Convective Heat Transfer

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Lecture – 03
Governing equations II – Energy Conservation

So, last class we laid down the foundations that, how the internal energy I mean the control volume approach? And, how it can be used to basically cast the conservation of energy?

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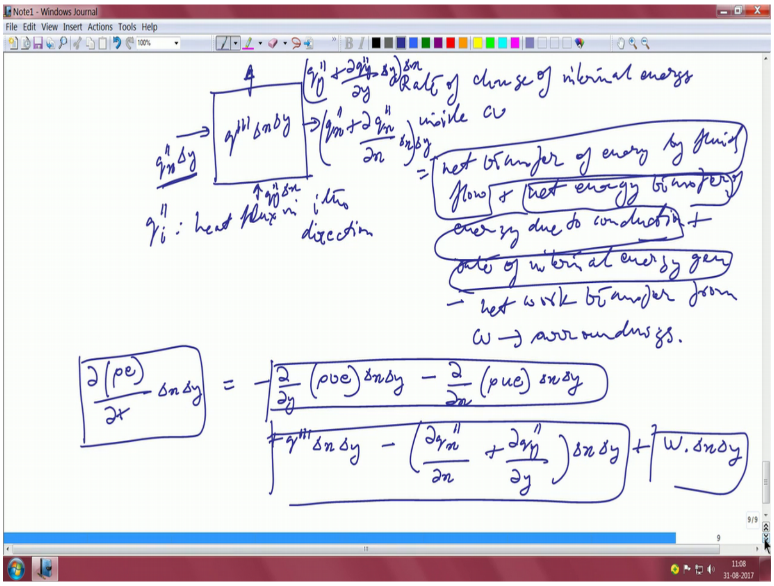

Now, if you look at this particular control volume. Now we want to cast it in the form of heat first or the heat fluxes. So, let us write the conservation equation in a slightly more rational form, the rate of change of internal energy, energy inside control volume is equal to net transfer of energy by fluid flow right.

Which is which we covered in the last class, that that was the fluid flow; that means, energy that is brought in and out, plus the net trans net energy transfer due to conduction energy due to conduction. Conduction means it is diffusion that if how the energy will be transferred, plus rate of internal energy generation. Now this can be important what we mean, but within the control volume say for example, we have some source of heat which is generating energy. So, that is also possible and then minus the network transfer from cv to surroundings.

So, we basically in the previous slide, if you or in the previous class, what we did we you saw that this was the expression that we laid down. So, this was basically the energy which was transported due to flow right. So, we covered one basis of this right so, that is the first thing that we covered that this is due to flow what is the energy that is transferred right. Now what is the net energy that is transferred due to conduction, let us look at that particular part of the problem. So, here it is q x x into delta y, where q whatever it is qi is basically the heat flux, flux in ith direction got it right, and it is leaving the control volume with qx double prime plus dq x prime divided by dx into delta x, this entire thing is multiplied by delta y got it. So, that is the other term.

So, similarly you have q y into delta x, whatever is coming out on the other side of the control volume, q y double prime y delta q y double prime by dy in sorry into delta y 2 delta x all right, it is a little bit messy the way, but your thinking can still identify that. So, this is basically and there will be heat generation within the control volume. So, that is given by that. So, that is this we have covered this term now, this entire thing plus, what is the rate of internal energy heat generation got it. So, the generation or dissipation it can be a sink also. So, depending on it can be positive or negative right.

So, positive can be positive it put a plus sign, negative can be negative. So, we understood. So, this is the basically the heat flux in the x direction, this is the heat flux in the y direction, heat flux in the x direction comes with it the you once again do the Taylor series expansion, and you got that. There is similarly the heat flux in the y direction you once again do the Taylor series expansion and get that all right.

So, entire thing if you now put it together, what it will look like it will look like rho e delta t delta x into delta y right, that is the rate of change of internal energy within the control volume, d by dy rho ve into delta x into delta y all right, that is what we got from the net transfer of energy by fluid flow, all right minus and I have taken into account all the all the negatives; that means, you basically simplify the form I am not showing the steps for the simplification that ideally you should practice.

