Convective Heat Transfer
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Lecture – 37
Heat Transfers Regimes

So, we saw that how the thermal boundary layer thickness behaved at the end of the time tf where the convection effect has become very important from the initial stage when the heat was being principally used for thickening of the layers, right.

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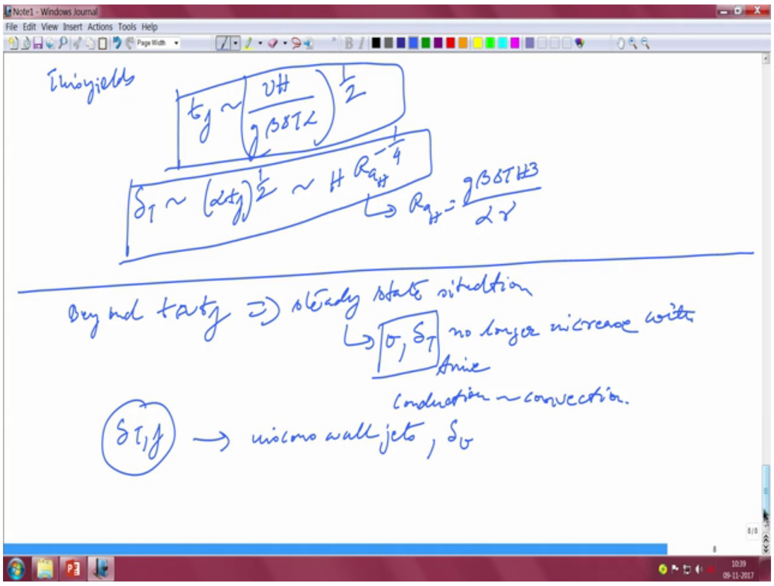

Now, so you can see that after say that beyond t equal to tf or t scaling as tf. You should ideally have a steady state situation; that means a steady state situation in which your v and your delta T should no longer increase with time. Thus unsteady term has become redundant at that particular point right. So, it is basically a balance between conduction and convection. With these velocities and the boundary layer thickening is not happening anymore. So, in addition to this delta Tf, if we call that as delta Tf, we also have the viscous wall jets, right, correct because this is the same wall jet, this is the velocity wall jet and the thickness of these jets is say delta v, we already did that earlier right that what delta v is, right.

Now, outside the thermal layer where buoyancy effect is kind of minor, we have a balance between; if you recall your natural convection external natural convection class.

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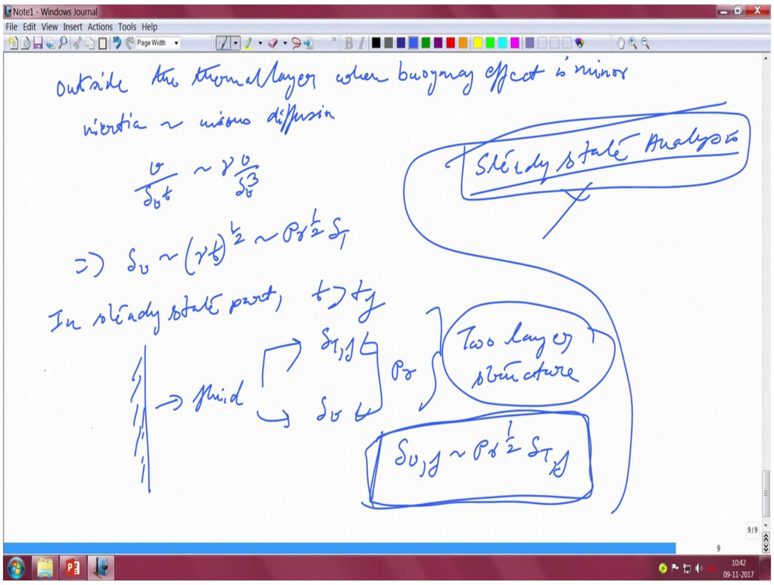

You will find that outside the thermal layer when buoyancy effect is minor. We have a balance between inertia and viscous diffusion or in other words, v and delta v into t, gamma v delta v cube, this actually leads to delta v is gamma t to the power of half leading to prandtl number half to delta T. We already know; what is the expression for delta T. So, in the steady state part when t is greater than tf, the fluid near this wall has got basically is characterized by 2 layer structure. One is basically delta Tf and one is delta tv which are linked with each other through the prandtl number. This is basically called 2 layer structure; basically called a 2 layered structure where delta vf is basically prandtl number, half delta Tf this is the 2 layered structure.

