Convective Heat Transfer
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Lecture - 38
Regime III

In the last lecture, we found out that; what are the different regimes of natural convection, internal natural convection and we identified regime 1 2 3 and 4. Now in this particular lecture we are going to look at that how these regimes, we are going to analyse a few of these regimes, and try to see that how these regimes actually translate into, I can analyse the heat transfer in a little bit of a better way. So, if you recall the original regime map which we had earlier, let me see if I can pull that up. So, this was the original regime map.

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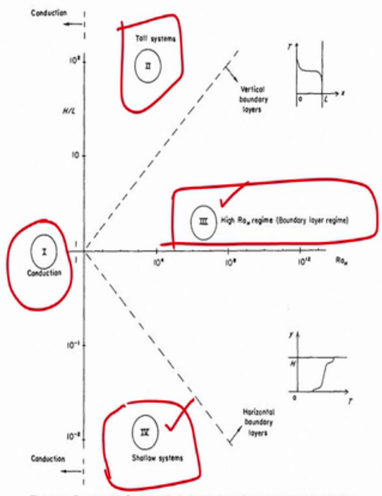

Figure 5.3 Four heat transfer regimes for natural convection in an enclosure heated from the side.

If you look at it here. So, you if you recall this was the pure conduction limit, this was a high Rayleigh number regime, this is the shallow enclosure regime. These are the tall system regimes. So, these are demarcated by the vertical boundary layers and we did a very detailed analysis and saw that when these vertical boundary layers are going to become very important. And when they are going to be distinct, and when they are not going to be distinct. So, this was already covered, and we went through it in a in a large amount of details, right?

So, in this particular class we are going to take 2 of these, and we are going to analyze it in a little bit more depth, and see what we can extract out of it. So, this was what we did in the last class, and you can just recall and see what we did.

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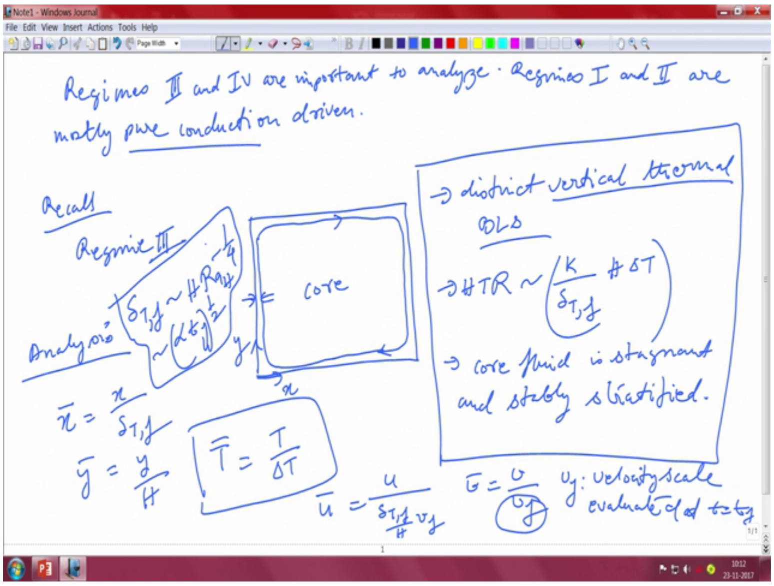

So, regimes 3 and 4 are important to analyse. They are very, very important. Because regimes 1 and 2 are mainly conduction driven, mostly pure conduction driven. So, they do not really carry that much of a you know analysis per say. So, mostly conduction driven. So, it is very important now to see that how the regimes 3 and 4; which is basically the high Rayleigh number regime, and the shallow enclosure regime that is what we are going to do in this particular class. So, recall let us take regime 3. If you just recall your regime 3, this was something like this. You had very, very distinct and thin. So, it is marked by distinct vertical thermal BLS or thermal boundary layers, got it?

So, this is basically your x and y, and the axis is the heat transfer rate. Let us call that HTR. Scales as k the boundary layer thickness H into delta t, right. So, that is the so, the core fluid; that means, the core fluid is here, this is the core, right? The core fluid is stagnant, right. And stably stratified right. So, these are the main things that we take out from this. It has got distinct vertical thermal boundary layers. The heat transfer rate as we know it is 1 over delta T dependent, right. The core fluid is basically stagnant, and it is stable stratified right. So, these are the very basic assumptions that we always have made.

