Convective Heat Transfer
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Lecture – 28
Introduction to external natural convection

The welcome our lecture from now onwards will focus on natural convection. So, we have spent quite a bit of time in understanding what we call the forced convection and we have seen both internal as well as external force convection. In this particular class, we are going to see how natural convection actually works and there are some fundamental and subtle differences between natural and forced convection to begin with.

Now, in any fluid flow that we have seen the fluid dynamic systems are usually dissipative in nature, because of the presence of viscosity and the fluid systems, it requires some kind of a mechanism to drive the fluid motion. Now in the case of forced convection it was usually some external influence like for example, take the flow through a duct there is an internal forced convection. What was driving the flow was basically a pressure head, you had a pressure head or you had a pressure gradient which basically drove the flow through those ducts.

That is why flow through that internal pipe was maintained by that pressure gradient you remove the pressure gradient the flow basically stops after some time and taking into account that that everything settles down the flow should naturally stop. At least in the in the in the fully developed and in the other regions now in the case of external forced convection like for example, the most common example that we saw was the flat plate. Now, in this flat plate what happens is that it is just the reverse problem say for example, you have some kind of you know some kind of a motion of a body through water. Say for example, it is a ship that is moving through water. That is akin to a flow over an obstacle in that case, if the if you assume that the object is very slender it can resemble like a flat plate, it can be any other obstacle as well it can come at any angle. We have already seen the Falkner skan class of solutions.

In that case also in order to push that object you need some kind of a mechanism. In this case if it is a ship or a boat it is basically the propeller or the engine that is basically driving the body through the fluid medium. It can be in the case of an aircraft in that case the fluid will be air in in all these cases there are some energy that you are spending in order to drive the flow and one other interesting thing that we saw in the case of forced convection, was that the momentum and the energy equation where essentially decoupled from each other.

Decoupled not in a fully decoupled way the energy equation still requires a momentum equation to be known a priori or to be solved a priori you need to know the velocity field, in order to solve the heat transfer part of it, but the momentum equation; obviously, did not depend on the energy equation at all unless of course, you are dealing with something like a chemically reacting flow or in flows where the density and properties like that actually starts to vary.

Now, in most of the flows that we did we treated the fluid as an incompressible fluid. Remember the sequence of steps we first always solve the fluid dynamic part of it; that means, we solved the flow velocity which is essentially what you have learnt in your any fluid dynamics course any advance fluid dynamics course and then what we use do you use that velocity information into the heat transfer or energy equations to solve the rest of the problem. That was the standard methodology there are variations, but most of the time we used this kind of a methodology to solve we never considered that, the density and other things are kind of temperature dependent. There was a coupling.

But things are going to change a little bit, when we are going to study natural convection. It is because natural convection is a is a slightly different beast to begin with. So, in forced convection we saw that everything was driven even though we did not explicitly specify it. Then when we said that there is a flow over a flat plate, we did not specify what creates that flow over a flat plate. It is understood that there is some mechanism by which either the plate is moving or the flow is moving.

Basically, there is some mechanism some propeller some engine something which is basically driving either the flow or the flat plate or whatever and similarly in the case of a pipe flow we just mentioned that the flow is entering through the pipe under a pressure gradient. we did not mention and how that pressure gradient was explicitly created.

it could be created in multiple ways, let us not bother about those things, but it is the hallmark of a forced convection mechanism. Something is forcing the flow through it. Because if you do not force the flow in those kind of cases, because of that general dissipative nature of the whole system, the flow will slowly attenuate and come to motionless. Say for example, in the pipe if you stop, whatever is creating the pressure head there may be a pump, which may be attached, which is basically driving the flow like the pump that you use to draw water from the reservoir to your overhead tank in your houses.

