Convective Heat Transfer
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Lecture – 36
Internal natural convection- Scaling analysis

Welcome. In today's lecture, we are going to cover we are going to start with internal natural convection. Now, internal natural convection is a very common phenomena and is a very important and a challenging task also. For example, anybody wants to design a room like, the one that you in your house. Any auditoriums, any buildings, things from solar collectors, all the way up to fluid mechanics phenomena's like Rayleigh Bernard convection, all this involves basically natural convection in enclosed cavities or enclosures, as we call it.

So, we will focus in our discussion in this class and beyond; this is natural, this is internal natural convection. So, that would be the topic of today; that internal natural convection that is what we are going to focus on.

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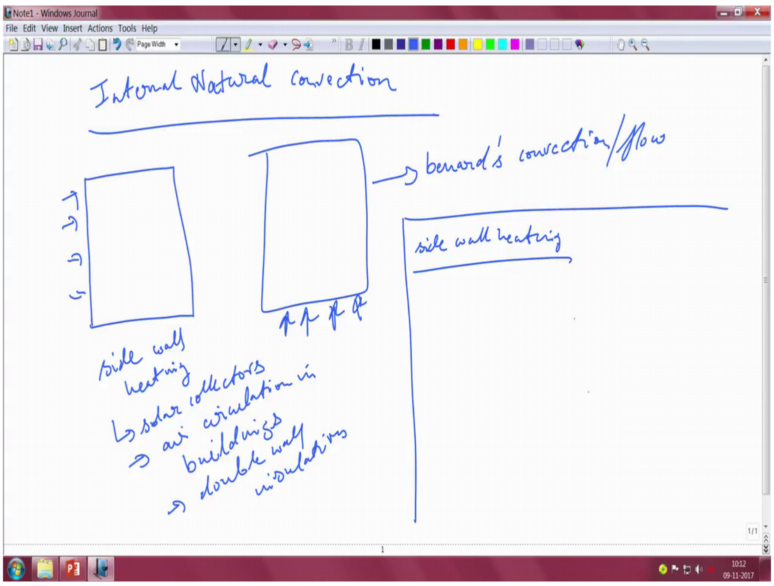

There are 2 types of natural convection. You can have scenarios where; the heat is supplied from the side walls. There can be scenarios where heat is supplied from the bottom. Now, side walls where heating is from the side; side wall heating. Now, when you are heating it from the side these are this is applicable in applications like solar collectors. For example, very classical problems in solar collector's air circulation in buildings.

When you do a room design, as I said, if you want to do an auditorium design for example then double wall insulation, the bottom cavity that particular kind of problems has got other applications also. One of the very famous appeal, one of the very famous phenomena you can say either Eddy Bernard's convection. So, that is very common in the fluid dynamic power bonds that you get the (Refer Time: 02:42) Bernard or the Bernard flow or convection.

So, that happens when you actually have this kind of an arrangement. So, it is a very common problem and it is useful in a multitude of applications, but how to attack this problems, is what we are going to do in the in the next few classes.

So, let us take the problem with the side wall heating. And we will focus more or less on this side wall heating. And we will try to first analyse this problem in the next slide. So, as we say it before that, we want to see both the transient as well as the steady-state.

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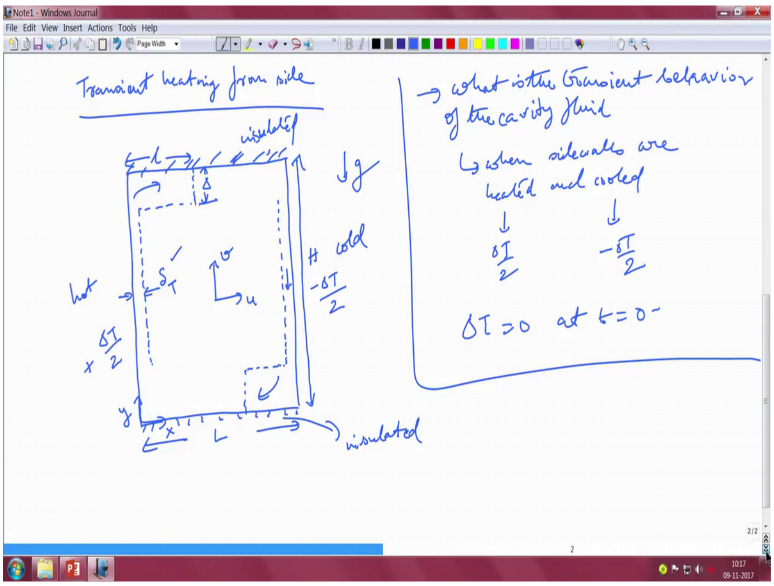