And so, the analysis now we have to do based on this. So, this is just a recap that how the your system actually looks like, right? Now if you now start to do the analysis, let us define some non-dimensional variables right now. So, for example, x bar can be written as x divided by dTf. So, I am dividing x by the corresponding boundary layer thickness, right which is that.

Then y bar is basically y by H, H being the height. Because the scale of y is basically H. T bar which is basically T divided by delta T; which is delta T being the temperature difference. U bar is basically given as u divided by delta Tf by H into vf. Similarly, v bar is equal to v by vf v being the vertical velocity once again.

So, what is vf? Vf is basically the velocity scale, scale evaluated at T equal to Tf. Velocity scale evaluated at T equal to Tf. Similarly, your delta Tf is given as, if you recall your old notes H into Rayleigh number to the power of minus 1 4th is also proportional to Tf to the power of half. Remember the 2 short transient analysis that we did. So, these are my non-dimensional variables.

Let us see x bar is basically normalized by the boundary layer thickness. Y bar represents y divided by the height of the enclosure. Delta T is nothing but the normalized temperature. U bar and v bar has been divided by the corresponding vf, right? Where vf corresponds to the velocity scale if you recall, which was evaluated when the layer become fully convective, right. So, at T equal to Tf the layer becomes fully convective. So, that delta Tf is therefore, given as H into RaH to the raised to the power of one 4th and that is proportional to alpha ty raised to the power of half. So, this is the basic premise that we have as of now.

So, let us look at using these parameters right so, the steady state conservation equations u bar x bar.

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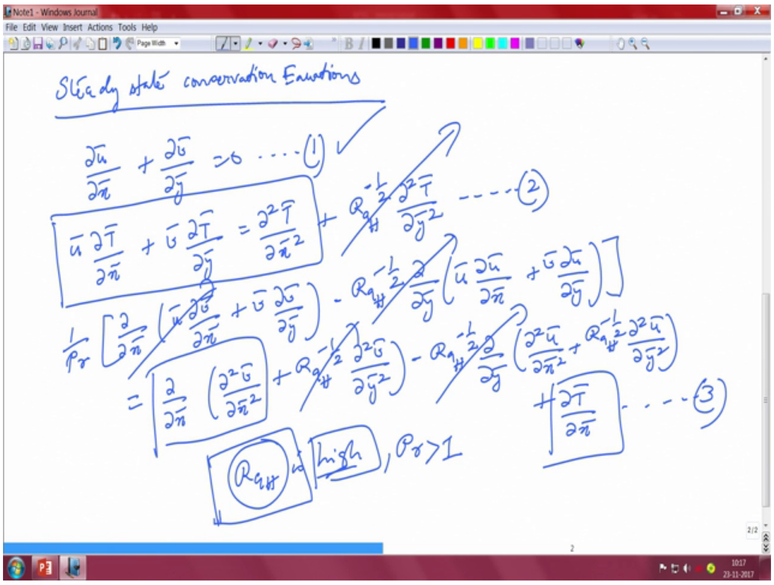

So, I am writing it in the non-dimensional space. That is your continuity, support that is one. Similarly, u bar dT bar by dx bar plus v bar. This is to write the conservation equation. So, this is basically your energy equation.

Now, we move to the to the momentum equation. Now the momentum equation comes with a catch. We have d we have decided to eliminate pressure, right. If you recall in the last lecture only we did that, that we eliminate pressure from the 2 momentum equations, right. So, we write it in a composite way now. So, 1 by prandtl number, that is u. So, see how the non-dimensional numbers are aiding.

Because you are getting them in the equations. So, and for high Rayleigh number you can see where we are going with this. So, this serves an excellent purpose of you know in streamlining the equations, and getting significant insights out of them. This is equal to; it is a long equation, but do not worry I mean most of the terms will drop out. So, that would be like your third equation. So, basically you have 3 equations now, coming into the picture, right 3 equations. And this is a combined momentum equation, the third one.

So now the topic of discussion will be that using, but this equations looks very hard and it is kind of unsolvable in it is current form. So, can we make some assumptions and try to see if those can be relieved a little bit. So, for if youre Rayleigh number is high, which is basically regime 3 right relay number is very high, and if your Prandtl number is greater than 1, you can readily see the continuity equation will remain the same, there will there will be no harm done. From the energy equation, this term will go out, right. Because Rayleigh number is high. So, therefore, 1 by Rayleigh number to the power of minus half or Rayleigh number to the power of minus half will be a very small quantity. So, this term will be basically drop out.

