Convective Heat Transfer
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Lecture – 34
Thermal stratification

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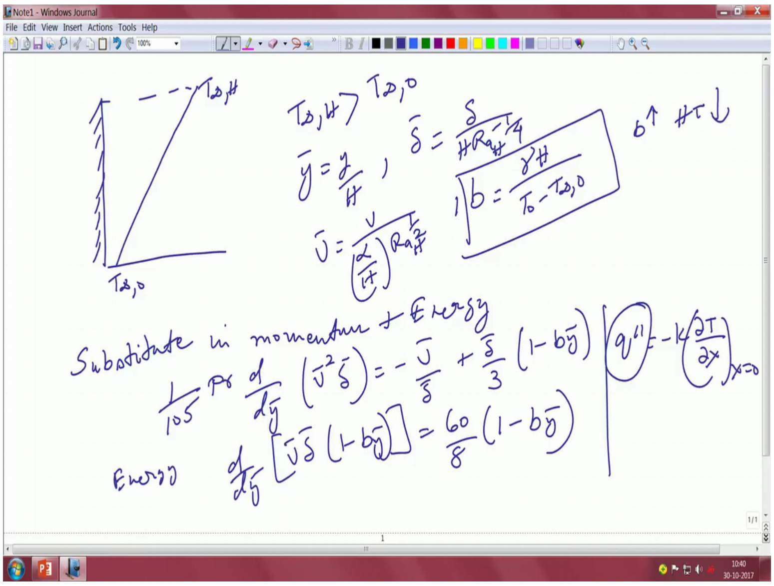

So, in the last class, what we did was that we for understanding this thermal stratification. If you recall the problem that it was still that vertical plate and one thing that happened was that the temperature of the ambient actually increased from T infinity zeros to some T infinity, H if you recall so which decreases basically the heat transfer potential. So, this is obviously we say it is going to be greater than this. And we define parameters which basically convey the meaning of this. So, our definition by our definition y was equal to y by H if you recall, just consult your notes delta was normalized by H R a H to the power of minus one-fourth. V bar is given as a V divided by alpha by H a Rayleigh number H to the power of half; and b which is basically signifies what is the increase in basically the room temperature or the ambient. So, this is given by a non-dimensional parameter b, where we said that as b goes up, the heat transfer actually decreases right. So, up to this point we did.

Now, we also cast the integral equations if you recall from last class that we cast both the energy as well as the momentum equation in integral forms. Now, we substitute in momentum plus energy equations, so in a nutshell what we get is, so these are the expression, this is the first expression, this is the momentum obviously. Then comes the energy equation.

So, these are the two expressions that we get. And of course, your wall heat flux is given as k dT by dx computed at x equal to 0 right that was what it was. Now, what will happen is that now if you do the integration right I mean you solve this equations keeping the variables in consideration and your main target is to find out what is going to be that q double prime that you have over there.

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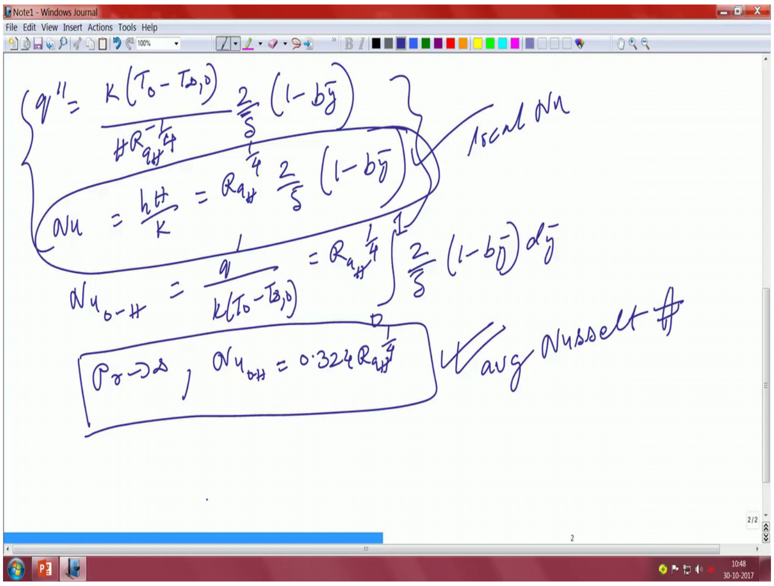

So, if you do it so the q double prime will come out to be let us go to the next one. So, the q double prime will be given as K T naught minus T infinity naught divided by H R a H to the power of minus one-fourth 2 by delta 1 minus b y bar, this is delta bar, so that is the expression that you get. So, naturally your Nusselt number this is the local Nusselt number is given by this right, so that will be nothing but R a H to the power of one-fourth 2 by delta bar 1 minus b y bar. So, these are the two expressions that you are going to get for q double prime and Nusselt number.

