Convective Heat Transfer
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Lecture - 30
Scaling analysis – II

In the last class we started doing the Scaling Analysis and if you remember correctly that we had two ranges, one was for Prandtl number greater than 1 and one was Prandtl number less than 1. And we said that in these Prandtl number ranges which particular terms become most important. So, let us write the governing whatever we did in the last class. So, it was something like this.

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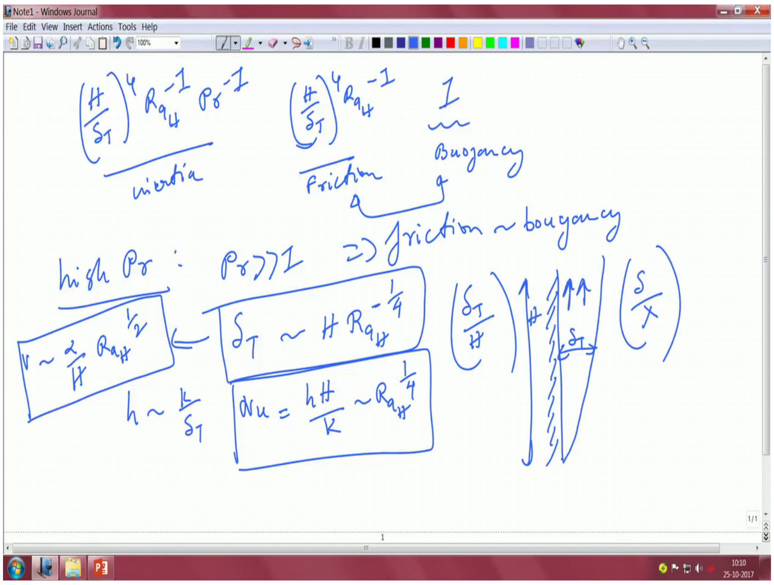

So, then there was the friction term and of course, this is buoyancy. So, this is basically was our buoyancy term, right. So, this was the friction or the viscous term and this was the inertia, right.

So, depending on the value of your Prandtl number we can have different types of scenarios, right. So, let us take the high Prandtl number fluid which means its Prandtl number much much greater than 1, we are going to propose a two extreme type of solution over here. So, when Prandtl number is much much greater than 1, friction sorry, friction is basically balanced by the buoyancy. So, friction and buoyancy kind of balances each other. So, in other words these are the two terms which basically balance each other. So, in that particular case your value of your delta T which is basically the boundary layer thickness will scale as H, R a H to the power of minus one-fourth right.

Remember the scale it is not Rayleigh number that is important it is Rayleigh number raised to the power of one-fourth minus one-fourth that is the term which is important. So, the correct scaling comes from that. So, delta S or delta by H in other words is nothing, but some kind of a slenderness ratio. If you recall that if this was your boundary layer this is the plate correct. So, this is your delta, delta T right if you recall. Then delta T by H is basically nothing, but the slenderness ratio.

This is the same type of thing that you would see in your flat plate boundary layer forced convection where it was delta by x right. It was in that case the plate was basically oriented in the horizontal direction and it was forced in nature. So, delta T by H and delta by x basically represents the kind of slenderness ratio. So, here the slenderness ratio essentially scales as Rayleigh number to the power of 1 minus one-fourth. So, the heat transfer as we know the heat transfer coefficient h basically scales as k over delta T this is still the same it is inverse of the boundary layer thickness right.

So, the Nusselt number in this particular case is this where H being the relevant length scale of the problem and this represents R a H to the power of one-fourth right. So, this is an interesting thing. So, the Nusselt number varies as Rayleigh to the power of one-fourth the boundary layer thickness varies as R a H to the power of minus one-fourth all right. Now, there are the problem does not really end here. The problem is a little bit more complicated than this. So, let us before we go into that let us also calculate that what is going to be your v; that means, the velocity scale that is imposed by this temperature gradient. Now, that velocity scale is given by alpha by H into Rayleigh number H to the power of half all right.

So, v is alpha by H multiplied by Rayleigh number H to the power of half. So, these all comes from the scaling arguments this all comes from the scaling that we did earlier right it is just a substitution of delta T that is all that we have done ok to get from here to here. So, now, that you know this now the question remains what will be the velocity profile look like. Because remember last time we kind of sketched and we said that we are going to change it that the velocity profile is going to look something like that, but in reality does the velocity profile exactly looks like that or is it something a little bit more different than this right. Because last time what we said was that at the wall it must be 0 at the edge it must be 0, so it must have a maximum somewhere in between right. So, that was the whole argument that we pursued right.