If you want moment you stop that flow moment you stop that pump what happens the flow they basically attenuates very quickly. A similar thing happens in the case of a flat plate also. If you stop the engine of the ship or if you stop moving the plate it will just basically be a plate. You know will be reservoir say for example, well there is no flow being created, but that is not what you normally experience always you might ask the question oh no there are other instances also like for example, I take if you if this is your room say for example,

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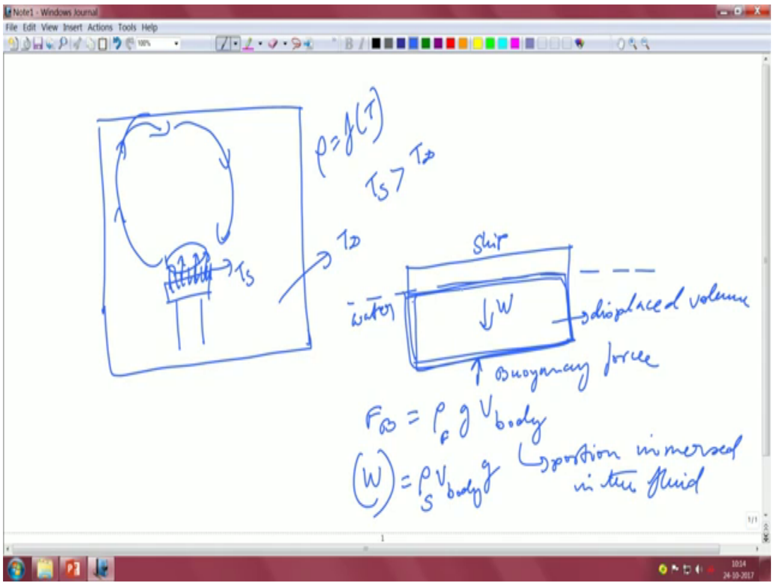

And you have say a stove situated in the middle of the room this is the canonical kind of a problem.

this this stove is actually heated at whatever temperatures, let us not get into those kind of things and the temperature of the room initially was at some temperature. So, naturally what happens normally what people experience is that here, the air becomes heated locally. Because of the heat that is given out by this stove because it is at a higher temperature than ambient. This is stove this is T infinity t s is greater than T infinity to write it in a more formal way what happens to the air? The air basically gets heated layer by layer and their density in usually these fluids is a function of temperature.

As we know that air when it gets heated becomes lighter. You know this from your climate knowledge, that hot air becomes lighter that is how your hot balloon actually goes up in air. What happens is that as soon as that happens the air starts to move up. What it does is that as it moves up it displaces that the air on the top the cold air on the top. You tend to generate what we call a recirculation pattern. The cold air comes down gets heated moves up and then this entire sequence kind of continues this kind of a motion that is being created would you call this convection I think we should call this convection as well.

But here the convection mechanism is brought about by density. Spy subtle changes in density and because of that you are creating a motion, which is naturally created because of the presence of a heated source, within the room now this is something that you have commonly seen in your weather patterns. Very hot air gets heated it goes up and then you create what we call a convection, but here the driving mechanism is not a forcing not something that you are actually using, it is basically the density gradient or that change of density with temperature that is actually creating the whole thing.

Whenever you place an hot object anywhere in a room say for example, it will actually create this kind of a pattern. It may be weak, it may be strong, that depends on a lot of things what is the temperature of that body? What is the condition of the ambient? And all other factors also the fluid properties to a certain extent you can have a whole gamut of things, that in that can actually affect things over here.

What is this motion called? If I call that this is likely to be a buoyancy driven kind of an effect. It is a buoyancy, let us just recap that what is buoyancy do you recall the famous archimedes principle that, when you have an object say for example, this is a ship. Ship is the most important I mean easily understandable thing, this is the water level, this is the part of the ship that is actually submerged in the got it. Naturally you know that there is a weight of the ship and then there is a buoyancy force and this particular section that we have given over here, this is nothing but the displaced volume. Correct is not that so? That is kind of the displaced volume.

What is the magnitude of the buoyancy force? If somebody asks you the magnitude of the buoyancy force is basically the weight of the fluid that is displaced by the body buoyancy force if I turn this as FB what will that buoyancy force be buoyancy force will be the rho of the fluid, that it has displaced into g, into the volume of the body whatever is the submerged volume of the body. That is the body proportion of the volume of the body, that is immersed in the fluid. Portion immersed in the fluid. That is what it is, that is the force.