Now, the Nusselt number of course, if you have to find out the average Nusselt number which is from 0 to H right, you need to basically integrate the whole expression. So, this is one-fourth to integrate it from 0 to 1 2 by delta bar 1 minus b y bar and d y bar right, so that is the integration. Now, for Prandtl number approaching infinity this value of the

Nusselt number o H approaches something like 0.324 into Rayleigh number to the power of one-fourth, if Nusselt number approaches infinity.

So, this kind of this from the integral formulation if you do more sophisticated numerical exercise on this, you will find that there is almost 11, this is close to almost 11 percent of the more you know what we call formidable and more complicated analytical or numerical formulations in a stratified enclosure alright. So, as you can see that obviously your Nusselt number if you look at this particular expression, so as you change b, the Nusselt number actually changes accordingly, the Nusselt number change.

So, previously this b was not there. So, if b is equal to 0 say for example, there is no gradient, so you basically recover back the original value. But here, however, since b is there, so depending on the value of b, your Nusselt number is going to be altered accordingly, so that is the most important part that you should kind of note.

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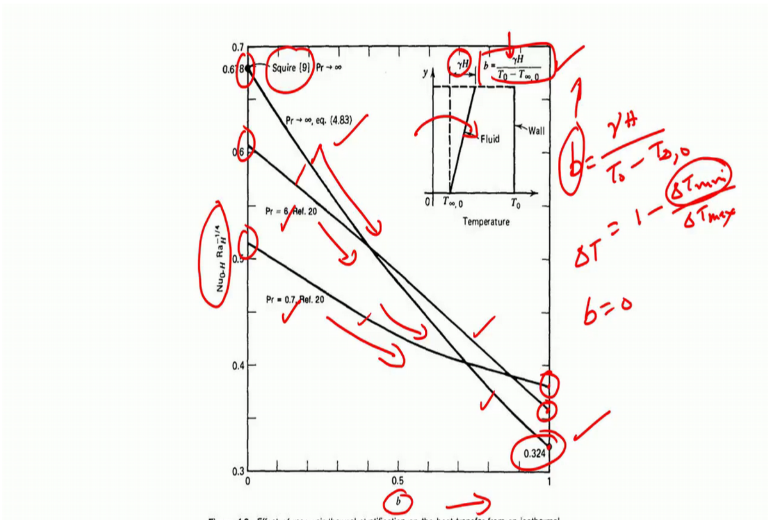

Figure 4.8 Effect of reservoir thermal stratification on the heat transfer from an isothermal vertical wall.

We also would like you to look at the generalized expression over here. So, these are the graphs that were plotted taken from Aderene Bejan. So, what we can see over here this is once again the same thing that we did. So, once again you can see this is the parameter b this is the extent of stratification what stratification has done. And what we have done over here is that we have taken the Nusselt number average and we have multiplied it out by the Rayleigh number to the power of minus one-fourth. So, you will be left with basically whatever is the b dependent term, b and y dependent term right, so that is then we have plotted it versus b.

So, as you can see when b is equal to 1, let us take first b equal to 0, b equal to 0 means there is no stratification at all. So, when b is equal to 0, you basically get back all your original settings, what you had earlier, those were the graphs that you had earlier. But as we move on to b equal to 1, b equal to 1, so that means, higher and higher values of b which would imply that your gamma that you have pointed out over here that is going up that means, the slope is actually going up that means this extent of stratification is increasing quite a bit. So, naturally you see that there is a decay in your Nusselt number value which is kind of expected.

So, for Prandtl number approaching infinity this we already said was about 0.324. For all the Prandtl numbers this particular value actually comes down quite a bit as you can see from 6, 7 all the way up to about 0.324. So, it is a loss of almost half the value. So, all the graphs actually show that for several orders change. So, this is Nusselt number 0.6, so it is one order. This is six and this is infinity. So, all of them show that this is how the numbers actually decrease, but it is highly a little bit non-monotonic in nature as you can see. Of course, this shows the largest drop followed by this and followed by this. So, something that you have to, so there is no real similarity in the pattern per say. So, there is a only thing that we can say that there is a gradual decrease in Nusselt number as the stratification degree which is b actually increases. So, in the isothermal reservoir limit they fall slightly below the corresponding results of square, square was a more analytical solution.

