by the KH delta T divided by L, L being the width of the enclosure. So, it is basically if you have like a constant diffusive type of a heat transfer that will be what it will be, right.

But, on the other side we are interested more in this. So, basically this translates to because you already have that q 1 now fetched from here. So, it will be 0.364 L by H Rayleigh number to the power of 1-fourth. So, that is the average Nusselt number that we are getting over here. So, that is the average Nusselt number.

So, now we can look at now that we know that the Nusselt number we know, what it is going to be in the high Rayleigh number limit this is the Nusselt number, but there is a lot of confusion regarding this particular aspect of the result. So, the result is particularly significant that is because you know that when Gill actually solved these 2 equations that is d bar dq bar by dy bar, he used what we call an arbitrary condition. So, let us write that arbitrary condition.

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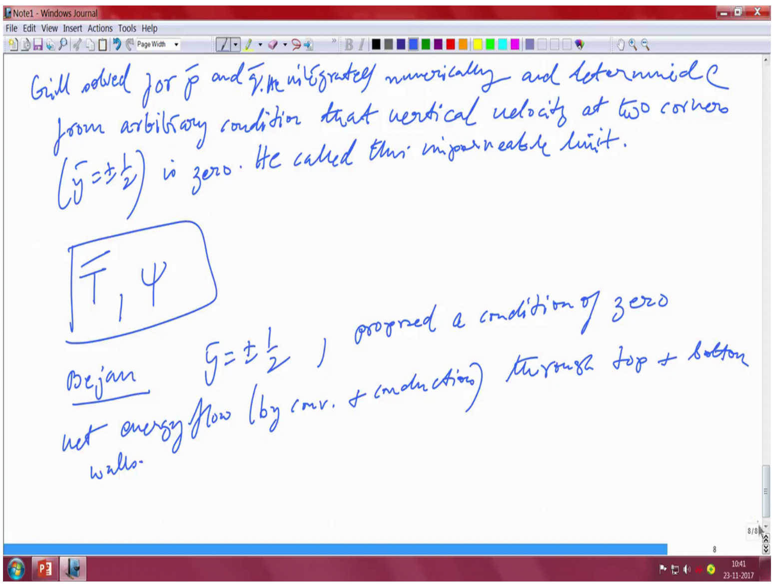

So, when he solved for, when Gill solved for p bar and q bar basically. So, that is what he used. So, when he integrated this in this expressions; he integrated these expressions numerically of course, numerically and determine the constant c, the constant c is this constant, this constant over here. So, that constant c and determine constant c from the arbitrary condition that the vertical velocity at the 2 corners. So, which is basically y bar equal to plus minus half those are the 2 corners is 0. So, this he called as the impermeable limit, he called this impermeable.

But, however, there is a lot of argument regarding this because the boundary layer strictly the assumption strictly does not hold when you go to the corners, the corners of the of the enclosure. So, it is only v bar, the scale of v bar or the expression for v bar is only valid in the boundary layer, so not at the corners. So, there is some dispute regarding that, but whatever we showed before this is based on gill's assumption that he solved for p bar and q bar by using numerical integration and he determined the constant using the arbitrary condition that the vertical velocities at those 2 points are basically equal to 0, in those 2 regions are basically equal to 0.

Now, what we are going to show is some data in which we are going to plot T bar and the stream functions. So, is stream functions is nothing but the velocity field, as you know the velocity is always tangential to that stream line.