Plus, q triple prime which is basically the rate of generation on dissipation, minus dq x double prime by x dq y double prime by y into delta x into delta y, plus there is some work term which is given by delta x into delta y right. So, the work term is whatever the work that is done to the surroundings, or the surrounding does work on the on the body.

So, this is the full expression, we have just assembled all the terms now these came from the flow, this came from the conduction part, this is basically the work part, and this is the rate of change of internal energy within the control volume all right, up to this I think it should look pretty familiar all right.

Now, we can go on simplifying the thing a little bit more.

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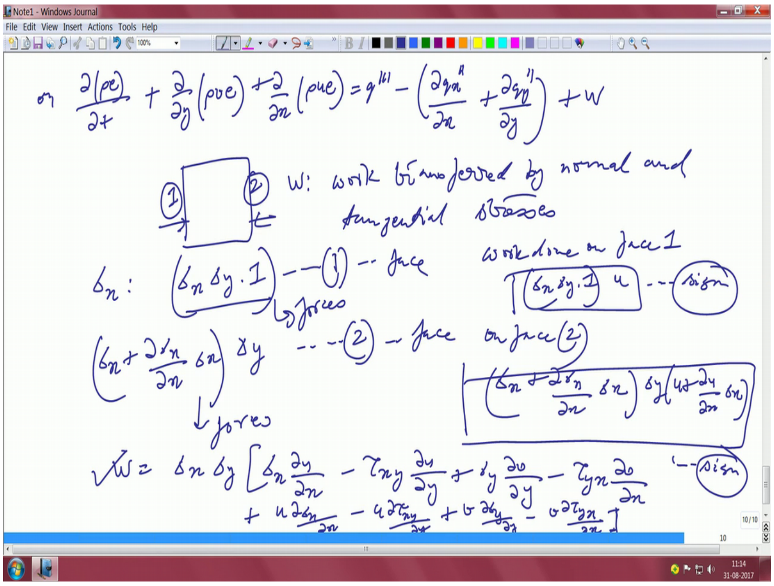

So, d rho e by dt plus, rho v e plus, d by dx rho ue is equal to, q triple prime minus, d q double prime dx double prime dy plus, this W all right. So, I have got rid of the delta x and the delta y kind of term all right. So, the work what is the nature of the work let us look because that W is something that we need to have a handle on right, but what is that W now if you take the control volume together all right. So, the work transferred by normal and tangential stresses right, what else the work can be, it will be the work that will be covered by the normal and the tangential stresses, these are the 2 forces that are acting on it so, these are the ones that should be doing the work all right.

So, the work done by sigma x sigma x is basically the normal stress that needs to be evaluated all right. So, that work done it should be something like sigma x into delta y into one all right. So, that will be the work done. So, similarly the so, this is the value of the work that will be done by the normal stress. So, similarly if this is phase one this is the work that is done, if this is phase 2 of the control volume, what will be the work done on that particular phase that will be sigma x plus, d sigma by d x into delta x right.

So, that is the other part of the thing, and that it will do a work sorry, delta y that should be the other part of the work that will be done on phase 2, this is 2 this is one, this is the face slow all right. So, one of the work. So, this will be the this will be the other part of the work. So now, is this going to be going to be all or is there a unit problem that we have over here. So, as you can see that, though this gives you the force this is basically the force right this is not the work right. So, this is the force that is acting on phase, this is the force that is acting on the other phase right. So, how do you get from force to work, you basically have to multiply it with some kind of a velocity scale in this particular case right.

So, in this particular case the velocity scale we have a ready velocity scale which is u right. So, for this particular work the total work done on phase one right, will be sigma x into delta y into one, multiplied by u right, on phase 2 all right the total work will be sigma x plus, d sigma x by dx into delta x, into delta y, all right he multiplied by u plus, d u by d u dx into delta x correct. So, this is the work that is done on phase 2 am I right. So, these are the forces this is basically the work correct.

So, once these work paradigms are kind of leveled out, now what we can do similarly there will be expressions for shear stresses as well all right, these are for the normal stresses similarly there will be expressions for the shear stresses as well not going to go through the whole thing, but let us put the final expression for W that we have will be, sigma x into sigma y, this is all the terms combined once again you should take the effort of going through each and every term and try to work out that how this is coming, plus there is a long expression, d y got it there is a full expression for the work all right, this is the full expression for the work that is done on the thing. Now you should also spend some time thinking that what will be the sign of these quantities, I have given you the final expression, but this is left as an exercise to the to all the participants, that what will be the sin of this will this be positive or negative.