So, in all we can say that there is no local similarity as such, but we can say for sure as expected that there will be a decrease because stratification is reducing the delta T potential so to say. It is reducing the delta T potential for this particular system. So, if b as you can see is gamma H divided by T naught minus T infinity naught, so that can be also written as 1 minus delta T minimum by delta T maximum. So, something like that. So, as this delta T minimum actually goes on, goes on, goes on changing, so we will have that the b parameter will increase and naturally this will hurt in the amount of heat that is coming from the wall to the fluid. So, this is not unexpected.

Only thing that is here that we have provided some fixed values based on the integral analysis that we just did. And it is like we already established that what will be the temperature profile what will be the velocity profile and things like that. But it is a simple substitution, but we are able to predict the slopes. And these slopes are not like these might look a little linear this looks like this has got some kind of a parabolic or a power law kind of dependence or polynomial kind of a dependence in this particular case.

So, let us move to the other one. So, based on this, it is quite obvious now that we have been able to say that what would. So, just remember these values, this is the local Nusselt number, and this is basically the average Nusselt number. So, these two things must be kept in in your mind when you actually attack the problem. So, let us now look at the situation where the two vertical walls are basically a little bit closer to each other. So, this problem is an interesting problem in the sense that it is applicable in a wide variety of applications ranging from fins, electronic cooling because where the two walls I mean the two fins may be placed very close to each other.

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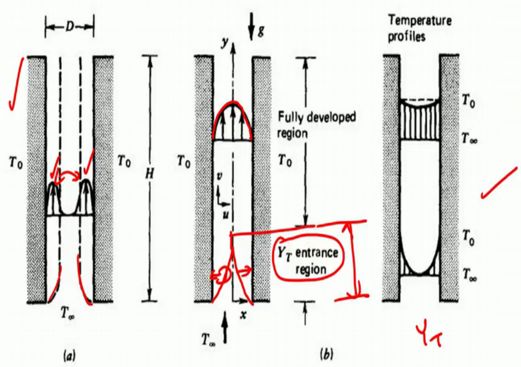

Figure 4.11 Natural convection in the channel formed between two vertical hot plates.

So, before looking at it, let us look at what I am actually talking about. So, let us look at this particular profile. So, as you can see that this is the normal vertical wall that we were actually dealing with. What happens if you place another vertical wall right next to it? Now, if the distance between the two walls is much, much greater than the thermal distance or the thermal boundary layer thickness which is delta, delta T or delta, whatever you call it.

If they are sufficiently placed far apart, so what do you expect to get, you expect that these two velocity profiles will remain almost distinct with respect to each other alright throughout the length through which they traverse. So, these are the wall jets right. So, these wall jets will remain distinct from each other, very distinct from each other, they would not cross talk with each other because of the simple reason is that the distance between the walls is sufficiently far apart. They are actually sufficiently far apart.

Let us look at the scenario two when the distance between these walls are very close to each other alright, they are very close to each other, that means, these two velocities will now start to cross talk. So, by cross talking we essentially mean that they will kind of merge right, and you get a very similar velocity profile like what you would normally expect in a duct right in the forced convection paradigm. So, the flow will be like that, there will not be any distinct, you would not be able to distinct the two wall jets, they are basically cross talking with each other.

And similarly, so you can see what happens when you actually have the temperature profile distinct versus not so distinct. So, in these cases also as we know that the boundary layer takes a little bit of time to develop right that is the same as in the case of a pipe flow or any other flow right, the boundary layer takes a little bit of time to develop. So, there must be some kind of an entrance length right before these two layers actually merge with each other must be. So, you mean to say that there should be something like this probably right, before these two layers actually merge with each other and that merging length is given by this parameter which we call as y T, see it is y T is the parameter which governs this.

So, let us now see that how we can analyze the problem. In the case of a duct flow, remember it was a pressure driven flow. So, there was a pressure head which was actually maintaining the flow right. Here of course, it has to be the temperature difference. So, the temperature difference creates a flow which resembles which kind of mimics right what we actually see in a duct. And actually whether the velocity profile will be parabolic or not that we will see shortly, but in a nutshell if these two profiles merge, they will look something like that but there would be similar to the forced convection counterpart. There will be some kind of a merger of the two layers and we will see how to analyze a problem like this.

So, let us go to our little notes. Now, so having laid down the problem and this is widely used in the case of electronic cooling and things like that. In fact this is one of the key pieces of design parameter that one would normally use, because when you try to design a coolant a cooling circuit right, fins especially in your computers and other things. And if you are relying on forced convection to do the effect, then how these two layers are cross talking with each other, and how the heat is actually transported it forms a large part of the problem. So, it is very characteristic of vertical fin to fin cooling channels in electronic equipments remember that in mind.