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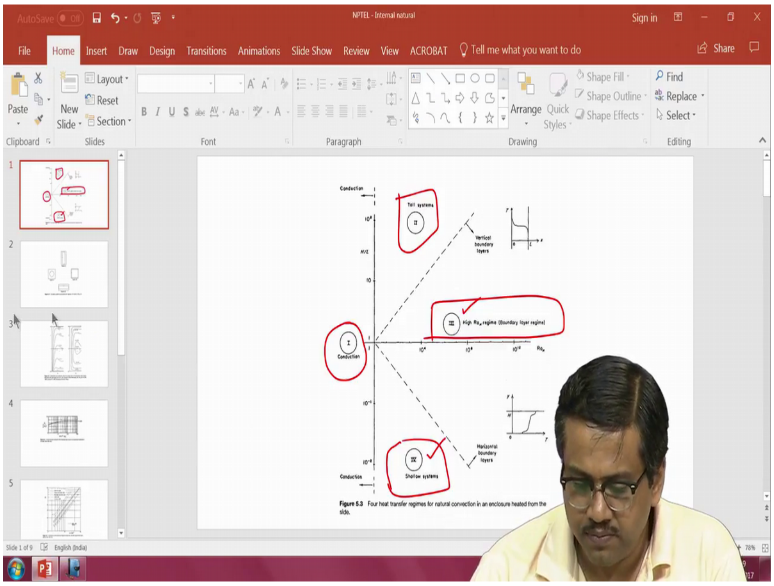

So, we are going to show these 2 results and this is taken from that Gill's work.

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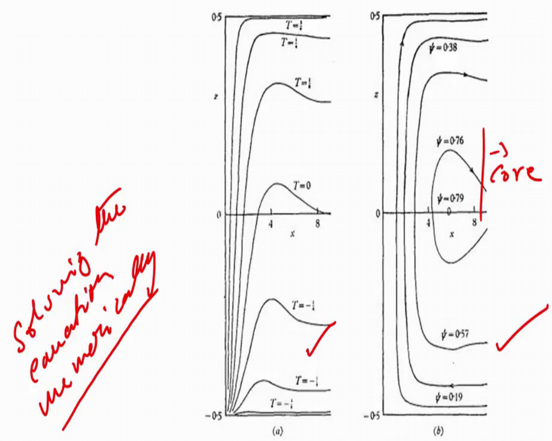

Figure 5.5 Streamlines and isotherms near the vertical wall in the boundary layer regime. (Reprinted with permission from A. E. Gill, *Journal of Fluid Mechanics*, Vol. 26, pp. 515–536, 1966. Copyright © 1966 Cambridge University Press.)

So, if you look at this now. So, these are the stream lines, so these are the isotherms and these are the corresponding stream lines that you have seen an enclosure of that sort. So, this is from A. E. Gill Journal of Fluid Mechanics very long time back, this work was done very, very long time back.

So, you can see, so this is the core region, these are basically the mounted there; so the core region lies somewhere there. So, you can see from this particular expression. So, this is what the isotherms looks like and this is what the stream functions looks like after solving the equations that I wrote, equations numerically. It is not a complete numerical model, but the integration and other stuff are done numerically. So, and from this it was further extended by Bejan and others in which they calculated now the heat flux or the heat transfer rate using this expression. So, that was what we coated in this particular here, this is what we coated here, so that was the Nusselt number that Bejan found out and what they were built up on that condition.

So, Bejan actually did, so this was done by Bejan. So, he proposed something as a substitute for this impermeable limit where y equal to plus minus half, the wall was taken as impermeable. So, Bejan proposed and condition of proposed a condition of 0 energy flow or net energy flow rather 0 net energy flow that is by convection and conduction through top and bottom wall. So, it basically takes into account the adiabatic wall condition as well as the same time it takes into account the impermeable limit as well what Gill actually did.