I have given you the final expression. So, you can see that one to sin ultimate sig will be if you work through the steps, but then you have to understand get a feel that why one should be positive on one should be negative, or is it both should be positive or both will be negative, that you have to kind of find out from this. Now you have a huge expression for this you have a huge expression, but fortunately on unfortunately you will find, that most of these terms are basically it does not make much of a contribution, but it is important to know what is the nature of these terms though. So, that is what we have done.

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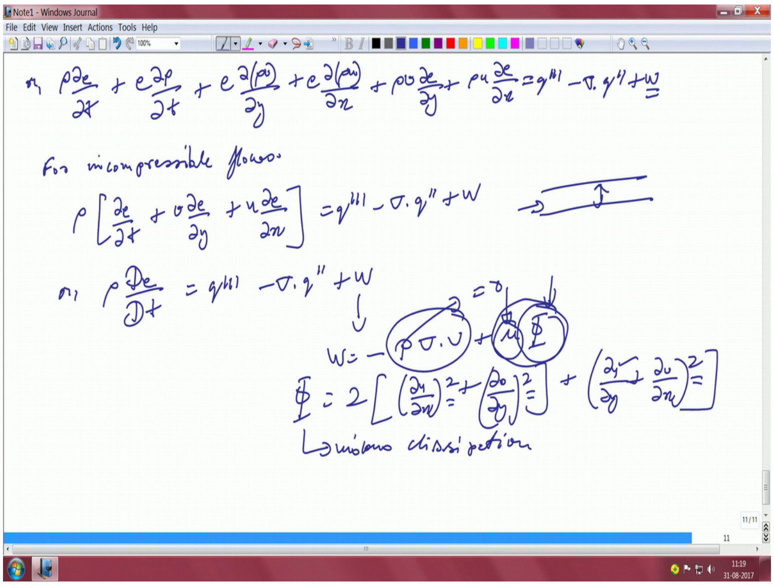

Now going back to the original formulation, and we will keep W as these we are just breaking up the whole expression.

Let us put this properly. So, that you can see, this is I have written W, W is that expression that we wrote in the previous this, what we have done is that we have basically broken up the whole expression into a series of individual quantities, unfold it once again. Now if the flow is incompressible that is what our main conjecture was and what I said that most of the course will be on incompressible flows, for incompressible flows. So, that is the expression that you get, or if you can write it in a more compact form that is W all right.

So, W is that big expression that we wrote, you can technically show that the W if you work through the math once again in the previous page what we showed, it is actually written by p into the divergence of v plus, a quantity called mu into phi, what is phi? Phi is basically 2 into du dx squared plus dv dy square plus, du dy plus, dv dx square, got it this is all can be derived from the previous expression that you had you just have to go through the steps, this is what we called viscous dissipation. Now for an incompressible flow, this first term will basically go to 0 all right, but the first term will go to 0, but the viscous dissipation term depending on the flow that you are dealing, with may or may not actually survive.

So, may or may not that is the key important part right, that if you are dealing with flow say for example, in very constrained geometries; that means, when there is a large say this is a channel there is a large flux, or a high flow rate through a very constrained channel high aspect ratio channels viscous dissipation can become a very key quantity. So, especially in lubrication and other places also where the viscosity also, there are 2 parameters as you can see one is viscosity can be very large that can lead to a high level of viscous dissipation all right, that expression can actually go up, or your velocity scales has to have a very strong correlation, and it is always the squared quantity that is that you can see over here right, all these quantities are squared; that means, this is always positive, phi is always positive all right, because it is always squared no matter what is the gradient.

It is always squared of the gradient all right. So, it is always a positive quantity that is the first thing and second diffuse look at these are basically all the slopes this is the slope of velocities in different directions all right. So, is the slope of you velocity and v velocity in different directions. So, if the velocity gradients are very sharp; that means, there is suddenly a sharp decay of velocity or a sharp rise in velocity, you can actually increase this phi, but remember it is a squared quantity.

