Convective Heat Transfer

Prof. Saptarshi Basu
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture – 35
Mixed Convection

Having established the velocity scale for this particular problem.

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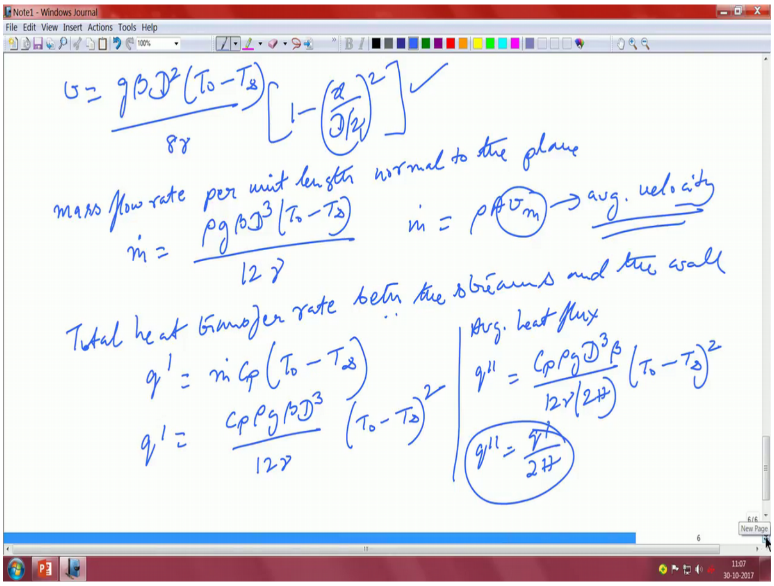

Let us write down the velocity g beta D square T naught minus T infinity divided by 8 gamma 1 minus x by D by 2 square that was the velocity scale that we extracted.

Now in order to calculate the heat let us calculate what is the mass flow rate per unit length normal to the to the plane? M dot therefore, will be equal to rho g beta because it is basically rho a v and that kind of stuff. It is basically D a cube T naught minus T infinity divided by 12 gamma. M dot basically we have founded through rho a into Vm Vm is the average velocity. In order to calculate the average velocity, you need to just integrate this guide right. That will give you rho a into the corresponding Vm. That is how that that integration you can do as an exercise it is just integration from 0 to d by 2 that is all the integration is there to it.

Now the total heat transfer rate between the stream and the walls. Which is given as q prime say is m dot Cp alright multiplied by T naught minus T infinity. It is m dot Cp T naught minus T infinity. In other words, q prime therefore, becomes because we already evaluated your m dot for u. It is basically Cp rho g beta D cube divided by 12 gamma T naught minus T infinity. Already there was a T naught minus T infinity in the m dot term. It is basically square of the 2.

The average heat flux heat flux that flux term q double prime will be equal to Cp rho g D cube into beta divided by 12 gamma by 2H T naught minus T infinity square. It is basically nothing but q double prime is basically equal to q prime divided by 2 H. That is all that is how you should remember this particular problem.

Now that we have found out what will be the average heat flux? We know what is the heat flux in general or the total heat transfer.

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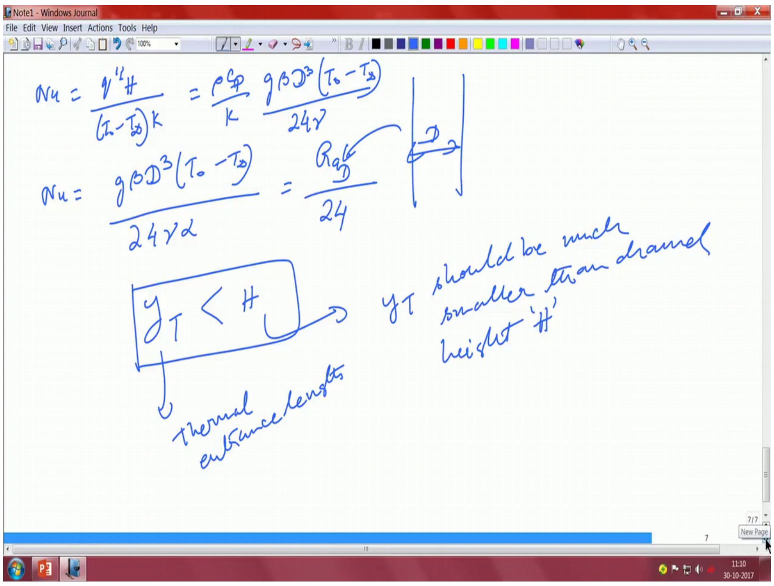

Nusselt number will be q double prime H divided by T naught minus T infinity into k. This is t infinity into k therefore, it will be rho Cp by k g beta D cube T naught minus T infinity divided by 24 gamma. Therefore, the nusselt number becomes g beta D cube T naught minus T infinity divided by 24 gamma alpha. This is given as Rayleigh number based on the diameter a diameter of the separation between the 2 plates separation between the 2 plates was D. It is the Rayleigh number has been. The length scale for the Rayleigh number has been taken as d instead of H.