And similarly, when you look at the momentum equation, multiple terms are going to drop out, because you all the terms with Rayleigh number of this, this. So, there will be many terms this one, this one, many terms will actually drop out. Because of the reason that we have taken that Rayleigh number is high. Otherwise these terms would not drop. It is only because of this that the terms will drop out here, right?

So, let us look at it. So, the terms that will survive over here are these 3, right? Here if we look at it will be this term and this term. This will be the only 2 terms that will survive, right? So, our equation looks now very simple.

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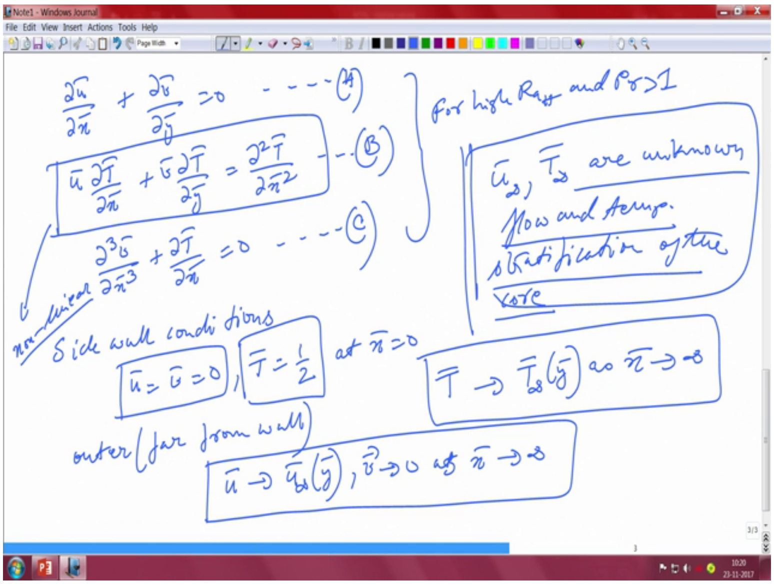

That is equal to 0, that is remains the same. So, the let us call this a equation A, this is B, equation C is basically this is your C, right? So, this is a very simple set of equations only for high Rayleigh number and prandtl number greater than 1, right? These are the 2 limits in which we have evaluated this.

So, the side wall conditions. So, side wall conditions means, u bar should be equal to v bar should be equal to 0. And T bar will be equal to half at x bar equal to 0. So, x bar is the left-hand side of the wall, right? This is the hot wall right. So, there the temperature will be half as we know; is plus minus delta T, right on both sides. That is a typical enclosure problem.

And you have u and v equal to 0, because of the no slip right. Outer; that means, far from the wall basically at the core. So, that is what we are taking. U bar goes to u bar infinity comma y bar. And v bar goes to goes to 0 as x bar approaches infinity. This is an important statement. U bar approaches u bar infinity, right. V bar approaches 0 as x bar approaches infinity, right.

And similarly, your T bar should approach T bar infinity at y bar as x approaches x bar approaches infinity, this is the second one right. So, these are basically far away from the wall right into the core actually, right. So, here if you can see that u bar and T bar infinity are unknown. They are unknown, right flow and temperature stratification of the core. But you should also pay attention that this equation is non-linear, right. Right this is a non-linear equation as well. So, you cannot solve it, without solving for the velocity field right.

So, as you can see these are unknown flow and temperature stratification of the course. So, this we do not know. And this equation are non-linear. So, in spite of having nice equations and a few boundary conditions worked out, we are basically nowhere, right.

So, this problem was solved by Gill.

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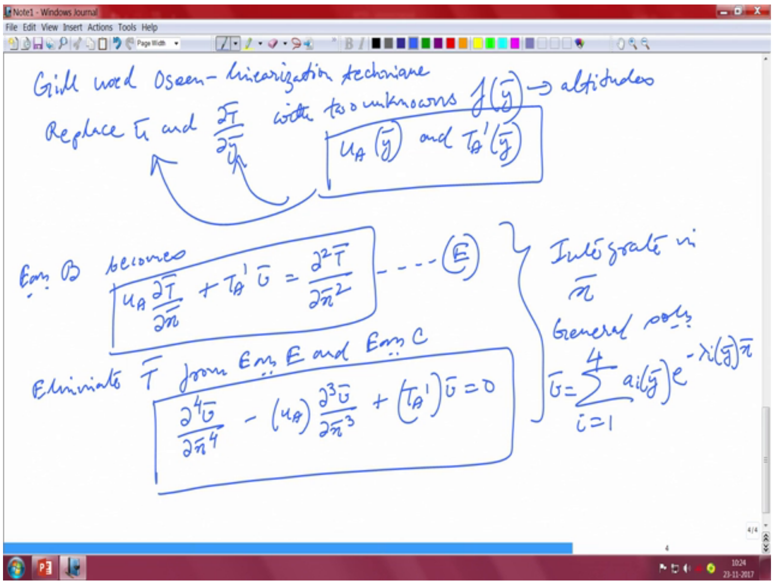