So, in that particular case to answer this particular question let us see the situation a little bit more carefully and this is where the plot actually thickens a little bit.

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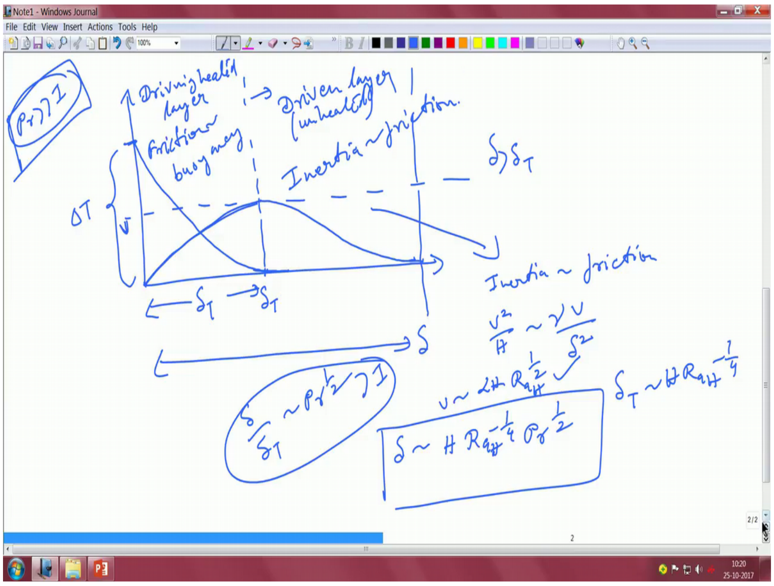

So, these are the two things. So, this is let us mark something like a delta T here, delta T is nothing but the temperature differential that is creating the problem this is your delta T which is basically the boundary layer thickness

So, in this particular zone for Prandtl number much much greater than 1 right, for Prandtl number much much greater than 1, if you look at in this particular situation towards the left side of this delta T or within this delta T or like delta T from here to here right. What you are going to get? The temperature is going to start at delta T that is the maximum it slowly goes down to something like 0 here, it is more asymptotic ok. So, that is the temperature profile correct. So, here in this particular zone in the driving heated layer this is the driver right, because beyond this there is no temperature gradient it is T infinity its T infinity here right.

So, the temperature decays naturally it takes heat from the wall, it naturally decays and it goes the decay amount is basically 0; that means, you go and reach the T infinity after the delta T is over right that is standard whatever we know about boundary layer. So, therefore, the driver for this particular problem is essentially this heated layer right, whatever the air, whatever gas that you are talking about, whatever is getting heated in this region around the wall is basically the driving mechanism for this buoyancy right. And here you have a balance between friction because it is a Prandtl number much much greater than 1 problem, balancing the buoyancy correct that is the nature, right.

Now, what about the velocity? Now, the natural question is that now we have a situation in which you know the temperature decays its gone right. But what about the velocity now. So, the how the velocity boundary layer because remember it is a Prandtl number much much greater than 1 situation, naturally. So, what happens in the region which is to the right of this delta T? Of course, the temperature does not exist anymore. So, in this particular region if we say that this is a driven layer almost like an added mass effect right that when you have something moving it carries some of the added mass with it.

So, this is a driven layer which is unheated correct is unheated because there is no temperature gradient over at there. So, in that particular case what is actually driving that driven layer what will be the forces that will actually act in that particular region. One thing that you can be sure it has to be inertia because there is a motion that is created right. So, that motion now has to be balanced by the friction, there is no buoyancy here right, that motion has to be balanced by the friction right. So, in this particular case what will happen is that if we mark the scale of the velocity, if this is the scale of the velocity this velocity will grow, will grow and this will decay at a certain distance which is given by delta right. So, this delta is this.

So, delta is definitely greater than delta T. It is a Prandtl number much much greater than one situation, remember now the scale of the velocity and the scale of the velocity is driven. Remember here two things, one is that the velocity scale is determined by the heated driving layer right. But on the other hand the driven layer thickness or the total thickness that you are actually dealing with is basically determined by this balance between inertia and friction outside of this delta T.