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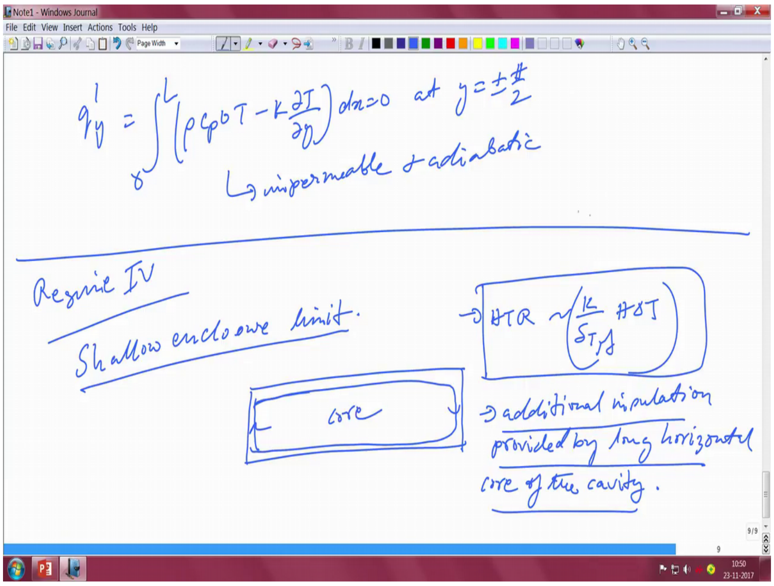

So, what happened was that his qy prime it looked something like this rho cpvT minus K dT by dy dx is equal to 0 at y equal to plus minus half or plus minus H by 2 this non dimensional space. So, this basically is both impermeable plus adiabatic, it kind of encompasses both the conditions; that means, you use both the parameters. So, the result that he got is actually shown in here.

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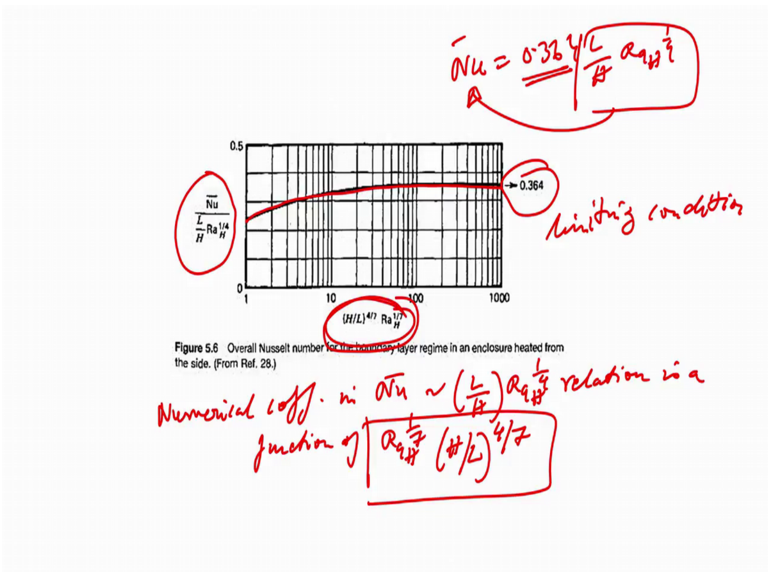

So, if you look at it here, it goes to 3.64 what Gill got, but it is at as a; it emerges basically as a limiting condition, it emerges somehow as a limiting condition actually the numerical coefficient of the factor because you see here it has been normalized by LH by Ra H to the power of 1 fourth. So, basically if you understand what Gill did or from gill's work what people found was 0.364, L by H Ra H to the power of 1 fourth. So, if you divided up by this you are supposed to get this factor only, which is what you get in the large limit in the limiting conditions.

However, here you can see that it is a trajectory which seems to be a like a function of this, that is what you are trying to get. So, the numerical coefficient in you see Nusselt number bar L by H Ra H to the power of 1 fourth relation is a; so, this Ra H by 1 fourth relation is no longer 1 single constant value. So, it is basically is a function of Ra H to the power of 1 by 7th; H by L to the power of 4 by 7. So, that is kind of the relationship that you get because that is what the factor is over here.

Only in the limiting condition we can see that it goes up to about 0.364 over here. So, that is the case that we see over here, so this you can remember, this is not very simple like what Gill did; it not very simple in that particular way.

Now, what people have done is that people have done a survey of different types of solutions that are available. So, you can look at this particular graph for example.

