Convective Heat Transfer

Prof. Saptarshi Basu
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture – 33
Uniform wall heat flux

Let us look at the in the previous class we looked always we looked at the problem of uniform wall temperature right. This is at isothermal wall kind of a situation, all the analysis the integral scaling everything was done based on that right.

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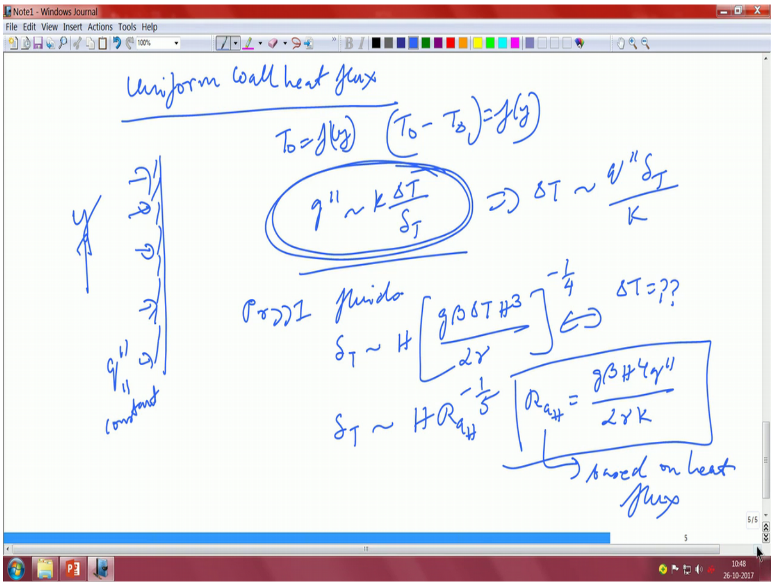

Let us look at for uniformity sake, what is a uniform wall heat flux this problem is a little bit more complicated because you would now have a wall which is no longer at a uniform temperature. You are supplying it with some heat flux which is q double prime which is equal to constant that is what you have done and this brings about that your T naught now will be a function of your y right, it has to be it has to be a function of your y now right; that means, this is the y.

So, it has to be a function of your y in this particular case right. So, in other words this T naught minus T infinity will be a function of your y correct, but few things do not change and we will just show the approach, few things which do not change for example, q double prime scaling as k delta T by this thing. This of course, does not change because this is this is the very definition of the wall heat flux right, that is what we have always used because at the wall you know it is a conduction problem basically. So, this is basically that right d T by d x right. So, that is what we already know that this particular scale will be applicable regardless of whatever.

So, in this particular case let us see that for prandtl number once again do the methodology for prandtl number much greater than 1 fluids, we always solve for the 2 extremes right. Let us see that what will be the corresponding delta T and things like that, we know that delta T the basic scaling argument do not change. It is just that what we are substituting that will change a little bit. So, this g beta delta T H cube divided by alpha gamma to the raise to the power of minus one fourth, that was what our delta T was right.

If you recall that was what it was except we wrote it in terms of Rayleigh number in this in that case right, now in this present problem your delta T is we do not know it is a variable quantity right, but on the other hand you know that the delta T the q double prime is now actually known right. So, why not use this expression right here and from here delta T becomes q double prime delta T divided by k correct. So, that is what we are going to substitute it now into this particular expression right.

So, in this particular expression moment you substitute it, it becomes equal to H r a H to the power of minus one fifth right where this r a H is different from the r a H that you are you have dealt with earlier except that it is given in terms of the heat flux rather than in terms of the temperature.

So, this is the definition of this R a H now several books will differentiate between this R a H and other R a H it is your choice you can either write it as R a H star as Adrian Bejan has done or you can write it in any other way that you want, but it is basically the Reynolds number the equivalent definition of Rayleigh number I am sorry. So, where it is given in terms of g beta H to the power of 4 q double prime divided by alpha, the kinematic viscosity and the thermal conductivity that is how this has been cast.

So, it is Rayleigh number we call it based on heat flux that is the only major change that you have essentially you have written it in exactly the same way because nothing should change, except that you have added one particular scaling which is basically q double prime is equal to k delta T by the thermal boundary layer thickness right from there we are eliminating delta T the or the temperature differential that is because that differential is not constant anymore, it is the q double prime which is the constant over here right. So, that is the only change that we have made over here right.