On the other hand, the body has got a certain amount of weight this ship must be having some amount of weight. That some amount of weight will be nothing but the corresponding solid, whatever is a solid density into the Vbody into g that is the weight that is the weight of the of the body the weight of that particular section correct. The Fnet which is basically nothing but w minus F buoyancy.

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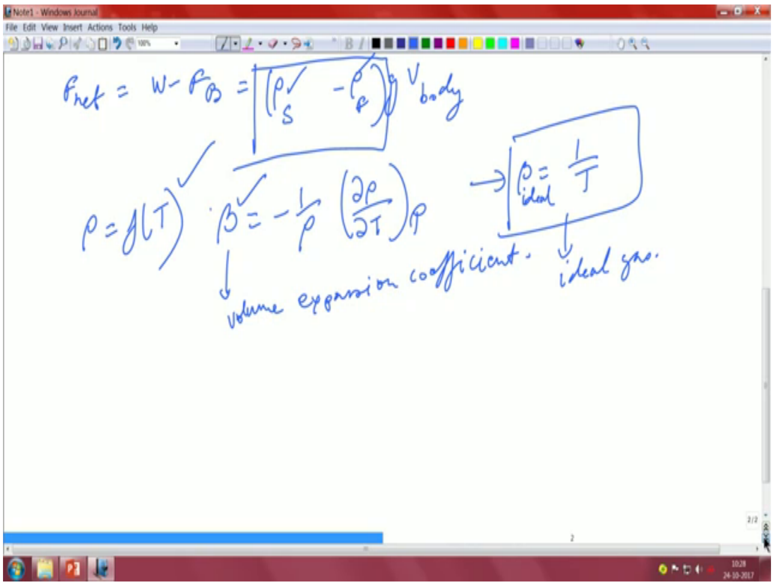

$$F_{net} = W - F_B = \boxed{(\rho_s - \rho_f)\, g}\, V_{body}$$

$$\rho = f(T) \quad \beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \rightarrow \boxed{\beta_{ideal} = \frac{1}{T}}$$

volume expansion coefficient.

ideal gas.

Right that is the total force that is nothing but FBody minus f not FBody, but FS minus F fluid into g into Vbody. In a way all it says that the net force is basically proportional to the difference in the densities of the fluid and the body. That is what it is, this is what is the famous archimedes principle, this is you have heard the story for a long time.

It is the difference in density of the 2 bodies; that means, the volume that is displaced, the fluid that is displacing the volume or the body is displacing some volume of fluid. What is the density of that? Versus the density of the body? If these 2 densities are basically the same, then there is of course, no net force that is being created. Densities are usually functions of temperature. Densities are usually functions of temperature, which is the situation in our case that is the variation of the density of a fluid with temperature at constant pressure can be expressed by something called beta. Beta is given as 1 over rho d rho by d t at constant pressure.

So, beta is nothing but what is called the volume expansion coefficient. So, what is it? It is a rate of change of density with pressure at constant pressure, is a rate of change of density with temperature at constant pressure. So, that is what it is it can be shown for an ideal gas this beta is basically equal to 1 over T for an ideal gas, that is what it is that is. Only valid for ideal gas we will see how we use this beta in a bit.

Now that we have established that there is something called natural convection and natural convection does happen in a in a certain way, because of the change of density and we already have seen that the difference in density as can be seen from the archimedes principle. Imagine in that particular case instead of S and F what we have is basically the same fluid, but at 2 different temperatures correct in the case of archimedes principle of course, is a ship and the fluid. You can there is a very stark contrast in the density between a solid object and the and the fluid a concerned, but imagine in the case, of the example that I showed you that there is a stove in the middle of the room. The fluid volume that is being displaced, the density change is happening because of temperature.

In that particular case rho S and rho F are basically the same fluid, but at 2 different temperatures and because the density is a function of temperature because there is something called a volumetric expansion coefficient, the density of the 2 fluids will be this will be different as a result of that you are supposed to create a net force. That is what it is. This is the thing that we are going to culture during the course of the next few lectures.