So, basically you have transient heating from the side; will resort to the scaling analysis as we did earlier. So, let us draw the cavity. So, this is a standard cavity that we have drawn. The length of this cavity is L; that means, between the 2 walls it is L and the axis over here is this is your x; that is your y axis, the height of the cavity is basically H. So, this is an enclosed cavity now. H by L can be any ratios that we will see how is it dependent on H by L type of ratios. This is the gravity, the direction of gravity. So, what happens is that let me draw a simple schematic? This is basically your delta T. This is basically something like a delta. This length scale is small l. So, basically the flow is like this. And this part has got a temperature of plus delta 2, delta T by 2. This has got a temperature of minus delta T by 2. So, this part is hot, this part is cold. And the velocity scales are as follows this is u, this is v.

So, apart from this, this is insulated, this is also insulated. So, it is adiabatic on 2 sides top and bottom. Only the sidewalls of a left and right walls are heated. One part is heated one part is cooled by an amount that delta T by 2. So, we are interested in the transient behaviour. So, what is the transient behaviour of the cavity fluid, right? What is the transient behaviour of the cavity fluid when the side walls are heated and cooled, right? All of a sudden to heated by plus delta T by 2 cooled to minus delta T by 2, got it where the top and the bottom walls are actually insulated. So, initially, the delta T was equal to 0 at t equal to 0 minus. So, just before the initial time, the entire cavity did not have any thermal gradient going inside it at all, right? So, that was the situation. And based on this situation, now we are supposed to analyse this particular problem.

So, that is a simple enough thing, we are interested in this transient behaviour. We are interested in what happens immediately after you start doing this. So, based on this, so normally what we would do, we would write down the equations of motions and energy first and then we will see how to analyse the problem. So, this part is a very important diagram, which basically shows you that how the flow actually is happening, how the circulation is taking place. And we are interested in these parameters right, delta T; one of the crucial parameters, what is the velocity scale how delta T is growing, all those answers.

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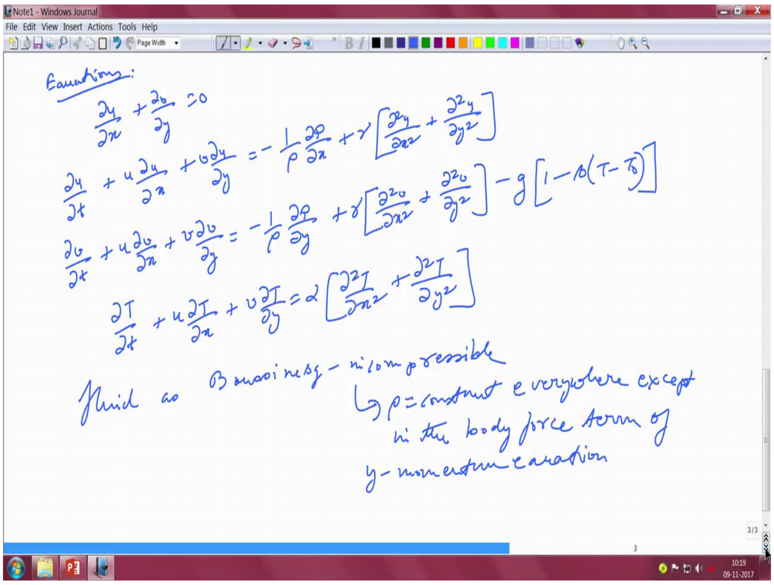

So, let us look at the equations. So, the conservation equations are as follows; as continuity, I am retaining the transient terms because, that is what we are going to do here the 2 momentum equations x and y right. So, these are the 4 equations that we are interested in, we note that we have modelled the fluid as we said earlier, as boussinesq's in incompressible; that means, that the change in density only comes in the body force term and not everywhere the rho is constant rho is constant everywhere except in the body force term y momentum equation, that is an important thing this we already have explained earlier. So, this is no nothing new just to reiterate that that is what we did.

Now, instead of solving all these equations now what we are going to rely is we are going to rely on the scaling arguments.