Now, if you ask what will be the steady state analysis, you can easily refer to the previous chapter where we actually did the analysis. There is nothing new about that there will be of course, the energy balance between conduction, convection between buoyancy and to the viscous diffusion terms, we already did that. So, we are not going into the steady-state analysis in that particular way, right. What we have shown that how the transient events actually lead to the steady-state and why; where does the boundary layer thickness attain its steady state configuration which we have done over here and how the velocity of the viscous wall jet, how is it related and that there is a 2 layer structure of the fluid near to the wall depending on the prandtl number from random number equal to one these are basically the same.

Now, let us look at now the criteria of the distinct vertical layers because this is an enclosure, where there are 2 jets on 2 sides, 2 thermal boundary layer thicknesses on 2 sides. So, again like we did in our external natural convection also; when are these 2 jets actually well separated from each other when they are actually have a tendency to merge what is the criteria for the same. So, that is the part that we need to address over here and this address is not very difficult.

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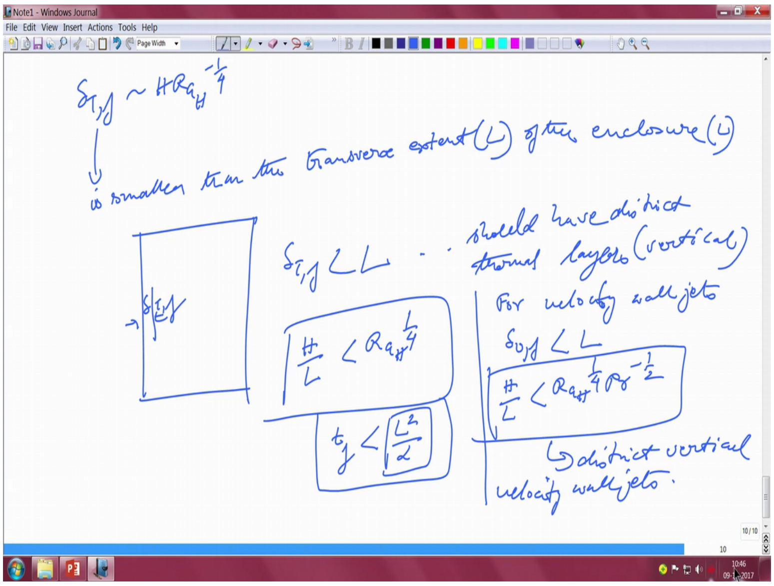

Because we already know that your delta Tf is basically H, R a H to the power of minus one fourth, this we already know is we already established in our previous class.

Now, for the final boundary layer thickness, if the final boundary layer thickness which is delta Tf is basically smaller than the transverse extent, transverse extent which is L of the enclosure L, the thermal layers are supposed to be distinct it is a simple, right. There are 2, this is the cavity if this layer thickness delta Tf, it is actually smaller than L right, then we should have distinct thermal layers, should have distinct thermal layers; at least in the vertical direction and come to the horizontal part in a little bit, at least in the vertical direction they will be separated. So, this would translate to H by L is less than Rayleigh number raised to the power of 4. So, this is the inequality.

Now, this inequality also means that the vertical layers actually become convective in a time scale which is much shorter than the thermal diffusion time between the 2 walls; that means, that Tf which is the time at which the layers becomes convective, fully convective, right, must be less than this particular time scale right this is nothing, but the diffusion time scale between the 2 walls. So, by the time the effect of the heat conduction reaches the other side of the wall, if that particular time is called T prime. Before this time, if the layers becomes convectively not unstable, convective in nature and then it is shorter than that particular time, then you basically have a situation in which the vertical boundary layers will be well separated, this is a simple thing. So, conduction effect if it reaches the other wall before these layers becomes convective then of course, we will have a merging. So, this timescale has off convective the layers becomes convective has to be smaller than the time that it needs for the conduction effect to be felt on the other side of the wall. So, that is the reason why we have got this particular term, that tf is less than the L square by alpha. So, that is important.