So, the effect is actually kind of minimized, but if you have a sharp gradient anywhere in the flow field, you can actually generate a lot of this work basically through work you can generate this viscous dissipation, if your viscosity is high, it can also have effectively the same effect; that means, if you try to push a very viscous fluid, through a very narrow opening you can actually generate a lot of heat. This is sometimes what happens in kind of lubrication in bearings and other things things can get hot also. So, that is the expression for that let us look at now the next one.

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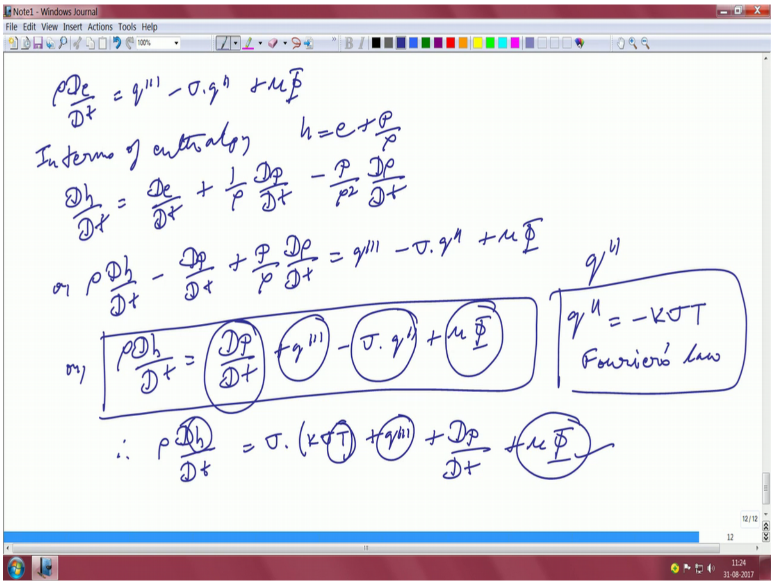

Now, we have this expression the combined expression, into phi so, you now in terms of enthalpy. So, enthalpy is some quantity that we are very familiar with so, enthalpy is basically e plus p by rho all right. So, Dh by Dt all right m will be De by Dt plus 1 over rho Dp by Dt minusm P by rho square D rho by Dt, see if you understood all right got it. So, this is just a simple expansion that is what we did over here all right, or in other words rho Dh by Dt minus, Dp by Dt plus, P by rho D rho by Dt is equal to q triple prime, minus delta double prime, plus mu to whatever is the viscous dissipation term all right.

Now, So, Dh by Dt you can just take this Dp by Dt you can take it to the other side, plus q triple prime minus u double prime all right and we; obviously, know that since it is incompressible all right, the rho is not varying all right. So, that other term goes to 0 that D rho by Dt term that is there that goes to basically 0 correct. So, based on this this is the expression that you are going to get all right. So, this cast is in terms of the enthalpy all right. So, the enthalpy this is the generation term this is basically the diffusion of the conduction term. So, to say this is basically the viscous dissipation term, this is like the pressure, pressure term that we have all right, now what we can do to get read of this q double prime, there is a readymade avenue right in conduction literature if you look at it, it is called the full years law, what it gives is basically q double prime is basically minus k into a gradient of temperature all right.

So, this is in other words is known as the Fourier's law correct. So, therefore, what we have is Dh by Dt, K Dt q triple prime all right, all we have done is that we have replaced now q double prime as a heat flux with the equivalent temperature field, coming from where the Fourier's law of heat conduction, this you already know and there is nothing more to say about that all right, but still if you see if you look at this equation there is an interesting problem associated with it, we still have h which is hovering around, there is t over here of course, these terms are still kind of dependent on the velocity field as you can see all right, here the velocity field is already hidden in the material derivative all right, because Dh by Dt is nothing by u da d I mean you can expand it in the full form, and you can write it that there is a velocity component of the velocity components are already hidden here all right.

So, but still that h needs to be cast in terms of temperature somehow all right, otherwise this does not make much of a sense right because enthalpy is not something that you can solve that easily.