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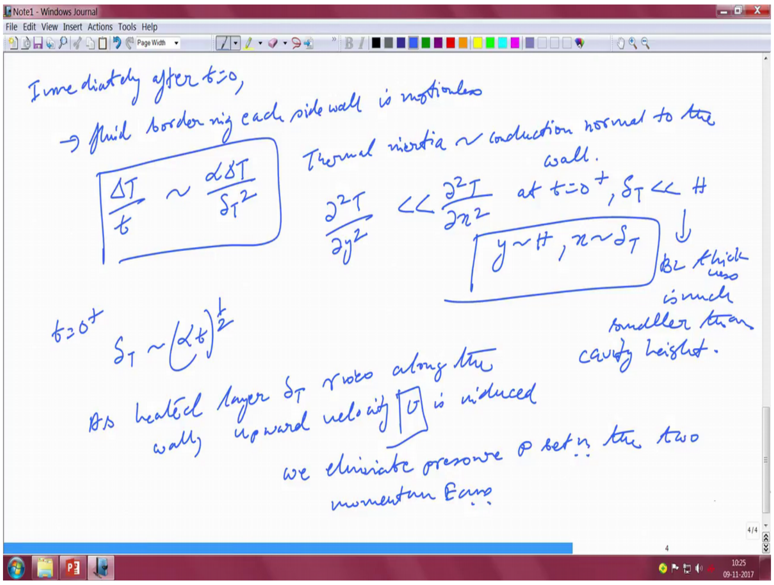

So, let us look at the situation immediately after t equal to 0. So, after immediately after t equal to 0, the fluid which is on the, which is basically adhering to the side wall each side wall is basically motionless. Just immediately after t equals to 0, we are assuming that a fluid bordering each side wall side wall is basically motionless; that means, when you look at the energy equation, the balance between the convection term is no longer important It is basically, the inertia thermal inertia or the unsteady term which should actually balance with the conduction normal to the wall right.

So; that means, if you put it in that particular context, this will be important because, this is the transient term. This should balance all right. So, it is a balance between thermal inertia balancing the conduction normal to the wall conduction normal to the wall. So, this is the scaling that we have put forward; obviously, recognizing that this square T by dy square term is much, much smaller than x square term.

So, because at t equal to 0 plus what happens is that the thermal boundary layer thickness delta T is much, much smaller than the enclosure height H and we already know that y should scale as H and x should scale as delta T right this was already kind of known right. So, boundary layer thickness is much much lower than the or the boundary layer thickness is much smaller than cavity height right. So, that is an important statement.

And so, from immediately this particular equation, following t equal to 0 plus right, each wall each sidewall is coated with a conduction layer right because it is a conduction balance basically where the delta T basically scales as alpha t to the power of half, very much like a diffusion problem you have seen this earlier in your unsteady heat conduction equations where, because, what we have said here, that a layer is motionless, but this heated layer now grows because of pure conduction with the rate of alpha t raised to the power of half.

Now, as this layer starts to heat, as this layer starts to heat, this delta T, there is a velocity scale that is trial that now gets imposed right, a velocity scale automatically originates. So, so as these layer as this heated layer this is a heated layer right heated layer delta T rises along the wall upward velocity v is induced. Now, we need to find out what will be the scale of that.

So, inertia conduction driven velocity has just started to come in right. So, understand that nature of the problem that, first it is a conduction driven problem immediately after time, with as time marches on you, induce this velocity scale fundamentally because, you have now have a heated layer which kind of rises now against the wall.

So, buoyancy starts to play a very important role in order to find that scale, what we do is that we eliminate pressure p.

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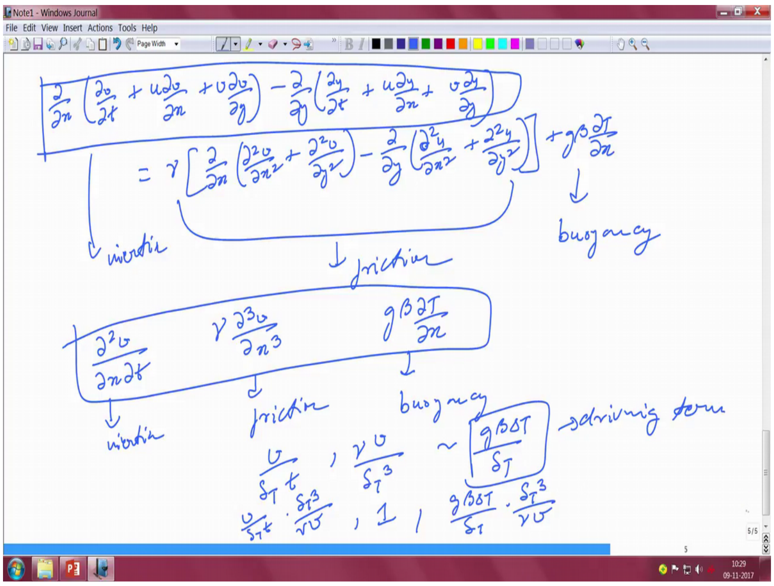

Between the 2 momentum equations all right, we eliminate the pressure. So, based on that we get, so you can see, there are 3 basic groups in this particular equation. You have all the terms on the left-hand side of this equations are basically the inertia terms correct; inertia terms and the four-viscous term, but plus the buoyancy term on the right-hand side; that means, these are all friction and this is basically your buoyancy.