And similarly, similar thing can be done for the vertical for the velocity wall jets also; it will be this dv must be less than L. Once again the same thing this would translate to H by L is less than R a H to the power of four into prandtl number half, right. So, this will also give you distinct vertical velocity wall jets. So, that is what you are going to get.

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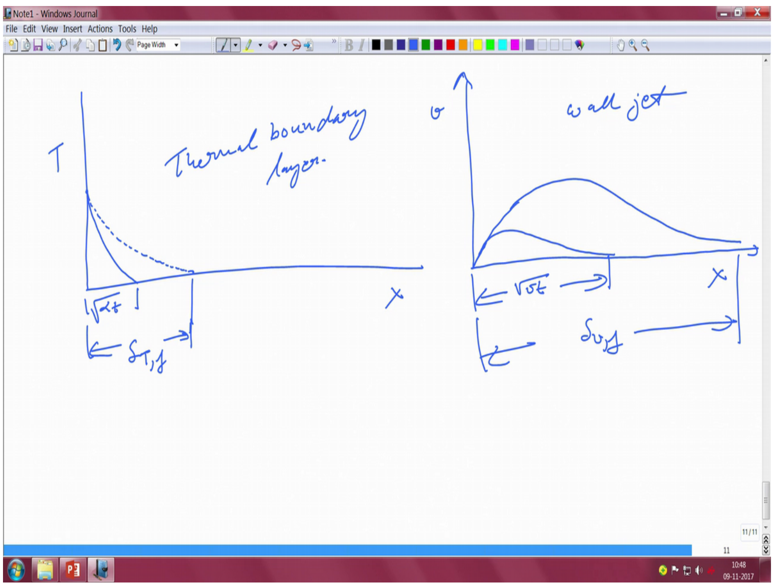

So, this can be put in a figure form and this is T, this is X. So, as you know initially, this is basically this intercept is basically your alpha t root over and then of course, like this has final delta Tf. So, this is how this mechanism works.

On the other hand, this is v, this is X, this is the velocity counterpart, right. So, you know that it is root over and this is the corresponding, it is the thermal boundary layer, sorry, it is a wall jet, thermal. So, these are the 2 situations now then we go to the criteria for distinct horizontal jets. Horizontal jets are the other thing that this is for vertical only that we did. So, let us look at the case of the horizontal jets. So, horizontal jets are also another slightly interesting thing.

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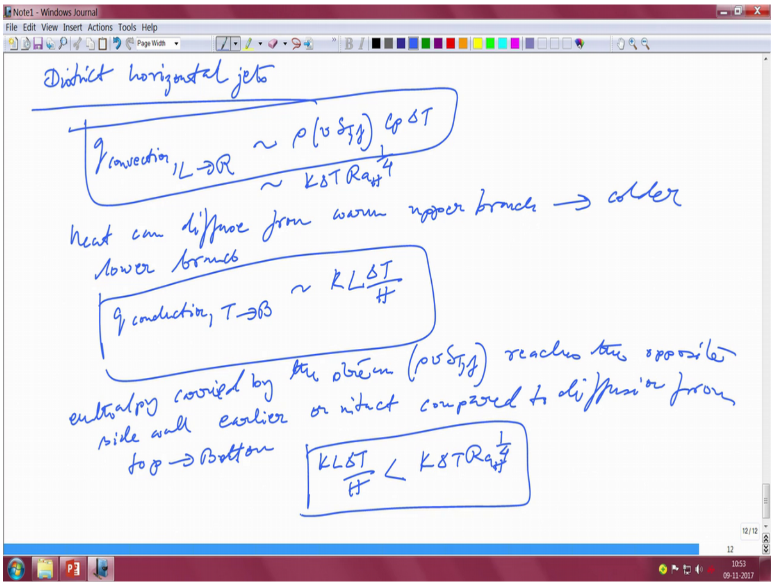

So, when you actually have for the distinct horizontal jets now your q conviction from L to R; that means, on the left to the right or the left to the right that is the convection that is taking place on the left side wall to the right side wall and that is given as rho v delta T f into Cp into delta T. This is the mass flow rate and whatever enthalpy that it is actually carrying, that is the enthalpy that is going in that particular direction. So, this can be written as k delta T Rayleigh number to the power of one fourth depending on after we do the substitution that is a scale that we get this is convection from left to right ok.