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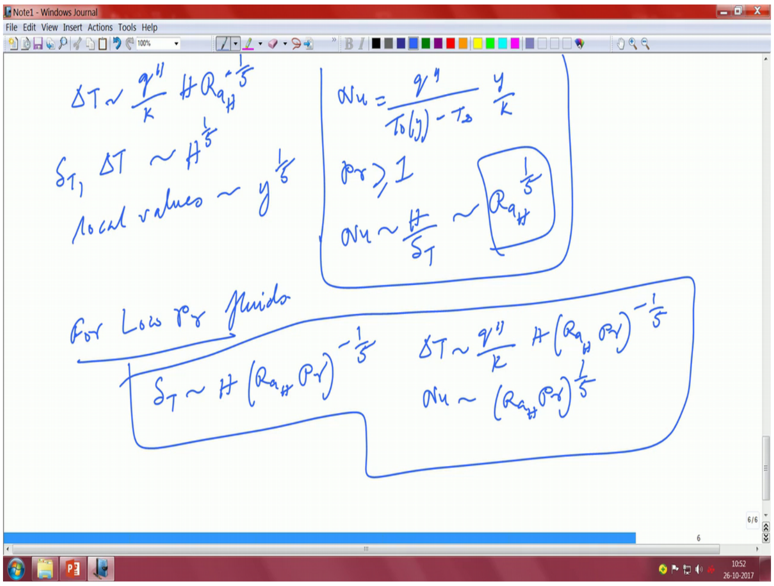

So, in other words this delta T now therefore, should become its not put.

So, you can see that both delta T as well as the temperature differential are proportional to H to the power of one fifth right, because H average quantity is a proportional to H to the power of one fifth. So, the local values are proportional to the local values. So, these are the way will be the average values the local values will be y to the power of one fifth right. So, that is the standard because we call that a Rayleigh number actually has got a H to the power of 4.

So, it is basically one over H to the power of 4 by 5 and you have H here. So, that is how this H to the power of one fifth comes into the picture, in case you are worried that how this H to the power of one fifth came right. Now, when you are dealing with local values that at any y along the plate this will become instead of H it will become y that is the dynamic variable that we are actually dealing with.

So, the nusselt number here will be q double prime divided by T naught which is a function of y now, minus T infinity y by k right. So, for prandtl number greater than to one, the nusselt number should scale as H over delta T right. So, its scales as Rayleigh number to the power of one fifth roughly right because your delta T scales like that. So, naturally it will be Rayleigh number to the power of one fifth got it.

So, this part should be very self obvious so right. So, nusselt number is like that and if you recall the other 2 parameters; that means, delta T is that right. So, H by delta T is basically Rayleigh number to the power of one fifth which is basically the scale for nusselt number and remember this definition of Rayleigh number ok.

So, there is nothing great that we did except that we have played with a little bit of variables. So, that it applies to the current problem right, now for low prandtl number fluids low prandtl number fluids, once again the methodology is almost identical now it is Rayleigh number prandtl number to the power of minus one fifth is the same thing there is absolutely no change. The delta T is q double prime by k H Rayleigh number H into prandtl number to the power of minus one fifth and nusselt number is Rayleigh number into prandtl number to the power of one fifth got it.

So, these are the 3 results that you will get again I am not going through the motion you just substitute it you will automatically get that Rayleigh number because the delta T now comes in the right hand side also. So, that is why when you take it away to the other side that is why instead of one fourth you have now one fifth dependence and this Rayleigh number has been cast in a particular way, these are the only 2 things that you need to remember or you need to you can do all the math also and you can come pretty much to the same conclusion, right.

The validity of the scaling results can be checked by you know more rigorous you know analytical solutions.