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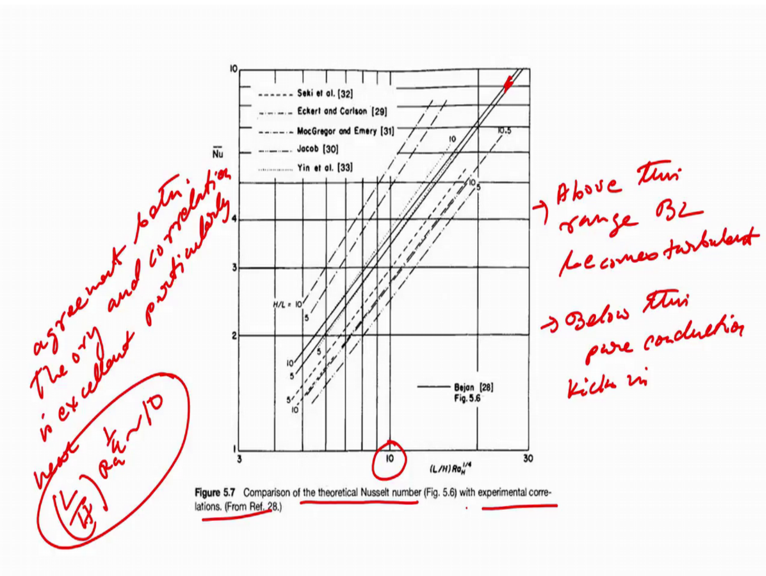

So this is for example, some of the theoretical Nusselt number with experimental work and experimental correlations basically take from reference and 28. So, here you can see that the agreement usually between the Nusselt number, I mean the agreement between the correlations. So, agreement between theory and correlations is excellent particularly near L by H Ra to the power of 1 fourth is about 10, so right around here, in that particular regime the relationship between the correlation and the experiments are kind of good because this is the line of Bejan, these are the 2 lines the rest are all different types of theoretical data.

So, around 10 this is particularly good the relationship as you can see over there, but there is a considerable variation below and above; above and below this particular thing. So, the main thing is that if you look at below this range, so at this 10 value is where the boundary layer assumption is particularly good look it right above this range the boundary layer. So, above this range the boundary layer becomes turbulent or at least it tries to transition to turbulence and below this we have what we call the pure conduction limit starts to kick in; conduction kicks in.

So, the match is kind of very good at around 10. So, below this and above this you have all these other factors that plays an important role over there. So, this is particularly useful task where we have reviewed some of the important work and we have analyzed the problem we showed how it can be solved, at the same time we have taken some liberty with respect to some assumptions, at the same time we showed that the heat transfer part is not that simple as far as what Bejan showed and what Gill showed. For example, is not very simple it is not a universal one, it does show a little bit of variation with this depending on how you are analyzing the problem. So, depending on your impermeable wall or adiabatic wall it kind of gets a little messy.

But, at the same time, but it starts to become constantish after a certain range. So, that part is particularly useful and you should remember that and also if we show that theoretical agreement between the different experiments and between the different theoretical results and this, you will find the agreement is particularly good for around 10 and then it starts to deviate as we go to the other limits. So, in this particular fashion we stop the at the Rayleigh number; high Rayleigh number limit before we go to the shallow enclosure limit after this because shallow enclosure limit is other end which is regime 4 basically.

So, let us look at a regime 4 now. So, next thing that we are going to look for is regime 4 which is basically the shallow enclosure limit. So, it is in this particular regime if you look at it the thing looks like this, very distinct boundary layers once again on the vertical sides. So, here the HTR or the heat transfer rate still scales as K delta Tf H delta T still the scaling is that, in addition you have additional insulation provided by the long horizontal core of the cavity. So, this is a shallow enclosure limit, the heat transfer rate is given by that, additional insulation provided by the long horizontal core. So, this is basically the core that is what you are getting.