It is almost similar to the reynolds number-based term in a in a pipe flow right it is dependent on D. That is what is the value of your nusselt number is going to be.

The Nusselt number is kind of done. Now one other thing that is important is that here also. We see that there is no grashoff number coming into the picture it is still Rayleigh number with respect to d that we are concerned about.

Now, it is of course, all these expressions are valid if and only if the entrance length y T that we have said earlier is less than h this is the critical condition right; that means, if the if the 2 boundary layers are developing for a very long or the if the plate heights are kind of very small; that means, they are short plates. Short is once again dependent on depending on how fast y T is actually developing. The thermal entrance length y T it is the thermal entrance length right thermal entrance length y T. Is much, much smaller than the channel height H. Y T should be should be much smaller than channel height H. That is one of the one of the crucial conditions.

Can we establish now that what will be that necessary criteria? Like we did in the case of your forced convection that what was the entrance length dimensions.

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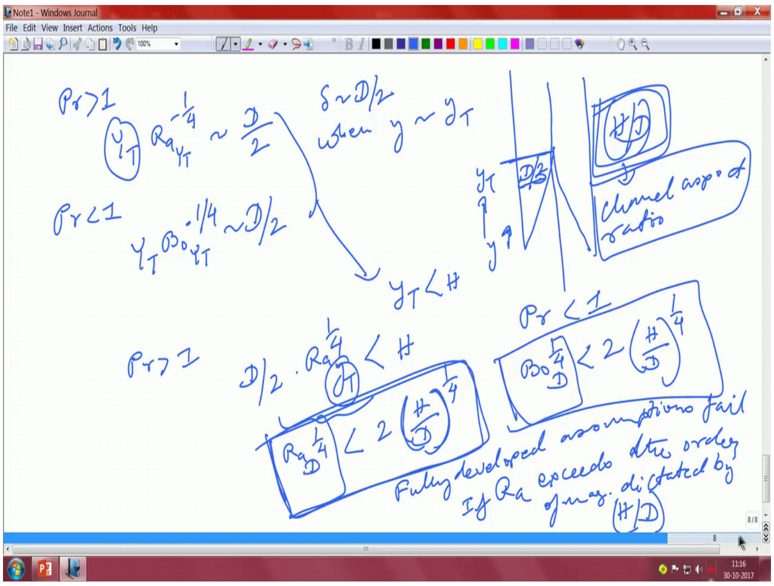

Now for prandtl number greater than one once again the 2 limits we are going to apply. Y T into Rayleigh number with respect to y T this is Rayleigh number with respect to y T remember that this is almost equal to the scale is actually D by d by 2 when the entrance length actually ends because delta becomes of the order of D by 2.

When y becomes of the order of y T right when this is in this regime right that the 2 layers are merging with each other I am talking about this. This is y. Y when y approaches y T. Delta should approach d by 2 also. That was the whole point right in the argument that we are putting forward over here alright. That is the first expression for prandtl number less than one this is y T we already know with respect to the boussinesq's number alright actually D by 2 it is all with respect to boussinesq's number.

Now, what we can do now is that we substitute y T and put it back in the expression where we said that y T has to be less than H. We put it there and establish an inequality of some sort. What we have done is that in the first case for prandtl number greater than one what should we do it will be D by 2 right into Rayleigh number with respect to y T to the power of one 4th alright should be actually less than less than H right.

Now, similarly for Prandtl number you can you can do the same thing for prandtl number less than one.

Now, if you now take this particular parameter out; that means, you try to convert it from the y T scale to the D scale the right means you cast it in terms of Rayleigh number with respect to d then you can show that this expression will now boil down as r a D to the power of one 4th is less than 2 H by D to the power of one 4th.

H by D with respect to the power of one 4th that is how it will pan out. R a D prandtl number less than one expression actually boils down to this. Similarly, prandtl number prandtl number greater than one expression boils down to this. Similarly, prandtl number less than one expression boils down to boussinesq's number to the power of one 4th less than 2 H by D to the power of one 4th.