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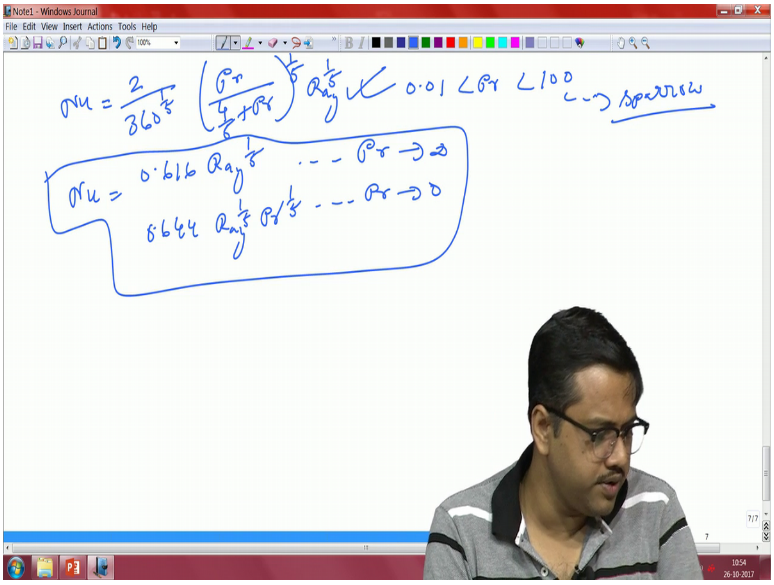

So, one such analytical solutions which is valid across by this was reported by sparrow, this is like an adequate, this is like a curve fit and this is valid in the range of 0.1 prandtl number and less than 100 about that. In the 2 limits 0.616 into Rayleigh number to the power of one fifth, 0.644 Rayleigh number one fifth, prandtl number one fifth this is the general scaling this is prandtl number goes to infinity, this prandtl number goes to 0. So, these are the 2 quantities that you would get, but this is the most general scale that was found by sparrow all right, these are the 2 most general scales that were founded by the sparrow.

So, as you can see over here now we have shown that uniform heat flux is no different except that the scaling parameters have been adjusted a little bit to make sense right. So, you can follow through the same motion the same type of similarity arguments the same type of you know integral analysis that basically come to the same fact, right. So, that is what is most important over here and we have finished this particular thing and I can show you one other important stuff which is worth mentioning before we move on to the other things.

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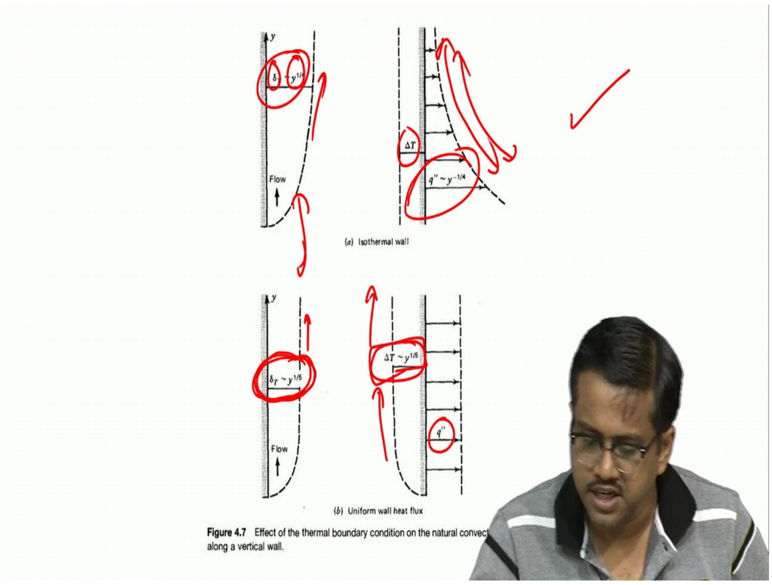

Figure 4.7 Effect of the thermal boundary condition on the natural convec[tion] along a vertical wall.

If you take the case of the isothermal wall alright isothermal wall that is the first part of it right. So, this is your delta T which scales as y to the power of one fourth we already told that earlier right.

This delta T is constant. So, naturally the fluid that is actually going up the wall, going up the wall there the heat flux that it will carry will progressively go on decreasing because the driving the temperature of the fluid is actually increasing at is as it is moving up with the temperature of the wall being maintained at a constant because it is an isothermal wall right. So, naturally you know the quantum of heat that it receives right the heat flux shows that kind of a trajectory right. So, it just is moving up like this ok.

On the other hand if you look at this particular situation delta T is now scales as y to the power of one fifth, not one fourth like in the case of an isothermal wall please look at it very carefully. What happens over here is that now the heat flux is a constant right, heat flux is a constant now to maintain that heat flux your delta T has to show a profile like this right because you have to have a profile like this because you have to maintain the same heat flux right.