So, first we focus on the core region and that is all that we are going to do.

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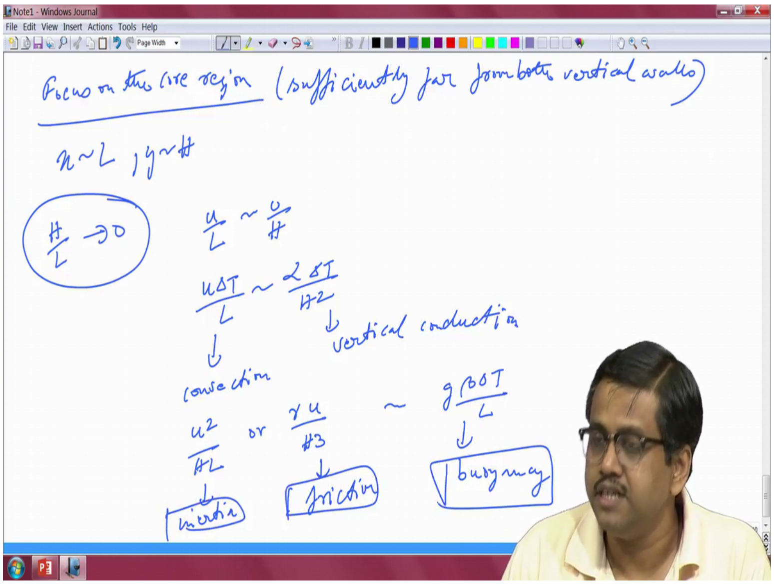

Focus on the core region that is the important part, where in the code region which is sufficiently away your x scales as your L and your y should scale as H because you are sufficiently away from the boundary layer. So, it is basically we are sufficiently far from both vertical walls got it.

So, basically; now, since your H by L is actually going to 0 because that is what the shallow enclosure limit is that your H is much, much smaller than your L, that is a shallow enclosure; shallow enclosure limit. So, therefore, if we write the different scaling from the continuity you will get u by L is equal to v by H that is a first scaling, then from the energy equation you are going to get the second scaling. So, this is convection and this is the vertical conduction, remember this is vertical conduction that is why the H the height has been taken into consideration over here.

Similarly, if you look at the momentum equation it will be u squared H by L or gamma u by H cube, I will write what these things are g beta, delta T by L where this is basically your inertia, this is basically your friction and this is basically your buoyancy. So, these are the 3 terms that you would normally get in any situation like this.

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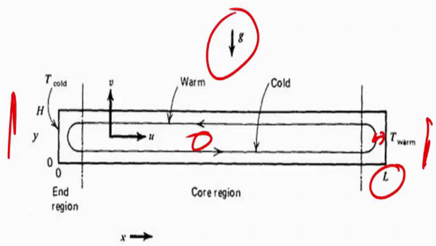

Figure 5.9 Shallow enclosure heated in the horizontal direction.

So, at this point we can take a look at the figure 5.9 over here, which basically gives an illustration of this particular problem again taken from page 1. So, this is the warm site, this is the cold site, this is u, this is v all the things remains the same; this is the gravity, this is the core region, this is the end region. So, as you can see the end regions are really thin it is a core, it is at core which is very dominating over here in this particular case, the height is really small compared to the length, so the length is L which is given over here. So, this is the basically the schematic of the problem. So, and this is basically what we are analysing here. So, I wanted just you to take a look at this.