So, in other words in this region if we just apply a balance between inertia balance is friction. So, what we will have is basically v squared H balancing gamma v by delta square correct then that will be the balance. Now, if you we already know what is the scale of v is right that we just did if you just go to the previous one. So, that is the scale of v this is remains unaltered alpha H by Reynolds number to the power of half. So, you substitute v equal to alpha H or Rayleigh number to the power of half you substitute that over here in this particular equation right, what you get is basically your delta scales as H R a H minus one-fourth Prandtl number half.

So, imagine this, what was your delta T per say? Delta T was H into R a H to the power of minus one-fourth right that was your delta T. So, basically the length scale is still H into R a H to the power of minus one-fourth is just that you have you have added a stretching factor with respect to the Prandtl number. So, Prandtl number basically acts like a stretcher, a stretching factor over here that that actually leads to this particular problem. So, in the Prandtl number much much greater than 1, situation to recap you have basically two zones right.

The driven the driving heated layer is basically a balance between friction and buoyancy right, where the relevant thermal boundary layer is delta T right and the velocity scale comes from this particular analysis which is given over here right and the Nusselt number is also given in the same way. However, the driven portion that is the layer that is driven because of this flow that is created and because it has got a Prandtl number which is much much greater than one effect basically, what we have is that we have a second region in which there is an inertia and friction balance because there is no buoyancy over there, there is no temperature gradient. So, how can there be buoyancy?

From there you find that your boundary layer your velocity boundary layer is basically stretched a little bit how much it is stretched is determined what is the value of your Prandtl number. If your Prandtl number is very very high you can see from this equation right here about delta. If your Prandtl number is very very high this velocity effect is felt for a very long distance got it. Even though this layer may be very small understand. So, the driving layer is very small, but the driven layer can be quite large depending on what is the value of your Prandtl number, if your Prandtl number is very high it can be felt over a very long distance right.

So, but ideally speaking, here of course, your delta a by delta T is basically a Prandtl number to the power of half kind of a relationship right, if you divide the two and it is greater than 1. So, this is one of the important relationship that you should recognize here therefore, unlike in momentum boundary layer the velocity boundary layer is determined by two scales, one is your delta and one is your delta T. There are two scales which determines it delta and delta T. This is obvious because your momentum equation now has got the inbuilt temperature right you have got the inbuilt body force term, in terms of temperature now or the buoyancy and the body force term combination in terms of the temperature in the momentum equation in built right.

So, it is very natural that your velocity profile is now going to be determined by a two parameter kind of a set. So, it is delta and delta T right. Unlike in forced convection where it was not it was decoupled from delta largely right. There is unless there is some property variation that you are taking into account. So, the velocity scale is reached within this delta T because you can see the peak is appearing within the delta T, whatever happens beyond that is basically the decay or the tail of the velocity because where inertia and friction are basically fighting among each other because you know the Navier Stokes is a dissipative kind of a system, got it.

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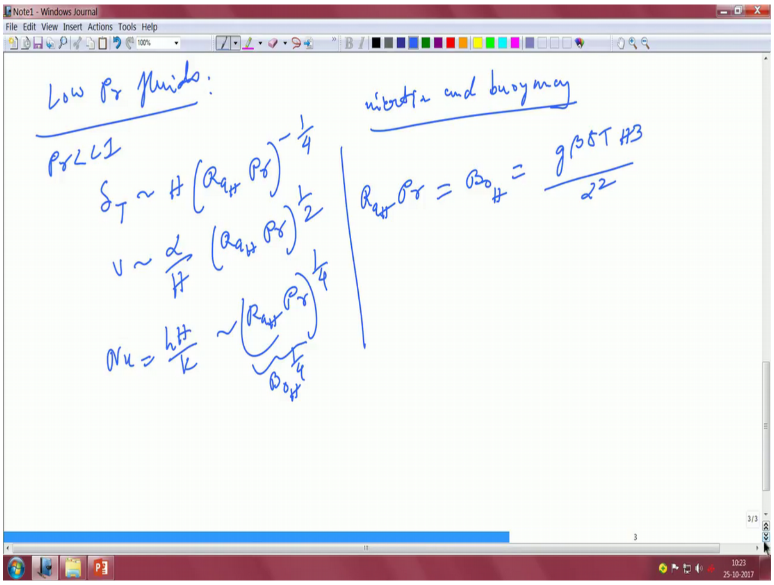

So, based on this we move to the next one which is basically the low Prandtl number fluid; that means, Prandtl number is much much less than 1. So, here of course, your delta T scales as H R a H Prandtl number to the power of minus one-fourth that was the old condition remember. And here of course, it is balance between inertia and buoyancy right that is what we told that it is a balance between inertia and buoyancy because

Prandtl number being much much less than 1 inverse of the Prandtl number is much much greater than 1.