At each point this is very similar to the if you remember your internal flow situation where to maintain a constant heat flux you had to do this. So, the temperature profile here shows as y to the power of one fifth dependents, the thermal boundary layer showed a dependence of y to the power of one fifth here of course, it was one fourth and here it was y the q double prime dependent as minus one fourth; that means, it shows that exponential nature of this decay right because it is a power law.

So, its y to the power of minus one fourth is like a power law right here of course, the temperature increases, but at a scale which is y to the power of one fifth; that means, the slope is not that sharp, that is why you have a more flattish type of a profile that is what we proved just now the delta T is basically dependent as y to the power of one fifth right. So, y to the power of one fifth means that the dependence is going to show a gradient which is very shallow. So, this is exactly how this has been drawn this gradient is very shallow right whereas, this gradient is large because this is minus one fourth variance right of q double prime got it.

And naturally the delta T also varies in this y to the power of one fifth kind of a variation. So, this is even flatter right, this was one fourth this is one fifth. So, that should have a slightly sharper kind of a increase compared to this, correct. So, that is how this these two figures have been drawn you can see that there is a little change this is a little this slope is not that high, but it is a little bit more than this because of the one fifth and one fourth variance.

So, it is more like these boundary layer profiles are almost like constant thickness as they move along the plate when you particularly. So, when you deal with the situation of a uniform heat flux right, but; however, uniform heat flux decays quite rapidly because of the minus one fourth dependence that delta T; however, does not increase that much it just increases a little bit because of the y to the power of one fifth dependence these 2 dependences are the same. So, these 2 profiles should also look like the same. So, you can just sweep it and you can with appropriate scales if you place on the one on the top of the other they will be the exactly the same profiles right. So, that is exactly what we are seeing in this particular case ok.

So, this finishes this gives you a good idea that how the constant are the isothermal wall and the uniform heat flux profile should look like how the temperature should vary how the heat fluxes should vary. So, this is an important visual concept because you have got all these expressions now you need some kind of a visual feeling that how this profile should look like. So, if you do the math you can easily identify these things yourselves, but even if you do not do the math these are the kind of profiles that you will get which all comes from scaling all comes from the mathematics that we did so far. So, based on this we next go to a slightly more realistic situation which is called the thermal stratification.

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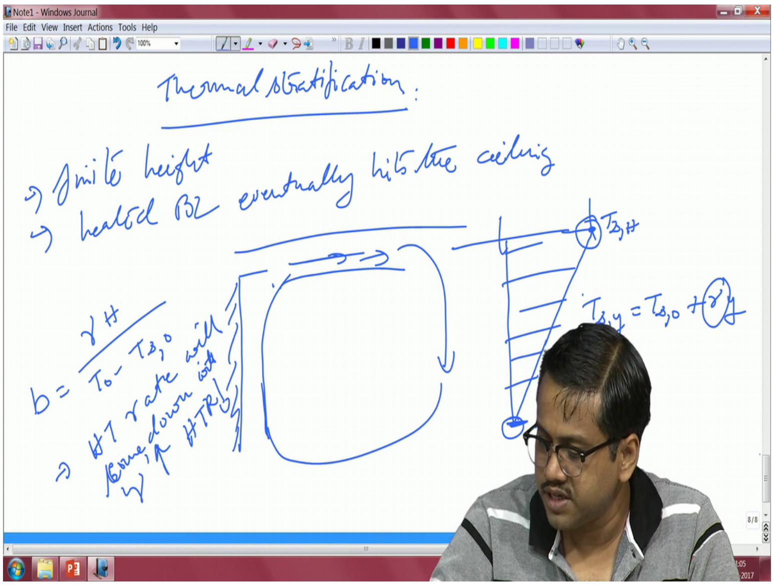

We will pause this problem and we will pick it up later as with time right. So, thermal stratification is an interesting thing to begin with right. So, you have we are talking about you know this isothermal walls placed in an infinite reservoir of fluids there is there is I mean it is not a not a shallow thing at all right, it is like a small wall which is placed in a large body of the fluid right, but this is of; obviously, the simplest model possible. But it is not exactly the case when you actually deal with realistic situations right, realistic situations there will be a sealing of some sort right there will be a sealing at the end of the day right. So, that sealing will pose a restriction so; that means, there is a finite height if I have to put it like that is a finite height.