Similarly, there will be a conduction that would happen from the top to the bottom. So, it is basically heat can diffuse from warm upper branch to the colder right lower branch and that is up and the top and the bottom of the cavity are at a different temperature. So, there can be heat conduction due to that ok.

Now, that heat conduction, now q from, this is conduction from top to bottom that is T to B, basically scales as how this is delta T by H the total height of the cavities H is a conduction problems, it is delta T by whatever is the thermal gradient that you have. So, delta T is the maximum gradient that you can create anyways in this particular cavity right. So, delta T by H the height of the cavity will give you a scale of the mechanism by which heat conduction is going to happen from the top to the bottom and k into L are basically the pre factors. So, in other words this is the q conduction, this is the q convection right now the idea is that if the q convection is faster than q conduction, then naturally you will have distinct horizontal jets. So, in other words, the enthalpy carried by the stream rho v delta Tf reaches the opposite side wall earlier or intact compared to the diffusion from top to bottom or in other words KL delta T by H is smaller than k delta T Rayleigh number to the power of half one forth right that is the scale that we have established ok.

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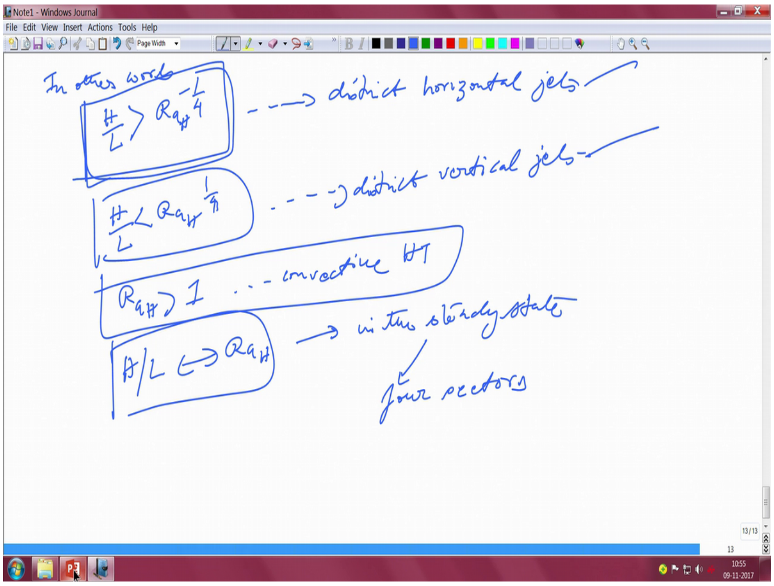

In other words, H by L is greater than R a H to the power of minus one fourth. So, when this condition is met, the horizontal streams along the adiabatic wall they retain their temperature identity; that means, they do not mix with each other. So, therefore, that is what the important criteria is right that H by L is actually less actually greater than Rayleigh number, this is four distinct horizontal jets ok.

Now, we already have the situation that H by L greater than Ra to the power of one fourth. So, that was for distinct vertical jets. So, these are the 2 criteria’s ok.

Now, what we can have thought is that the 2 criteria for these 2 that is distinct horizontal and vertical jets and Rayleigh number greater than one basically for the convective, Rayleigh number less than one is not convective anymore convective requirement; that means, convective HT requirement we can divide this H by L and R a H field into several sections. Each sector corresponds to a distinct regime in the steady state. So, in the steady state this in the steady state when I show it you will see in the steady state, this map can be divided into 4 sectors depending on these 2 and the fact that Rayleigh number is greater than 1 that particular limit.