These are the 2 principal expressions that you get for this in conclusion what we can say that in order for this kind of flow to exist; that means, your y T is much much smaller your Rayleigh number has to be less than certain aspect ratio the geometric aspect ratio of the channel that you work on that you are concerned with here. This is the most crucial form that you will that you. Your Boussinesq's number or your Rayleigh number has to be such that they are lower than the aspect ratio of the channel or rather the aspect ratio of the channel in this particular case.

If the Rayleigh number Rayleigh number exceeds you know the order of magnitude as has been predicted by this expression. Then of course, you are not going to have any such flow. I mean your assumption on which you have based your velocity on which you have based your nusselt number expression all are going to break down. The temperature assumption the fully developed assumptions fail. It is fail if the Rayleigh number on you know exceeds the order of magnitude dictated by dictated by H by D. What is H by D? H by D is nothing but the channel aspect ratio this is crucial; that means, therefore, when you actually design chimneys or all these kind of things all this fins and other things you have to be sure that the Rayleigh number is such that this is actually satisfied that we have to keep in keep in consideration.

Very high Rayleigh number and things like that can be actually pretty detrimental for the kind of problems that we are actually doing over here. This part was very crucial. I hope this part is actually understandable that it is H by D ratio channel aspect ratio plays a very, very major role and you can see that if this aspect ratio is very, very small; that means, the D is much much larger than H. Therefore, there the merger parameter may not be always satisfied. Similarly, you can have other such scenarios very short channels and things like that.

Now, we move on to one very crucial thing before we slowly move on to our next parts.

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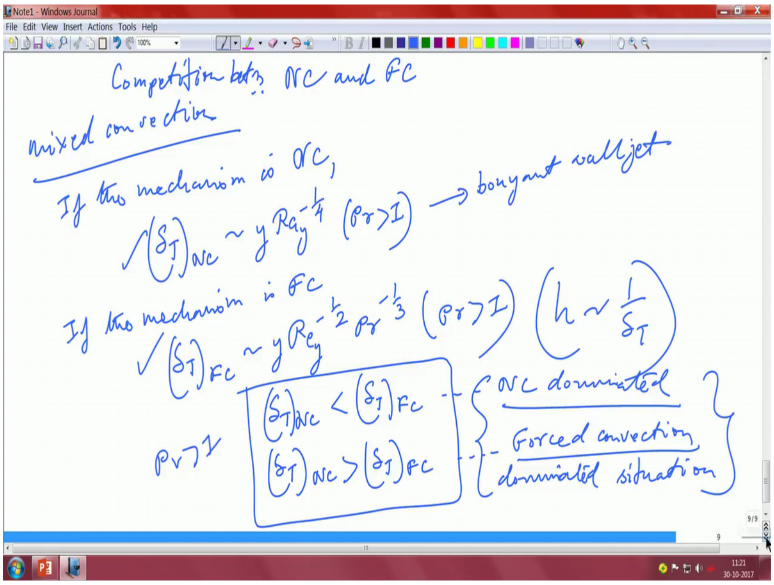

It is like basically the competition the competition between natural convection which I call as NC and forced convection which I am calling as FC. In a in a particular problem you can have what we call mixed convection. Mixed convection means that both the players are entering into the picture. There is for example, a gradient that is created due to the, whatever forcing that you are planning to do.

It could be flow it could be pressure it could be many other thing, but also the natural convection part cannot be ignored. There can be situations where the natural convection is small. Then it can be virtually ignored, but there can be also situations to the contrary that forced convection is so small that it can be ignored we are talking about situations in which the forced and the natural convections both are kind of important to a certain extent, but there may not be equally important, but on a relative basis they are kind of important with respect to each other.

Mixed convection and natural convection, when we are dealing with such a situation let us see the mechanism. If the mechanism is natural convection which I call as NC therefore, the thermal distance of the thermal boundary layer thickness in the case of natural convection; that means, as if natural convection was the only player in thus in this particular situation it is given by R a one 4th right this we already saw for prandtl number greater than one limit this was already there. The thermal distance between the fluid and this and the wall is basically given by that.

Similarly, on the other hand if the mechanism is forced convection this is corresponds to a buoyant wall jet, but if the mechanism is if the mechanism is forced convection right; that means, there is a driver of a different sort. Delta t forced convection or FC is given as y into Rayleigh number y or sorry Reynolds number y prandtl number to the power of minus 1 third for the prandtl number greater than one limit. This we already know from our boundary layer analysis in forced convection.