So, it is a finite height and the boundary layer and the heated boundary layer eventually hits the ceiling, it has to hit the ceiling the boundary the boundary layer. So, what happens when it hits the ceiling is that if you have this kind of a situation right what will happen is that it will go up, it will hit the ceiling right.

What we will do it will start to discharge in a horizontal direction, this heated fluid right because if it meets a ceiling on the top right there is no other way that it can go right. So, it will go and hit the ceiling and then it cannot penetrate through the ceiling right. So, it has to go in the horizontal direction or rather it is forced to go into the horizontal direction in this particular case and then it can climb down and complete the circuit right.

So, that is what we normally have, now this has got a long term effect this happens in any room that you will see. What will happen is that because of this you know finite size of the whole thing after certain period of time there is of course, a developing timescale during which all these dynamics happens what you will find is that the room will become stratified, stratified in the sense the bottom of the room will have the have a slightly lower temperature from an ambient perspective and the top of the room will have a slightly higher temperature right so; that means, here if I have to do it this way.

So, if this is the height of the wall or the height of the ceiling whatever you call it alright. So, that is the thing. So, here the temperature is T infinity 0 right so; that means, it is the ambient temperature at the base of the room so to say right it goes up assuming that it is linearly stratified it goes up to another temperature which is basically given as T infinity H all right. Some T infinity H and the temperature at any particular section right at any particular y is given as T infinity 0 plus gamma y, this gamma is not the kinematic viscosity this gamma is some slope right.

So, that is what the thing is so; that means, here at this point the ambient temperature is a lot higher than whatever is the temperature at the base that is because the room has gone stably stratified the lowest is the coolest at the top is the warmest, this happens in many of the rooms. If you will find that in many of the many of these rooms which operates on natural circulation you will find this kind of stratification is obvious that is because the heated air encounters the ceiling and it kind of stays there. So, it over time it basically increases the temperature of the room in that particular linear very linear way. So, that is the most important part of this particular argument. So, of course, when this slope is 0 you get back your originally non stratified situation; however, when this slope is not 0 is you basically have different levels of linear stratification. So, if this slope is very high; that means, you have a lot of stratification and when the slope is a little low you have a very little amount of stratification, one other thing that should be clearly mentioned over here is that as you increase the level of stratification like this right; that means, you have a temperature going up to infinity H, T infinity H what will happen to the heat transfer it must actually go down right.

That means, it is it becomes a more ineffective way that is because the driving potential is still your T not minus T infinity, right. So, your T infinity is going up. So, naturally this will affect your delta t. So, effectively or reducing the delta T by increasing the temperature of the ambient. So, naturally there is that correspondence right that that driving influence which drives the buoyancy, which drives this entire recirculation loop that is going to become a little weaker, for the same given wall temperature remember the wall temperature is still the same right.

So, that is a very important argument over here and we can write this particular parameter as b which is like a stratification parameter and gamma H divided by T naught minus T infinity naught alright. So, b is equal to 0 when this thing is fully this thing is not stratified at all and b is equal to 1, when it is fully stratified right got it. So, that is the most important part, now we will see now how to analyze this particular problem with the with the understanding that the overall heat transfer rate, heat transfer rate will come down, come down with gamma as gamma increases H T R comes down it has to be the case right.

So, let us look at now this particular problem and see what can be done..