You understand. It is not a stark contrast like a solid and a liquid that is just to give you an example that the density difference actually brings about the change. In a net force that is acting on any parcel of fluid. Next what we do is that now that we have established this.

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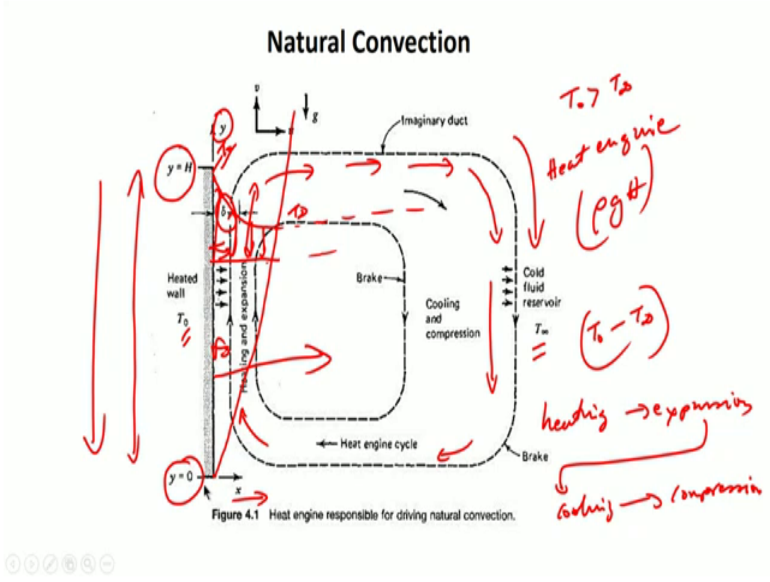

Natural Convection

Figure 4.1 Heat engine responsible for driving natural convection.

Let us look at the PPT and let us look at the first example of how to analyze natural convection? Take the example of this heated wall over here. This particular wall that you have over here, is actually heated. It is at a temperature T naught whereas, the ambient is at a temperature T infinity T naught being greater than T infinity it can be the other way around also we will see what; that means the axis here, this is the x axis this is important the direction of perpendicular to the wall is x. Where, the direction along the wall is y you can see it over here the height of the plate is h. This wall is heated there is a cold reservoir of fluid to the side of it. What happens is that. The wall transfers heat to the adjacent fluid as you can see what is given as delta t you will come across what is delta t and things like that in a little bit, but it transfers the heated walls transfers the heat to the adjacent fluid. Adjacent fluid gets heated it becomes lighter it moves up as it moves up forget about this imaginary duct that is that you do not have to consider. You it moves up and then it displaces the cold fluid, then as it goes up it becomes colder it drops it drops it is density increases, basically sinks and then it again completes the cycle.

The cold reservoir fluid is displaced downwards and the difference in temperature between this T naught minus T infinity is basically what drives, this entire convection pattern. What happens the air becomes or whatever is the fluid medium usually it is air. In most of the common application cases it gets heated, it rises, it becomes less dense, that is it becomes lighter and it rises. At the same time the cold reservoir fluid is displaced. It basically drives that particular pattern that we see over here. This particular pattern, this natural convection loop, that you actually create, is pretty much like a heat engine it is like a heat engine. How is it like a heat engine. Let us look at the steps from the heat engine. The packet of fluid is imagine a packet of fluid that packet of fluid is first heated by the wall it expands that it rises along the along the wall rises it rises to a lower pressure because from this to this, there is a hydrostatic pressure gradient that you have. As you know that as we go lower within a volume of fluid; that means, it is a reservoir of fluid, your hydrostatic pressure actually goes up because hydrostatic pressure is basically rhoGH at the bottom of the plate it will be rhoGH at the top it will be whatever is the value.

The point is that this is almost very similar, that you take a heated plate. Imagine it like this and you are immersing it in a reservoir of cold fluid. Understood? So, it is akin to your stove problem. It is like a like a heated plate that you are inserting directly inside a reservoir of it is a huge reservoir, of high very high thermal mass and you are inserting that particular plate inside that reservoir. That is exactly what you have d1 over here.