Which one will be the more dominant that will be given by if you look at these 2 quantities these are the 2 thermal boundary layer thicknesses. Whichever one is the smaller of the 2 basically should determine which particular mode is dominant because if you remember your H heat transfer coefficient was dependent as 1 over delta t always. It is basically the inverse; that means, smaller of the 2 is going to lead to a higher H with respect to the corresponding mode of heat transfer.

If H of with respect to natural convection is higher than the h with respect to forced convection therefore, you will have a national convection dominated situation or in other words I can put it like this delta T N C if it is less than delta T F C. Then it is a natural convection dominated problem on the other hand if delta T N C is greater than delta T F C. Then it is a forced convection dominated situation got it. Delta T N C is less than delta T F C whereas, delta T N C is greater than delta T F C means is forced convection dominated if delta T F C is greater than delta T N C. It is natural convection dominated. It depends on which mode you are actually choosing and these are all valid for prandtl number greater than one situation.

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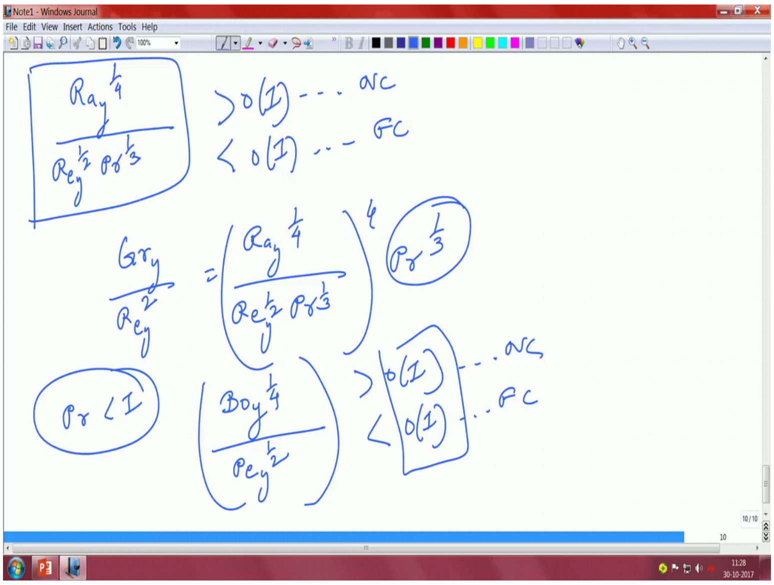

Now, if we cast it in terms of the non-dimensional numbers. R a y one 4th divided by Reynolds number 1 to the power of half prandtl number one third. Now if this is greater than order one it is natural convection dominated. If it is less than order one it is forced convection dominated. It is given as a ratio of the 2 in terms of the non-dimensional parameters.

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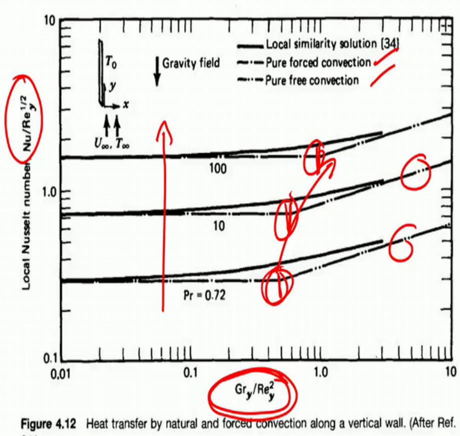

Figure 4.12 Heat transfer by natural and forced convection along a vertical wall. (After Ref. 34.)

This should be kind of pretty apparent, but what normally people do if we look at this particular curve. For example, what people have drawn over here? Is basically your nusselt number and basically instead of using the formulation that we just now advocated they have done it in terms of grashof number and reynolds number that is what we have d1 over here.

Now, you can see in this particular situation that as this number. For example, this is the total field which takes into account both natural convection, as well as the forced convection. If you look at this this this plots you will find that in these particular zones there is a kink in the curve right there is a knee there is a knee in the curve and this knee in the curve actually implies that. There are basically 2 asymptotic limits which converge roughly around the value of around close to one in that particular order. It seems to converge around that particular point right, but in that in and around that one order and this is plotted of course, with respect to prandtl number. Prandtl number is going up in this particular situation.