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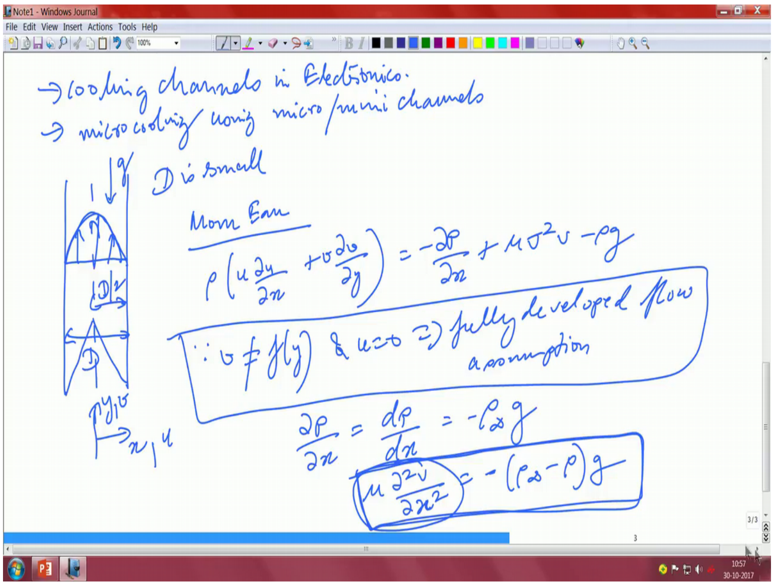

So, cooling channels cooling channels in electronic equipments in electronics it can be in other cases also, specially this era of you know micro scale cooling or micro cooling all those things can be very important and micro cooling using you know micro to mini channels. These are all very, very important over here. Now, let us now take the flow part and try to analyze the problem.

So, the problem essentially boils down is that this d which is the separation between the two plates is small, D is small. And if this is the axis let us define the axis, this is your x, this is your y. So, this is naturally this portion is basically D by 2; and obviously, these two layers merge with each other. So, all those things are very similar to the pipe flow situation. So, this is given by u, this is given by v alright, so that is the way that we have always defined it.

So, the momentum equation, if we write the momentum equation, it will be rho dx plus v and dv dy minus dp by dx plus mu, this is the momentum equation. So, similarly looking at this particular kind of a velocity profile what we showed right like that, we showed the velocity profile of course, this is the peak. The velocity profile that we showed, we can safely assume that perhaps your v that is the velocity in the vertical direction is not a function of y, it is very similar to the fully developed assumption, and your u is basically equal to 0. All this comes from the fully developed flow assumption, all this comes from the fully developed flow assumption that v is not a function of your y and u is the actually equal to 0.

So, similarly dp by dx this we already established earlier it is like a hydrostatic head that we develop. So, this will become that is the only term that is remaining. So, there is of course, gravity here, because gravity is always important right in this. So, as you can see of course there is no pressure gradient, but you have this hydrostatic and this buoyancy driven flow essentially which is basically trying to balance this viscous term, this is nothing but the viscous term right. Very similar in origin very similar how we have actually analyzed the problem that v is not a function of y and u is equal to 0, this is our standard definition of fully developed flow, that means, the flow is convectively non accelerating, so that is what we did earlier alright.

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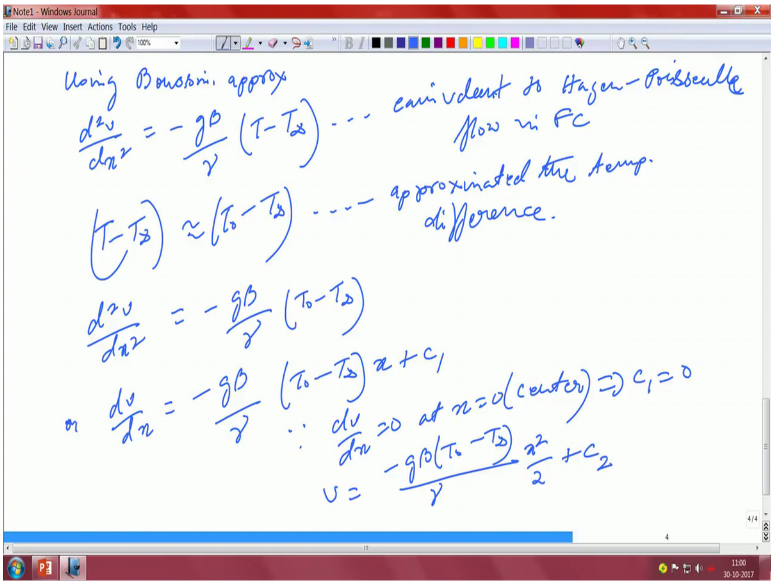