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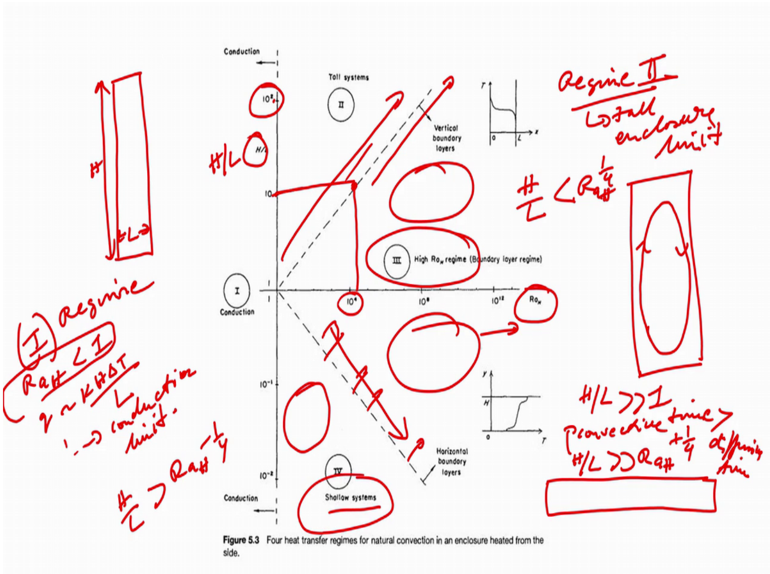

Now, let us look at the PPT for this particular purpose. So, you can see that these are the 4 sectors that we are actually talking about. So, this is the corresponding Rayleigh number this is the H by L. So, there are several limits when H by L value is very high, this is H by L by the way when H by E value is very high, it is called toll enclosures; that means, the value is 100 and beyond you can see, along this axis if I go up; that means, H is become H by L aspect ratio; that means, it is becoming an enclosure of that kind. This is your H this is your L aspect ratio is becoming more and more.

Then on the other side, you have what we call the shallow enclosures which is basically the reverse of this is something like that. So, these are the 2 limits that in the lower side you have shallow enclosures and the higher side you have tall enclosures H by L depending on the factor of H by L this side if you go on to this side you get higher and higher Rayleigh number right and if you move to the other side you basically go to lower and lower Rayleigh number which is basically the conduction limit. So, this vertical axis where I mean; when Rayleigh number is almost close to 1 right is basically what we call the conduction limit. For any case Rayleigh number less than equal to 1 is basically considered to be the conduction limit. So, based on that we have divided this whole thing into several sectors. These sectors are called as you can see regime 1, 2, 3 and 4. So, these are the 4 sectors that we have done ok.

Now, for regime 1. So, what is the key condition for regime 1, that regime 1 is basically corresponds to Rayleigh number less than equal to 1 where the q is basically kh, delta T by L. So, it is basically in the conduction limit, whatever heat transfer is happening it is happening in the conduction limit right. So, when you're when this situation arises when you actually have the conduction limit; that means, your Rayleigh number is a very small number right. So, that is why you have the all these things converge all these sectors in this is the region where we are considering the solution because it is in the relay number less than one range, right.

So, based on that this entire thing will be basically your Rayleigh number less than 1, I mean Rayleigh. So, this is wholly conduction this band, whatever it is from this onwards it is wholly the conduction band if you can see it properly now [vocalised-noise]. So, that is regime 1. So, regime one is done and dusted.

Let us look at regime 2 now. So, what is regime 2, regime 2 is basically a tall enclosure limit, it is called a tall enclosure limit. So, in the tall enclosure limit which is what we have shown over here redraw it once again. So, in the tall enclosure limit. So, the circulation something like right that is the directionality of the circulation. Now, in this stall enclosure limit we know that H by L is much much greater than one all right that is the tall enclosure limit. we also have the secondary criteria of H by L less than R a H to the power of minus one fourth for distinct horizontal [vocalised-noise]. So, this H by L is basically less than R a H to the power of minus one fourth.

So, if H by L is actually greater than if H by L sorry. So, if H by L if H by L is much much greater than R a H to the power of plus one fourth. You have the convective time for these cases. Convective time is more than the diffusive time. So, depending on where you are in this, while you are on this you can see that moment you are on this side of this particular thing that all enclosure limit right when you're on this side of this, you will have distinct vertical layers, if you're on the other side you will actually have merged, when your convective time is actually more than your diffusive time. So, as you can readily see that for vertical layers the scale is that your Rayleigh number H to the power of one fourth has to be greater than H by L that is the criteria right. So, when H by L value becomes very large for the Rayleigh number to become higher than that you need a higher liberal a number. So, if you look at this particular plot over here, let us look at it say you pick a value of 10. So, you can see this graph, you can meet that criteria very easily because you just have to be above 10 to the power of four, because what is the criteria for distinct vertical layers it is basically H by L has to be greater than R a H to the power of one fourth, correct.