As the fluid packet gets heated and it rises along the wall what happens is that it goes to a lower hydrostatic pressure. It goes to a lower hydrostatic pressure you know although in a tank as you go down, your pressure increases for example, in scuba diving when people actually do scuba diving, what happens you know that they go actually to a substantial depth. Where there is high pressure and they ask you to slowly come up. Otherwise, you develop all this pressure syndromes. It is very akin to that, but you are moving up.

The fluid being lighter actually moves up more towards the more towards a substantially larger height. Later along the down flowing branch, which is basically along this branch of this of this particular cycle, the fluid packet is cooled by the reservoir because it is a cold reservoir after all it is cooled by the reservoir and it gets compressed. Compressed means this density increases compression is like a density increase phenomena and it reaches the depth of this.

It sinks, it compresses density increases it sinks. It is like a pebble it just sinks. It sinks in a relative sense and it is it is and it is able to maintain this kind of a circuit that we have shown over here. Like a heat engine it is heating leads to expansion which further leads to cooling and then compression. This loop is equivalent to the cycle, executed by the working fluid in a heat engine something similar to that.

You can imagine it like this. The fundamental should be very clear. There is a small region, in which the heat is being transferred that region is basically that delta T that we mentioned. Where the heat is transferred from the wall to an adjoining layer of fluid moves up, gets cooled, sinks comes back it completes that particular kind of a cycle. Got it? This is kind of pretty interesting in a natural convection case.

Now, in a natural convection case, if you are supposed to see that how will there be the concept of a boundary layer? In a natural convection, you are because there is a wall there is a fluid packet very in close to it. Of course, as you move in this particular direction you arrive at a situation where the fluid is no longer heated. There is a gradual change in the temperature of the fluid from T naught all the way up to T infinity as we move perpendicular to the wall.

This is very similar to the case of a forced convection where if the plate was heated and if you move up perpendicular to the plate. What happens is that your temperature? Just takes a takes a beating. You go to back to the free stream temperature here of course, the free stream temperature is basically, there is no free stream here is basically the reservoir temperature and that reservoir temperature is given by T infinity. So, it is very similar akin to your forced convection problem.

So, naturally you should expect that there will be a boundary layer that will be generated and how the boundary layer will where will the boundary layer start? Of course, it will start from y equal to 0 in this particular part it has to start from there the boundary layer. It has to start and this boundary layer is going to develop in a certain way. That was what I would say will be the boundary layer, this is how the boundary layer should grow correct.

Now, whether there will be a velocity boundary layer and a temperature boundary layer? Now that is an interesting question that whether the velocity and the temperature boundary layers will be separate? Whether they will be the same? Whether the prandtl number is an important parameter over here or not? Those we will see very shortly, but as of now there will be a boundary layer of some sort. Because of the temperature because you are transferring temperature from a heated wall to the to the layer of adjoining it.

The temperature, you can pretty much foresee that the temperature will be high here and it will gradually come down. To some level correct outside the boundary layer the temperature will be the same as T infinity. This is your T naught that is how the temperature profile should loZ like correct. This is basically the temperature how about the velocity? Now without even analyzing anything, if we assume that the velocity and the temperature boundary layers are more or less the same we do not know whether they are the same or not?

In that particular case, what you can imagine? That outside this boundary layer probably and we will see that there are some mistakes, in what I am going to say now and I am going to correct it as we move on to the to the remaining part of the lectures. I think it is a better.