You have the 2 extreme limits right in one case it is pure free convection in one case it is pure a forced convection you can see from the from the 2 legends. When it is as you can see in this particular case when it is free convection dominated 2 forced convection dominated you have this kind of a kink that develops in the curve. And in one extreme it will be forced convection dominated in other extreme it will be free convection limit dominated the reason why there is a kink and this kind of kink actually shifts over here because this has been plotted with respect to grashof number and not with respect to the parameter that we just now defined.

In order to see, what that leads to let us put that grashof number y with respect to reynolds number y square basically gives you Rayleigh number 4 Reynolds number y To the power of half prandtl number to the power of one third raised to the power of 4 and prandtl number 1 third.

The reason that you had this knee which was shifting if you look at it once again this knee was actually shifting from here to here as a function of pandtl number. Basically comes because of this prandtl number one third dependence that we have included over here explicitly. That is not quite the correct thing to do because then you have a prandtl number dependence.

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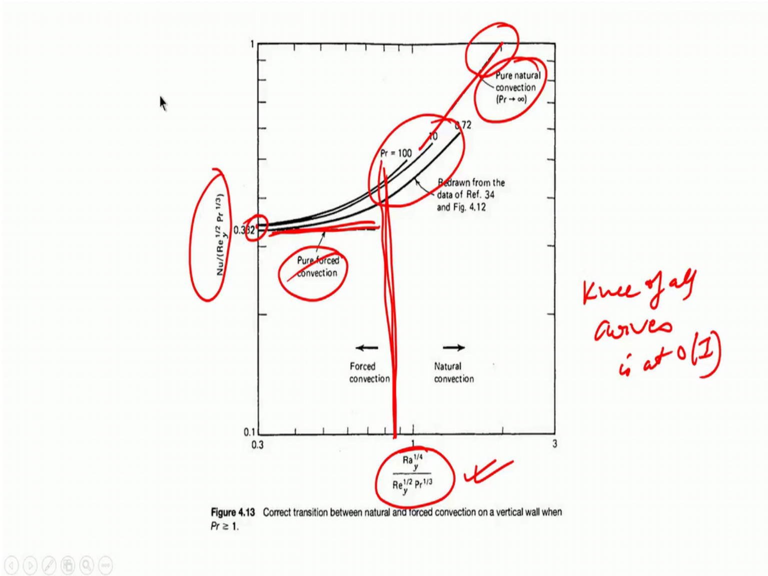

Now, coming into the picture if you draw the same plot as what bejan did in terms of the new similarity variable; that means, the new variable that we have done along with the nusselt number keeping everything the same. You will find that that all that for prandtl number greater than one the curves fall on the top of each other kind of.

There is the, and the kink is of the order one in that range. Because, that is where we go from pure forced convection to pure natural convection and this we know that it is going to be 332 that is what we discovered earlier. If you remember right that earlier we found out that for flat plate that is what it is like point 332 and here is the forced convection.

The pure natural convection limit and these are the 2 asymptotes, but here the knee does not shift and if they all fall around the value of around 1. They all fall the knee of all curves all curves is at order one on the abscissa we show that we can estimate the nusselt number with sufficient accuracy and efficiency and by doing this asymptotes we can actually predict that which mechanism is the most dominant of the 2, but we have already established what the scaling arguments are going to be.

For Prandtl number this actually covers the revised scaling arguments. To say now if we look at the prandtl number less than one fluid, we can repeat the same assumption that we did earlier. That is given by Bossiness's number to the power of one 4th and pecley number to the power of half. When this is greater than point one this is natural convection when this is less than 0.1 this is actually forced convection or 0.1 of order 1 this is actually forced convection.

There has been a lot of work that has been done in this particular category where we have revisited you know we have just done a simple scaling argument. People have done more you know authentic and complicated solutions of the same thing, but essentially it remains that whichever boundary layer is basically shorter or smaller. That basically gives you that mode should be the stronger of the 2 modes. Now, if you go to a situation where one mode is completely suppressed then you have to show that that order is kind of very, very low compared to the other order.

There has been lot of work particularly by sparrow. Sparrow university of Minnesota and there has been a lot of progress on this mixed and natural convection that has been done in by varieties of people from lorenzo and others. Who have done more systematic solutions of these kind of problems and have tried to show that what it actually what it actually means.

But; however, from our perspective we can actually show that these kind of simple scaling can give you an idea that when which parameter actually defines the mode of heat transfer. This can be kept in mind when you actually start to solve a problems. Basically, doing a back of the envelope calculations and try to find out how these things actually look like. See you in the next class.