In a famous JFM paper. He used what we call the oseen linearization technique. So, what I did was as follows. He replaced replace u bar and dT bar by dy bar with 2 unknowns. So, what are those unknowns? These unknowns are functions of altitude only, this is altitude. So, these functions were uA y bar and TA prime y bar. So, these were the 2 functions that he used as substitution for this and this, right. That was what Gill did.

So, using this B, equation B which we define in the previous page that becomes uA dT by dx bar plus TA prime v bar d square T by dx square bar. So, this becomes your say your equation E right. So, equation B becomes that, right.

So, what we can do is that now we eliminate T bar from equation E and equation C, right. C being the last equation, which is the momentum equation actually. So, in essence what we get is; this is the other equation that you get, right. First one and the second one, right. So, how these equations can be solved.

So, one simple way will be to integrate, right in x bar that would be one way to solve it in terms of integrate in terms of x bar. So, the general solution in general, the general solution is v bar. So, that is the general series solution. So, look at it is an alpha I y bar x bar, and this is ai into y bar. So, basically let us go to the next page.

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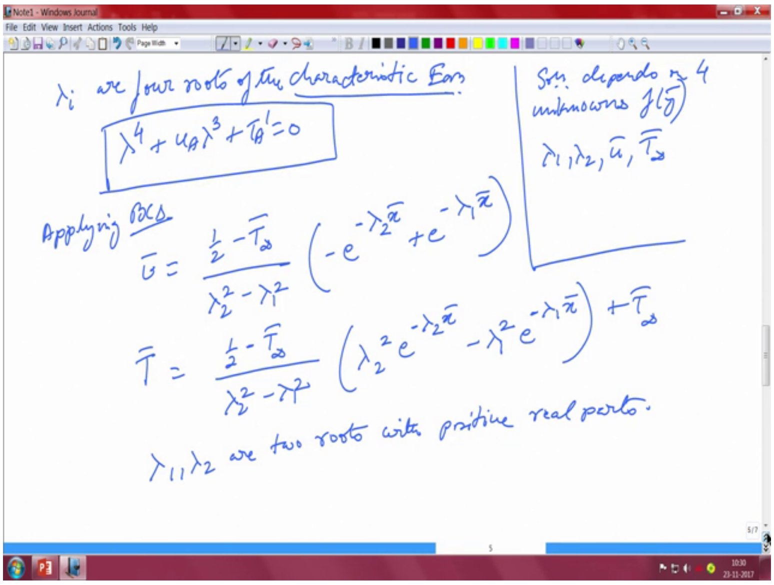

So, basically that is lambda I that you see are 4 roots, right of the characteristic equation. So, that is given as lambda 4 plus uA lambda cube sta a prime is equal to 0, all right. Lambda are the 4 roots of the characteristic equation lambda 4 into uA lambda cube plus TA prime is equal to 0.

So, if we apply now the boundary conditions; which is on which other condition which are basically the sidewalls and the core boundary conditions. We get v bar is equal to half minus the infinity bar, T bar is equal to half, T infinity bar divided by plus. So, these are the after applying the boundary conditions, these are the things that you get. And lambda 1 and lambda 2 are 2 roots with positive real parts, right. So, that is the thing that you get out of this.

After solving all the all the equations. So, the solution of this particular equation now depends there are basically 4 unknowns that you see over here. So, what are those 4 unknowns? So, the solution depends on 4 unknowns. So, which are functions of altitude, which are basically functions of y bar basically. So, these are lambda 1, lambda 2 u bar and T infinity bar, right. So, what Gill did was. So, in order to have a unique solution.