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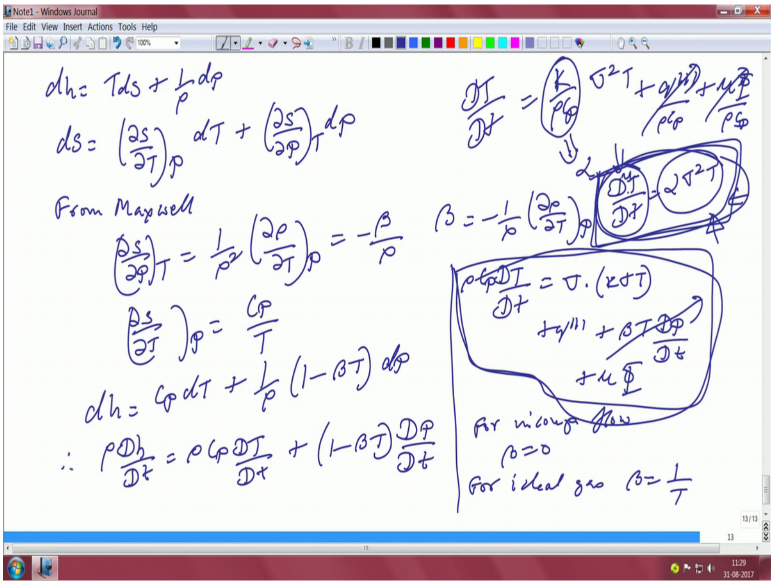

So, what do you already have d h equal to, tds plus 1 by rho dp this is already known right you already know this correct. Now d s which is the entha which is the entropy it can be written in this particular form if you are not familiar please look at the Maxwell relations. So, this is how it is written, now from Maxwell these are the Maxwell relations extremely powerful. So, d s by dP T is actually given as, nothing but beta by rho where beta is basically 1 over rho into d rho by dT at constant pressure all right.

Similarly, d s by dT at P is given by Cp by T all right, this is Maxwell relation all right. So, d h therefore, can be written as because we know already we wrote this particular expression in this particular way. So, as you can see is therefore, d h can be written if you substitute quantity by quantity substitutions, if you do because d s we have already cast it in this particular form, is basically Cp into dT plus 1 over rho 1 minus BT into dp all right. Therefore, rho Dh Dt the rho Cp DT by Dt plus 1 minus, beta T into Dp by Dt all right, that is what you will get, now we are in a perfect situation now to cast it in terms of temperature, now which will give you rho Cp in terms of temperature.

Now, for incompressible flow beta is equal to 0, for ideal gas; however, beta is equal to 1 over T all right. So, for incompressible flow whenever we say that beta equal to 0, that is because density is not changing so, naturally the beta will be equal to 0. So, as you can see that moment that particular thing happens this entire problem now is a highly tractable problem that we see over here, if beta is equal to 0. So, therefore, this term will drop out you will get the very familiar expression DT by Dt is equal to 1 over rho Cp, if K is constant you can take K out. So, this will be given in terms of the Laplace of temperature, and if the heat generation term either it can be 0 or it can stay plus mu into phi, both divided by rho Cp correct.

Now, K by rho Cp is basically given as alpha, which is basically nothing but the thermal diffusivity of the system all right. So, it is basically DT by Dt equal to alpha into del square by T all right. Now in most of the cases you will find that when there is no heat generation that term goes off, when there is no viscous dissipation effect that term also goes off. So, this is the most familiar expression that you have kind of familiar with which is basically this is the convective term, this is basically the diffusion term all right. So, as you can see we started with the conservation of energy equation, which slowly took off certain terms, and we showed that the final most simplistic version is given by this, now there are other versions which are given in this particular form this is the most generic type of things.

But you can take off quantities depending on your requirement your flow field, it does not say that this is the universal relationship, this is not the universal relationship all right, the universal relationship you have to start a lot earlier right here, there are a lot of assumptions that we have made before we got to the final form. So, that is the final form that you will normally encountered in this particular course and that is the expression that we will use, as you can see u or the momentum feeds in into this material derivative all right.

So, you need to know you need to solve the u or the velocity field to compute what we are going to be the temperature field. So, in the next class what we are going to do, now that we are done with the conservation equations we are going to look at the first set of canonical problems which will be the flat plate.

Thank you.