So, therefore, the friction term does not really become very important here. So, v or the velocity scale that is introduced is alpha by H into R a H into Prandtl number to the power of half, that is the velocity scale. And the Nusselt number is once again the same, but of course, it has got the Prandtl number now, right got it. In some cases this Rayleigh number into Prandtl number combination is given by a new number which is called Boussinesq number B o H. So, you can write this term as Boussinesq number to the power of one-fourth as well all the places where there is Rayleigh number into Prandtl number combination you can write it in terms of the Boussinesq number the Boussinesq number is basically given by g beta delta T H cube divided by alpha square right.

So, for low Prandtl number fluid it is a Rayleigh number which is important right. For high Prandtl number fluid is the Rayleigh number which is important for the low Prandtl number fluid it is a Rayleigh number and Prandtl number combination that is important got it, which we have cast and written it as Boussinesq number right. So, also here of course, once again if I try to draw now this particular system right. So, what this particular system will look like, so that is the natural thing, that the natural question that pops in one's mind. So, let us look at that situation.

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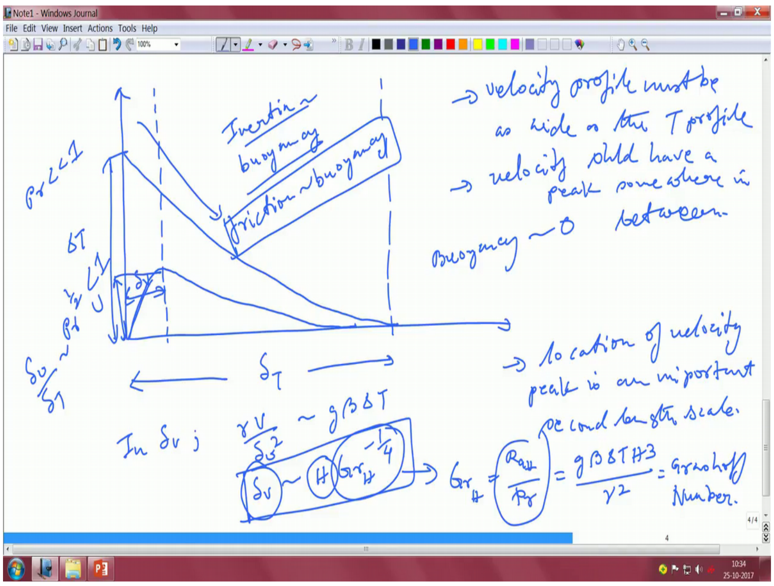

Once again it is a Prandtl number much much less than 1, system I will put a few lines and we will go through it as and when this is delta T, this is the temperature differential. So, it comes down like this kind of like that, got it, this is basically a balance between inertia balancing buoyancy alright inertia balancing buoyancy, we have not said anything about the velocity scale though. So, this looks very natural that once again it decays to some kind of a 0, the profile looks a little bit different it looks a more linearish kind of a profile, but that we will see that why that thing happens in that particular way.

So, based on this you have a very clear idea that this is how the temperature profile should actually be. Now, outside this delta T of course, because this is a situation in which we are dealing with a problem in which the friction is not important in this particular layer right. So, beyond this delta T remember the difference between that last situation and this beyond this delta T once again buoyancy goes to 0, the buoyancy goes to 0 alright. So, in this particular region beyond delta T because it is a low Prandtl number fluid right there is no velocity possible outside this particular layer correct because if it is low Prandtl number fluid its inertia driven by buoyancy, inertia buoyancy are balancing each other. So, beyond these of course, the buoyancy is; obviously, going to be 0 because there is no temperature gradient to support it. So, naturally there is no velocity scale that are possible.

So, that velocity profile must be then as wide as the temperature profile that is about it right it must be as high as the temperature profile right so that means, whatever the velocity profile we are going to draw it should have a maxima somewhere and it will be 0 on this side and 0 on the wall side. Wall side it has to be 0 regardless right, on this side it will be 0 because there is no driving mechanism and friction is not important in that particular region anymore and it is a low Prandtl number fluid right. So, the effect of momentum is not felt up to that particular region, got it.