Now, using our Boussinic approximation, if you recall the Boussinic approximation where we said that in the convective part it usually is the constant density and the variation only happens in the buoyancy part, remember that was what Boussinic approximation was. So, using Boussinics approximation, this particular problem therefore, becomes unsolvable when you write it like this, because it is g beta by gamma into the T minus T infinity is generally an problem which cannot be solved because you have velocity as an unknown, you have temperature as an unknown.

So, what we need to do is basically you need to couple it with your energy equation, and solve it kind of together right, together you have to solve these two equations. But of course, this particular expression is equivalent to the Poiseulle flow to the Hagen Poiseulle flow in forced convection. So, it is equivalent, but this natural convection equivalent of the Hagen Poiseulle flow, but it is cannot be solved in such a nice way as we did in the case of your Poisseulle flow.

So, what we have done, what we can do over here is that if we assume thus T minus T infinity, we assume that this particular thing almost equal to T naught minus T infinity. That means how what we have done, we have got rid of the problem of T minus T infinity and we are substituted it by a constant head temperature difference like T naught minus T. So, this T naught minus T that is what we have done we have approximated the temperature difference by this. So, this is approximated the temperature difference.

So, moment we approximate the temperature difference then the problem becomes solvable that is because your equation now boils down to minus g b gamma T naught minus T infinity. So, you can apply your first integration this is g beta by gamma T naught minus T infinity x plus c 1. Now, since you have dv by dx is equal to 0 at x equal to 0, which is basically the centre right, this should ideally lead to c 1 to be equal to 0 as well, so that will lead to c 1 to be equal to 0. So, similarly, so therefore, your v now becomes g beta T naught minus T infinity divided by gamma x square by 2 plus c 2.

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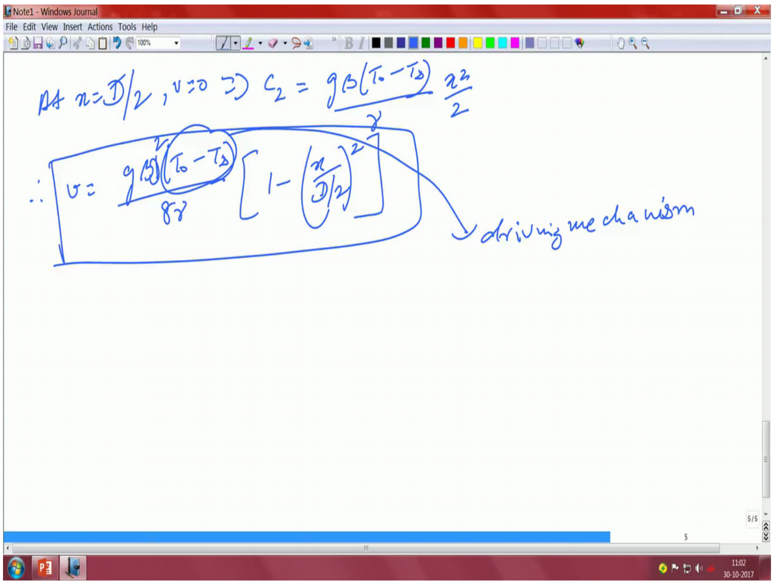

Now at x equal to D by 2, that means, at the wall your v should be equal to 0, because of the no slip condition this leads to your c 2 to be equal to g beta T naught minus T infinity divided by gamma x square by 2. So, your velocity therefore, your velocity becomes g beta T naught minus T infinity divided by 8 gamma T naught minus T infinity T naught minus T infinity sorry this T naught minus T infinity and there is a D square here

So, in this particular case, as you can see this is basically the driving mechanism, this is basically what is your driving mechanism right. So, like in the pressure head, we had in this particular case this is basically the driving mechanism of the flow. So, we have established what is the velocity scale going to be, now it remains to calculate what is the corresponding heat, and what will be the corresponding you know the Nusselt number and stuff like that.

So, let us look at that in the next class. So, here we have established up to the velocity scale of the problem.

So, let us see that how the Nusselt number, and the heat release comes into the picture when the two plates are very close to each other, and they are cross talking with each other, so that the two flows have basically merged with each other.

Thank you.