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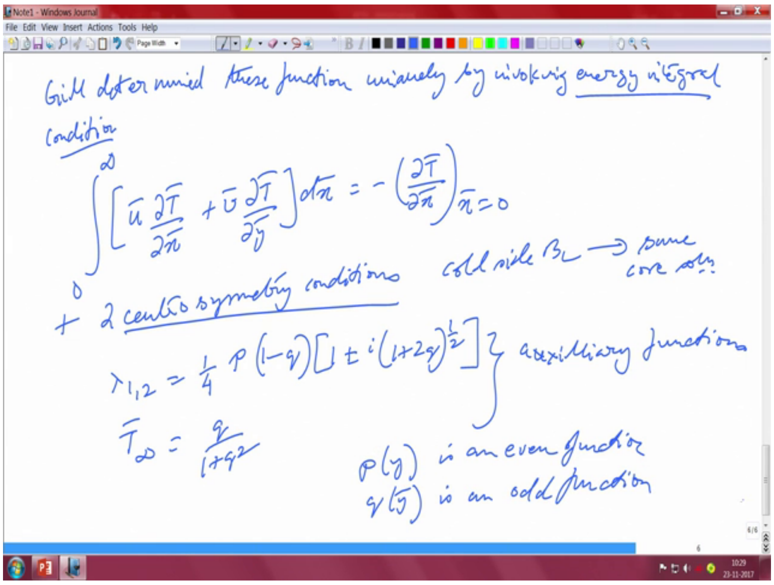

So, gill determined these functions uniquely, by invoking what we call an energy integral condition. So, he used this energy integral condition. In addition, he used plus, there were 2 what we call centro symmetry conditions.

So, what are the centro symmetry conditions? So, the centro symmetry conditions is that the cold side boundary layer, as we go to the core, it approaches the same core solution, same core solution as the hot side boundary layer, right. So, cold and the hot side boundary layer approaches at the same as we approach the core they approach the same limit. So, that is what is called centrosymmetric symmetry conditions.

So, lambda 1 comma 2 therefore, becomes and this is getting a little complicated, but we are trying to solve things analytically. So, that is the whole problem. 1 plus 2 q half. So, and T infinity bar is q plus 1 plus q square. These are called what we get the auxiliary, auxiliary functions. So, the auxiliary functions, these are the auxiliary functions. So, out of this p which is a function of y, because these are all altitude functions is an even onE or is an even function. And q y bar sorry this q y bar is an odd function. There is an odd function. So, it is even an odd that kind of a function.

So, what happens is that p is equal to 2 1 plus 3 q square 11 by 9 divided by c 1 plus q square 2 by 3 1 minus q square.

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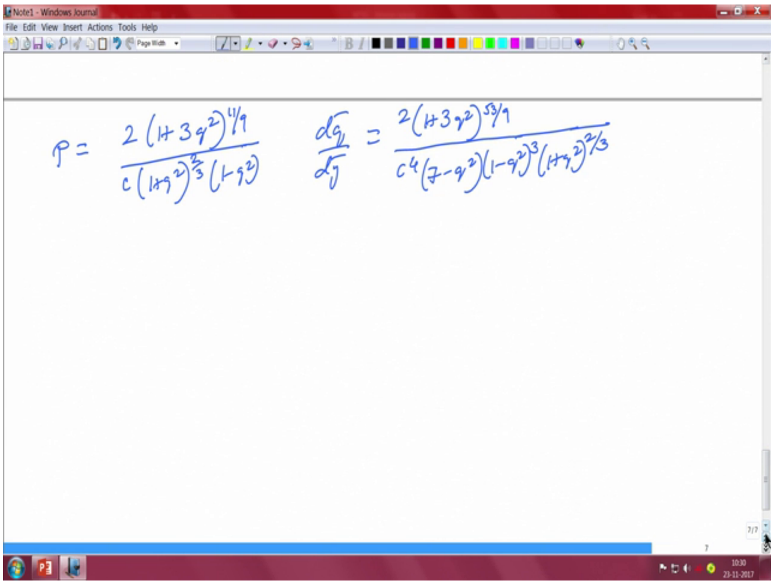

Similarly, dq bar by dy bar is equal to 2 into 1 plus 3 q square 53 by 9 divided by c 4 7 minus q square 1 minus q square cube, 1 plus q square 2 by 3. So, these are the p and the q conditions that we get. So, based on this as you can see that based on this we basically have been able to solve this part of the equation; which is let me just recap very quickly. So, we started with the standard equations.

And then in the high Rayleigh number limit and the high prandtl number limit. We were able to linearize them and solve them well still not linearize. Just have the have the simple forms, and then we linearized. We replace them with functions uA and TA, and from that we are able to get the 4 roots. Basically, solve the characteristic equations and then we used auxiliary conditions basically to complete the analysis of this.

So, in the next class we will see that what these numbers, I mean how the solution actually should look like. So, see you in the next class.