Now, if your R a H value is greater than 10 to the power of 4 and your H by one is 10. So, naturally if your value is greater than 10 to the power of four you satisfy that limit right. So, this marks that particular point on the graph. So, beyond this to the right of this you basically have distinct vertical layers. Now, why this graph actually goes up in that slanted way that is because as your H by L approaches say 10 to the power of 2 you need a Rayleigh number which is 10 to the power of 8 and above basically to satisfy the limit because all you need to do that you have to make your convective time lower than your diffusive time right. So, that is why this graph shows this kind of a slope; that means, as you increase the H by L you would need very high Rayleigh number basically to satisfy the same thing. So, that will be the thing. So, very high Rayleigh number therefore, comes in the regime 3.

So, the tall systems where there is no distinction of the 2 boundary layers right that, but this the vertical boundary layers are not distinguishable from each other, we get the tall system that the tall system limit to the left side where you have marked it as regime 2; that means, the layers are not separated from each other whereas, if you have the higher Rayleigh number you have distinct boundary layers; that means, the heat transfer is dominated by basically convection and you go to this particular region this whole region if you just look at the tall enclosure limit, do not look at what is below ok.

Similar thing you can heuristically see in the shallow enclosure limit. In a shallow enclosure limit is a very same type of thing. Here, of course, in the shallow inclusion limit this is instead of being vertical layers now it is horizontal boundary layers. So, horizontally, they will be well separated and what is the horizontal criteria, the horizontal criteria is H by L must be greater than R a H to the power of minus one fourth right, it is just the reverse it is basically L by H has to be greater than Rayleigh number one for less than Rayleigh number one for. So, it is the same thing. So, in this particular regime you will have separation of distinct layers because we are moving into the high Rayleigh number regime. This is basically the regime where the L by H ratio is such that and the Raleigh number is small enough that you get the shallow system kind of a configuration.

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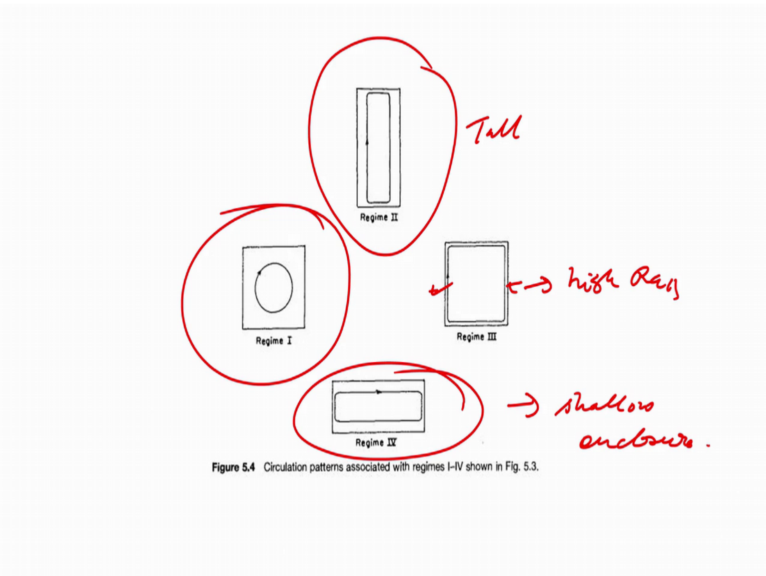

And before I end we will discuss a little bit more on this. These are the types of things that we will form this is regime one which is pure conduction type of a regime, this is a regime 2 which is basically your tall enclosure regime, this is regime 3 which is the high Rayleigh number you can see very distinct boundary layers here right and this is the regime where you actually have the shallow enclosures. We will spend a little bit time more in discussing some of these patterns over here, but this is just to give you an idea that how the Rayleigh number actually plays a role over here and how the shape and the size of the cavity plays a major role in ascertaining that which regime you are going to follow.

Thank you.