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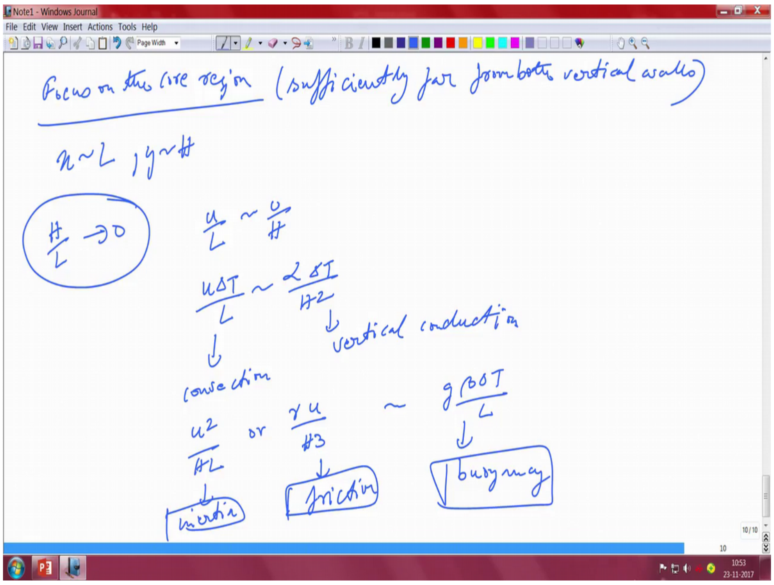

So, the question that remains that, what are the scales of u and v?

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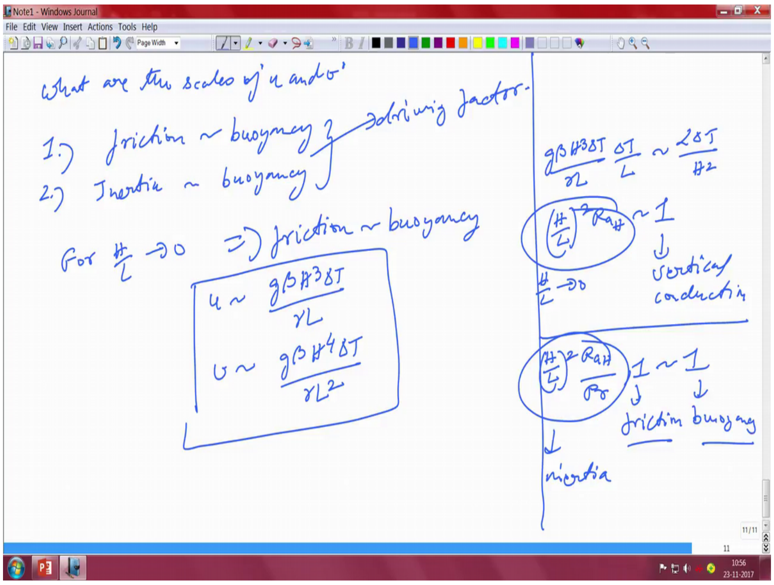

So, first option could be that friction is being driven by friction and buoyancy balance, the second one which could be inertia by buoyancy, buoyancy will be there regardless because that will be the driving factor regardless of what you whatever you do, these 2 terms remains the same.

So, for H by L tending to 0, where you have a very large core it is usually the friction which will balance the buoyancy, you are encouraged to look into the past notes to find out why that is the case and u scale of u will be g beta H cube delta T by gamma L where the scale of v will be g beta H to the power of 4 delta T by gamma l square, so these are the 2 scales for u and v. Now, what we can do is that we can take this u and v values and you can substitute them in the governing equations right the scales that we actually determined over there. So, what will happen if we substitute them in the energy equation this will become g beta H cube delta T by gamma L, delta T by L scales as alpha T by H square this will become therefore, HL square Ra H scales as 1. So, this is basically once again the vertical conduction.

Similarly, from the momentum counterpart what we are going to get is H by L square Ra H by prandtl number given as 1 scaling on the other side is 1. So, this is basically your friction, this is basically your buoyancy and so these are the things. So, these are the 2 things as you can see H by L goes to 0 because it is like a shallow enclosure, this term basically goes to 0, this term basically goes to 0 which is basically this was the inertia term anyways. So, it is basically a balance between friction and buoyancy all right.