So now, we can it is possible for show and this you can actually try to do it is that only 3 terms dominate in this basic group. So, the first term that dominates is basically this and the second term from the friction part and of course, the buoyancy is a single term event. So, this is basically your inertia, this is basically your friction, this is basically your buoyancy. So, these are the 3 terms we would normally associate if you do an order of this equation. So, you will find that these are the terms which are the dominant terms in this particular series.

Now, if we just put their corresponding scales, now into the picture soo, one will be v t v and delta T into t right that is the first term then of course, you have gamma v by delta T cube this of course, balances with gb delta T by delta T right. So, these are the 3 terms that in their scaling argument. So, buoyancy is of course, the driving term no matter what right, it is a driving term which is definitely not equal to 0. So, therefore, what we can do is that, we have to find out that whether buoyancy is being balanced by friction or by inertia like we did in our external natural convection, if you recall that particular problem.

Now, what we can do is that we can divide it by the friction scale all the terms and let us try to see how what evolves as a result of it. So, this definitely becomes of the order one now; because of the division. So, this becomes v delta T into t delta T cube gamma into v this becomes g beta delta T by delta T delta T cube by gamma into v.

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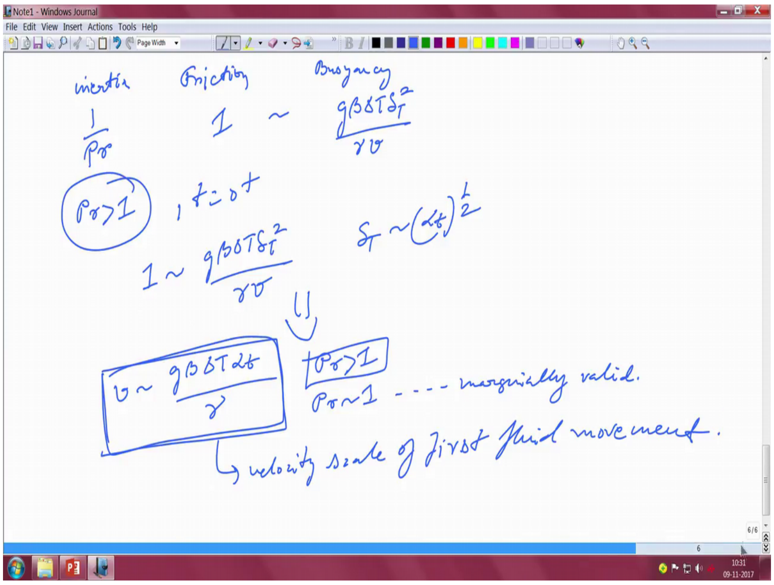

So, in the next one, so, the inertia now becomes 1 over prandtl number, friction is of the order one, buoyancy the scale against g beta delta T. So, these are the 3 terms in a series. Therefore, for fluids which has got prandtl number greater than one that kind of a fluid situation right therefore, it is a balance between friction and buoyancy. So, at t equal to 0 plus; that means, as this velocity scale starts to pick up you have a balance between friction and buoyancy which is g beta delta T delta T square and gamma v right ok.

You already know that, your delta T is scaling as alpha t to the power of half that we derived just earlier or. So, therefore, all these things leads to v scale as g beta and delta T alpha T divided by gamma, valid for prandtl number greater than one, closely valid for prandtl number almost close to one, marginally valid. Marginally valid for prandtl number greater than one it is of course, valid. So, this is the velocity scale that we induce. So, this is the velocity scale of the first fluid movement is given as that.

Now if we go back, now to the energy equation, now the heat that is being carried. So, previously the energy equation we neglected the convection term right.