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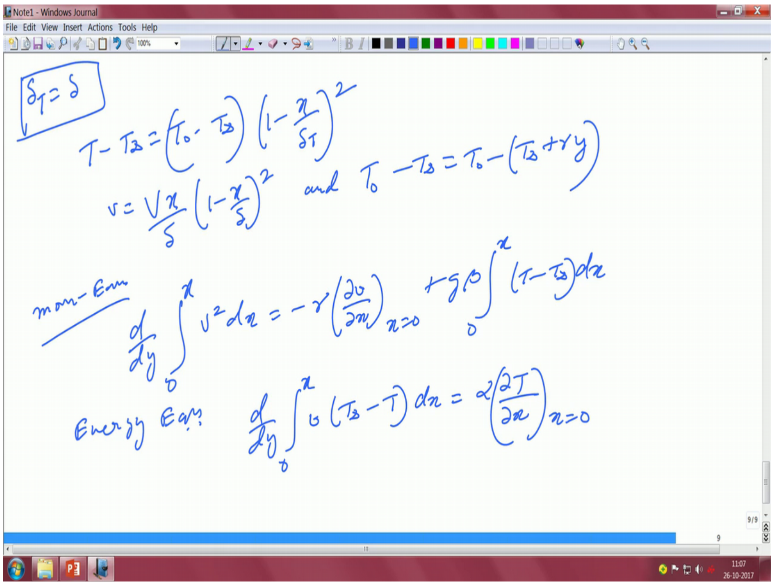

So, in this particular problem let us assume delta T is equal to delta to avoid any velocity and thermal boundary layer issues. So, T minus T infinity is equal to T naught minus T infinity into 1 minus x by delta T square right. V is equal to capital v into x by delta 1 minus x by delta square and you have that additional thing T not minus T infinity is equal to T naught minus T infinity plus gamma y, right.

So, now if we do the integral approach the momentum equation, the momentum equation 0 to x v square d x minus gamma d v by d x and x equal to 0 plus g beta. Check all these calculations when you actually do the entire exercise at home sometimes it is better if you do some of these things at home also. So, the energy equation becomes d by d y 0 to x v T infinity minus T d x equal to alpha d T by d x x equal to 0. Now, what we do the basic part after this is basically to substitute T is there, v is there substituted in the energy equation and in the momentum equation, both needs to be substituted in both the cases right.

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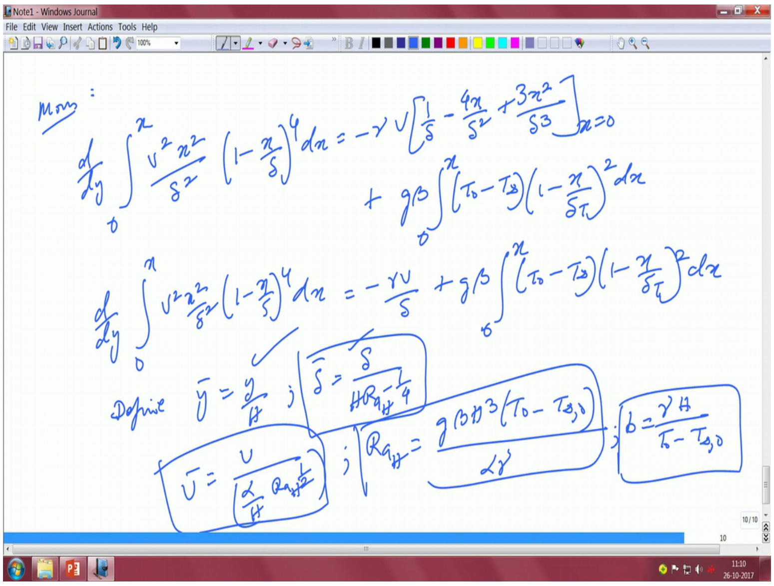

So, the momentum equation let me just write it quickly d by d y and this can be even shortened a little bit.

So, now that we have this problem let us just define before we end this class define the non dimensional parameters and then cast it and then we will pick it up in the next class and do a more detailed discussion on the results. So, we can define y bar is equal to y by H and delta bar is equal to delta by H into Rayleigh number to the power of minus one fourth right, v bar can be taken as v divided by alpha by H these are the natural scales of the problem. So, there should not be any issues in understanding that why this is coming out to be like that the real number h; however, has been defined based on the initial; that means, on the lowest temperature that is the T infinity this is the temperature at the bottom of the plate and b is of course, equal to gamma by H divided by T naught minus T infinity comma 0. So, these are the 4, 5 non dimensional parameters that we have defined. B is the new one these are common parameters and the y we have normalized it by these scales should be pretty apparent because these were the scales that we used in our measurements anyways right.

So, based on all these things in the next class what we are going to do, we are going to put the, these parameters in the momentum and in the energy equation and we are going to solve it and we are going to show that what the nusselt number and the heat transfer rate is going to look like. So, see you in the next class.