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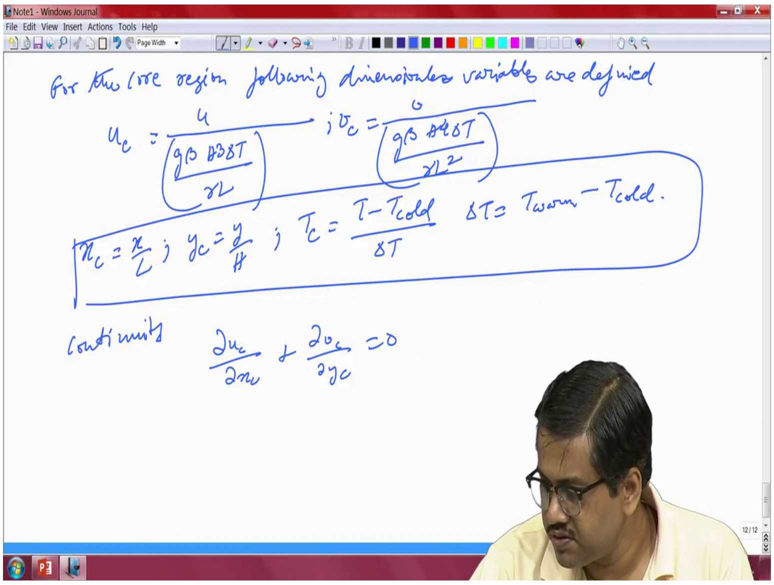

So, now for the core region, so you can understand this, now for the core region following dimensionless variables are defined. So, 1 is basically (Refer Time: 23:45) uc for core g beta H cube delta T by gamma L it is a first quantity v c g beta H 4 delta T by gamma L square; xc is equal to x by L; y c equal to y by H; t c equal to T minus T cold divided by delta T; t cold and T L we already defined in that diagram, delta T is basically given by T warm minus T cold, got it. So, for the core region these are the non-dimensional variables that we have defined and this is the generalized form.

Now, we can do is we can substitute all these quantities into our basic governing equations. So, the continuity will become, that is the first let us take the next page for the momentum equation.

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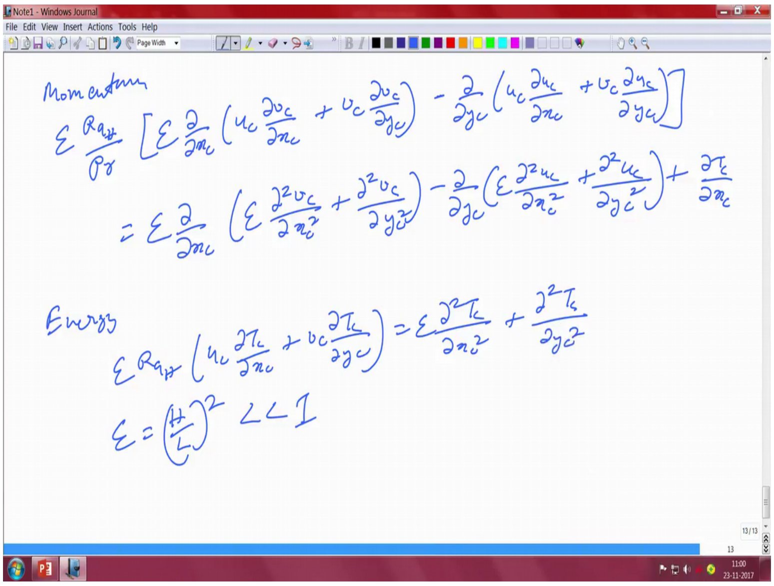

So, equal to epsilon, we will explain what epsilon exactly means. So, epsilon plus, so, similarly the energy equation now, where the epsilon is basically H by L square which is much, much less than ha 1.

So, what we will do in the next class, we will try to see how this equations are typically solved, we would not solve them because they cannot be solved in the conventional form, but we will solve a few and we will try to show you that how this data actually looks like.