So, the velocity profile must be as wide as the temperature profile and so we can write this velocity profile must be as wide as the temperature profile alright, it should be as high as the temperature profile. So, naturally this dictates that the peak of the velocity would be somewhere in between these two limits, these two 0 and delta T in this limit the velocity must be maximum in some particular, some location right. So, velocity should have a peak should have a peak somewhere in between right has to be, all right.

So, the location of the velocity peak is a important second length scale that comes in to this particular problem. So, location of the velocity peak, peak is an important second length scale. So, if we assume that delta v is that particular thickness it is a very small thickness of a layer right next to the wall just hear the arguments here very carefully that we are imagining that there is a thin layer of fluid very close to the wall right, and that we are defining as this delta v because we are trying to find out what will be the location of the peak, what will be the location of the peak and what will be the value of the peak right.

Now, this in this particular location in that small band what is important? The most important part over there is your friction can no longer be taken to be equal to 0 because it is a region very very close to the wall right. It is a region very very close to the wall where the shear is very very important right. So, there the friction cannot be neglected right. What can be neglected though in that particular region is the inertia, because it is a region very close to the wall right. So, by the concept of your boundary layer and whatever you have learnt in the course of this particular course it is basically the friction in that particular layer which is being which is balancing what we call the buoyancy.

So, friction and buoyancy basically balance each other in a layer which is very very close to the wall in a very thin layer which is called the delta v, in which the buoyancy driven fluid is restrained by the viscosity of the wall, that is essentially in a nutshell what friction balancing buoyancy exactly means right. So, there if we now apply our scaling argument, in that particular region, in delta v in that region if we apply the scaling that is the scaling that we have right that is buoyancy and this is friction right.

So, from there you can devise your scale will be something like this right where this grh this parameter that you are actually seeing is basically Rayleigh number divided by Prandtl number. In other words it is written as g beta delta T H cube divided by gamma square, in other words this is called the Grashof number right. So, this is called the Grashof number right. In other words the delta v that layered thickness that we have found out this second thickness is given by once again H into the Grashof number, Grashof number is Rayleigh number and Prandtl number.

So, once again you see that a scaling is still Rayleigh number H into Rayleigh number to the power of minus one-fourth and it has been compressed or changed by factor involving the Prandtl number. Once again that particular as I told you earlier it is a stretching or a compression term that is what Prandtl number is right. So, you have a scale using that you are just compressing it like this or stretching it out like this that is what you are doing with the Prandtl number by dividing it or multiplying it right and that is exactly what we have done over here alright. So, there are 3 numbers that emerges out of this discussion, Rayleigh number which is valid for high Prandtl number fluid analysis right.

Then for low Prandtl number fluid analysis there are two further numbers that comes into the picture, one is the Boussinesq number which determines what we call the thickness of the temperature boundary layer the first length scale of the problem. The second length scale which is the thin layer in which the velocity basically peaks that is taken care of by the Grashof number which is nothing but the Rayleigh number divided by Prandtl number right. So, it is a once again a stretching term that we have applied over here. So, naturally before we end this particular lecture is basically what will the velocity profile look like.

So, it will peak somewhere there and then it will slowly kind of decay like this something like that it will peak somewhere there. So, this is your velocity scale and that location of that delta v is given by the Grashof number. So, this is, this has been our grand findings. So, to say and of course, your delta v by delta T like we wrote before. So, once again Prandtl number to the power of half and this time it is less than 1, right. So, clear and up to this particular part. Now, we will show some observations based on this in the form of a table, but up to this particular point it is clear three numbers Rayleigh number, Boussinesq number and the Grashof number.

Now, in many of the books this will be represented that rather than stating these numbers as a ratio of forces. For example, in this case Grashof number you can see from the definition it is a balance between the friction and the buoyancy force correct because that is how we got it in the first place right. Similarly in Boussinesq number and Rayleigh number you can just look at the scale arguments right. But in some cases people look at it like what we call slenderness ratio; that means, the ratio of the boundary layer thickness versus the natural length scale of the problem right.

So, the ratio of those two is what actually gives you that is what the significance of these numbers are. But in many cases you will find the other interpretation that is because it is a ratio of the two scales, I mean either it the two forces basically buoyancy friction, buoyancy inertia and things like that. So, it is just the balance between all these forces.

So, in the next class what we will do, we will start with the observations and then look at the integral formulations in a little bit more details right. So, that was what we are going to do.