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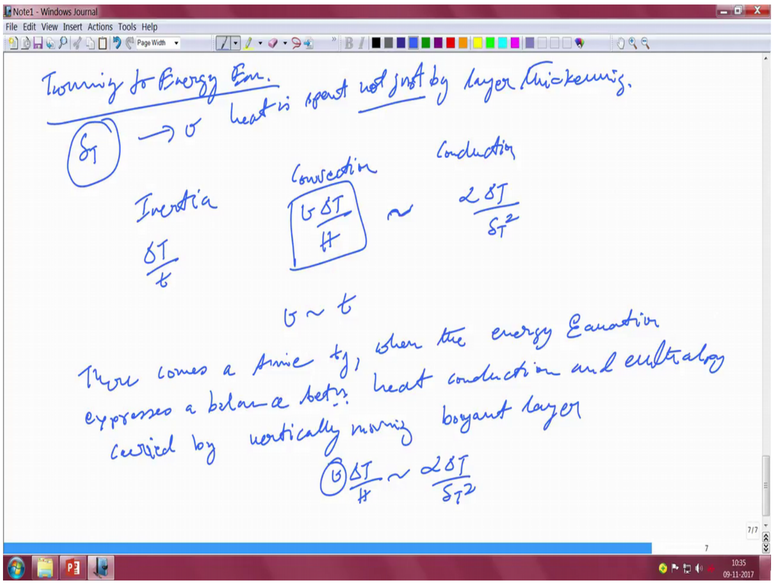

Now, if we're turning to the energy equation to energy equation, what we can show is that, the fluid layer delta T that the heat that is transferred is no longer going into just thickening of this delta T right, it is also being taken up by the layer delta T which is moving up with a velocity v right. So, the heat is spent not just by layer thickening got it. So, it is also based is carried then it is not just and it is also being carried by the velocity v. So, therefore, naturally in the energy equation, we now have a competition of the convection term also remember previously we neglected the convection term.

Now, once again, so, in the energy equation we have delta T by t the convection term is given by v delta h by h and conduction term alpha delta T by delta T square this is a balance between these 2 right and whatever gets balanced by whatever. So, as the t increases the as with time because, if velocity we saw, so, the velocity scale that we saw, was proportional to t right that is what we derived just in the previous slide it is dependent on t linearly right.

So, as time goes on, it is expected that the velocity scale should go up. As a result of that the convection term should become more and more important corresponding to the inertia term or the conductor corresponding to the inertia term. So, naturally what we can do here. So, it slowly becomes important. So, there comes there comes a time t f when the energy equation expresses a balance between heat conduction and enthalpy carried by the by the vertically moving buoyant layer got it. So, it will be there for a balance between delta T by h balancing alpha delta T by delta T square right.

So, based on this and knowing that the expression for v we can find out that what is this time scale t f is all about.

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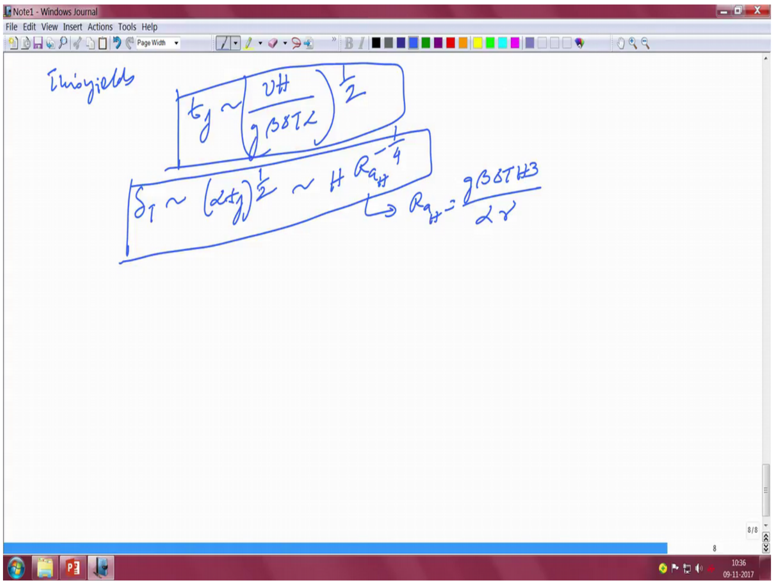

This yields t f is it is not v is that is gamma delta T into alpha raise to the power of half. At that time, remember your delta T is alpha t f to the power of half right at that point of time this would translate to H into Ra H minus 1 4th where your Ra H is given as g beta delta T H cube by alpha r right. So, so there is a time t f at which your convection term equates the corresponding heat conduction term in the energy equation and at that point of time, the value of the thermal boundary layer thickness scales as H into RaH to the power of 1 4th.

So, you end here in the next class, we will see that how the velocity boundary layer actually takes it is form.

Thank you.