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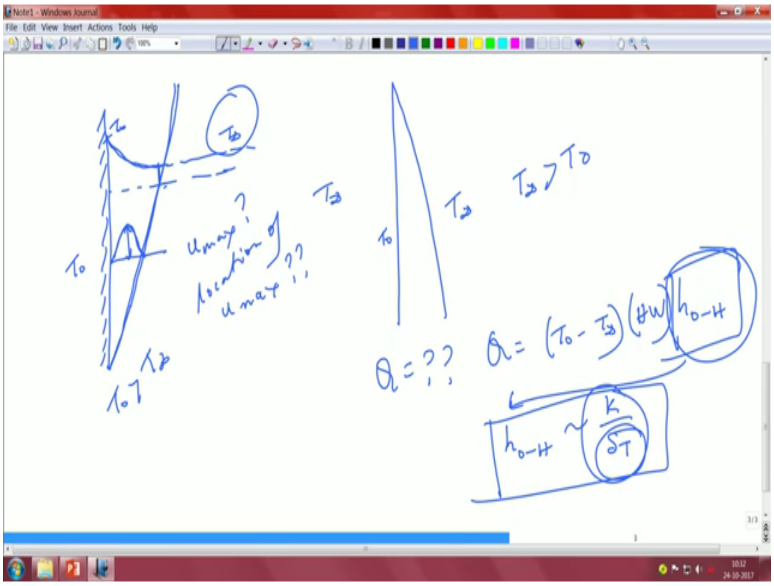

Way to basically go and look at the journal part. Here for example, this is the wall, it has got a temperature T naught and this is a temperature T infinity. I am assuming that the temperature boundary layer is moving up like this forget about my drawing it is not wiggly, it is just a monotonic increase.

We already said that if this is the temperature how it should come down. This is the point should come down and become the same as the ambient. This part I think everybody would agree, but the velocity is a little bit more dicey than that and I will make a statement over here as I said I am going to correct it. In the subsequent presentations what we can see over here what will happen to the velocity? What is the velocity at the wall? Velocity at the wall should be 0. And outside the temperature boundary layer I do not see that why there should be an existence of any velocity. Common sense because, the driving mechanism is gone. Temperature difference was the driving mechanism for the density when the temperature equilibrates; that means, there is no density change per say now there could be some added mass effect; that means, some part of the fluid could be still drag like for example, if you are pushing a mass of fluid you will also drag a little bit of mass of fluid along with you.

That effect can be there, but in principle when there is a flow, when there is that that mechanism which is responsible for the flow, when that is removed; that means, there is no temperature difference beyond this particular case in that particular case what will be the velocity? It should be also 0. The velocity essentially should show something like this. It is 0 on this side 0 on that side. It has to attain some kind of a maxima in between. What that maxima is? What is a location of that maxima? What is that value of that maxima? We do not know. Whatever is this velocity say you call it, u max say for example, that velocity is unknown and the location of u max is also unknown.

But we know that it is something like this can actually happen. That there will be a temperature profile that will develop like this there will be a velocity profile which will look like something like that, artistic rendition of the whole thing and there will be a boundary layer which will start developing like this now if it is a cold fluid it k if it is the reverse mechanism; that means, you would have the boundary layer which will start like that if the if it is cooled; that means, the plate is at a lower temperature than the thing is the same thing. It will still create up that buoyancy driven flow except that boundary layer will start from the other way around.

Whatever it is it does, not matter in this case T naught this is T infinity. T infinity is greater than T naught here of course, T naught is greater than T infinity these are the 2 cases that we have. The key question that we are supposed to answer in this particular case is the heat transfer rate Q. What is the heat transfer rate Q? What is the heat transfer rate Q? And heat transfer rate Q is written as T naught minus T infinity; obviously, and it is also given by whatever is the dimension of the wall H and WH is being the length W being the width which is perpendicular to the board or perpendicular to the screen in this case and what about the heat transfer coefficient from 0 to H. This is the heat transfer coefficient.

Heat transfer coefficient from 0 to H. That heat transfer coefficient we know scales something as k over delta T, this we know already from our old analysis from our flat plate external forced convection boundary layer analysis. We have to answer this question, that this 0 to H heat transfer coefficient is equal to this; that means, it requires us to find what is will be your delta T? Once we find our delta T all the things solves by itself. In the next class what we are going to do we are going to start addressing the problem taking from the navier stokes equation and try to work out and see that how we can arrive at the solution for this particular problem.

Thank you.