Convective Heat Transfer
Prof. Saptarshi Basu
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture – 27
FORCED CONVECTION – TUTORIAL III

So, in this convective forced convection question set. So we are going to look at 4 to 5 problems in this particular set. So, let us look at the first problem, if you consider the first problem is that, there is a Laminar boundary layer formed by the flow of ten degree Celsius water, over a 10 degree Celsius flat plate or a flat wall of length L, essentially this is what the water is doing.

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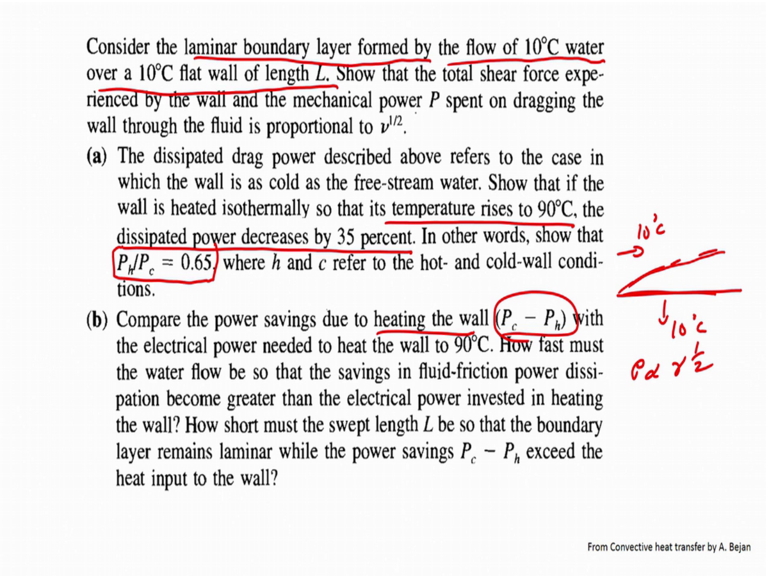
Consider the laminar boundary layer formed by the flow of 10°C water over a 10°C flat wall of length L. Show that the total shear force experienced by the wall and the mechanical power P spent on dragging the wall through the fluid is proportional to $\nu^{1/2}$.

(a) The dissipated drag power described above refers to the case in which the wall is as cold as the free-stream water. Show that if the wall is heated isothermally so that its temperature rises to 90°C, the dissipated power decreases by 35 percent. In other words, show that $P_h/P_c = 0.65$, where h and c refer to the hot- and cold-wall conditions.

(b) Compare the power savings due to heating the wall $(P_c - P_h)$ with the electrical power needed to heat the wall to 90°C. How fast must the water flow be so that the savings in fluid-friction power dissipation become greater than the electrical power invested in heating the wall? How short must the swept length L be so that the boundary layer remains laminar while the power savings $P_c - P_h$ exceed the heat input to the wall?

From Convective heat transfer by A. Bejan

So, this is at 10 degree Celsius the wall is also at 10 degree Celsius show that the total shear force experienced by the wall and the mechanical power is proportional to gamma to the power of half, then the other parts of the questions are that, in this case the wall is of the same temperature as the fluid. So; that means, both are at 10 degree Celsius, but; however, if the wall is heated isothermally. So, that its temperature rises to 90 degree Celsius. The dissipated power decreases by 35 percent or in other words Ph by Pc becomes 0.65 or in other words, the power that is required becomes like 35 percent lower than in the cold condition.

So, we have to show that and then the third part of the question essentially boils down, that you have to compare the power savings, because you are investing some power in heating the wall and then you are of course saving some power because of that because the drag force is actually you are able to reduce because you do not have to have a lot of power now in dragging the plate. So, therefore, you have to compare the two basically you have to compare the electric power that is needed to heat the wall to 90 degree Celsius and you also have to see that whether that electric power that you have invested, must be lower than the power savings due to the wall. So, these are the three things that we need to prove in this particular question.

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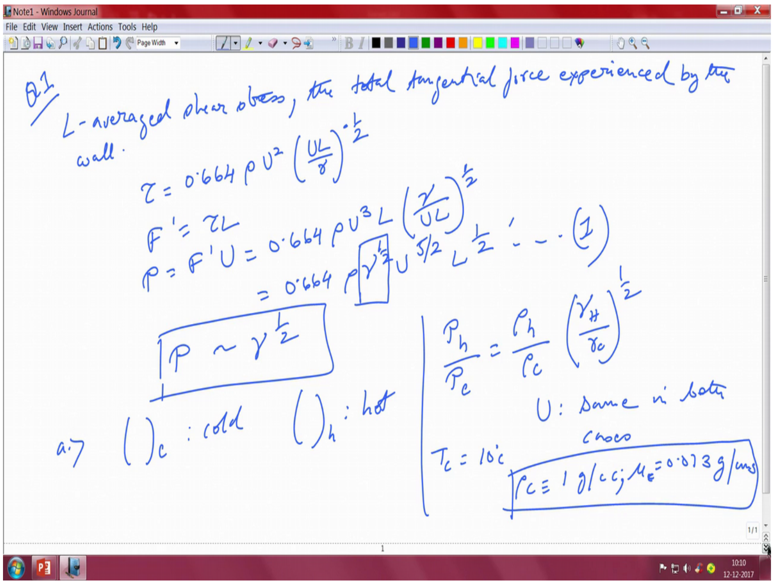

So, let us look at how to solve this. So, question 1. So, let us do the first part of the question that L averaged shear stress, that is the total tangential force experienced by the wall. So; that means, the tau 0.664 rho U squared UL by gamma raised to the power of half, this is the shear stress expression for the shear stress, these are again comes from all the relationships that we did in our class. So, the force is basically tau into L and the power is basically F prime into U. So, in other words is 0.664, rho U cube L, gamma by UL to the power of half. We just inverted the whole thing. This leads to 0.664 rho gamma to the power of half U to the power of 5 by 2, L to the power of half. So, in other words you get a direct dependence of P, it scales as gamma to the power of half. So, that gives you the first part of the question that is relatively easy we know that, it is dependent on root over of the kinematic viscosity.

Now, the a part of the problem, which is the key thing that we want to find out, let us say that this represents cold and this represents hot, this represents cold and this represents hot. So, if you take the ratio of the power between Ph and Pc. What will happen is that, you will get rho H, if you just look at this expression let us call this one. So, if you just look at this expression. So, its rho h by rho c, because the velocity is still the same gamma H by gamma C raised to the power of half, U is the same in both cases. Now, for Tc is equal to 10 degree Celsius. 10 degree Celsius that is cold water, rho c is approximately 1 gram per cc and mu c, because gamma c gamma c is nothing, but mu by rho c. So, mu c is equal to 0.13, these are all from the tables, gram per centimeter second. So, these are the 2 for Tc equal to 10 degree Celsius.

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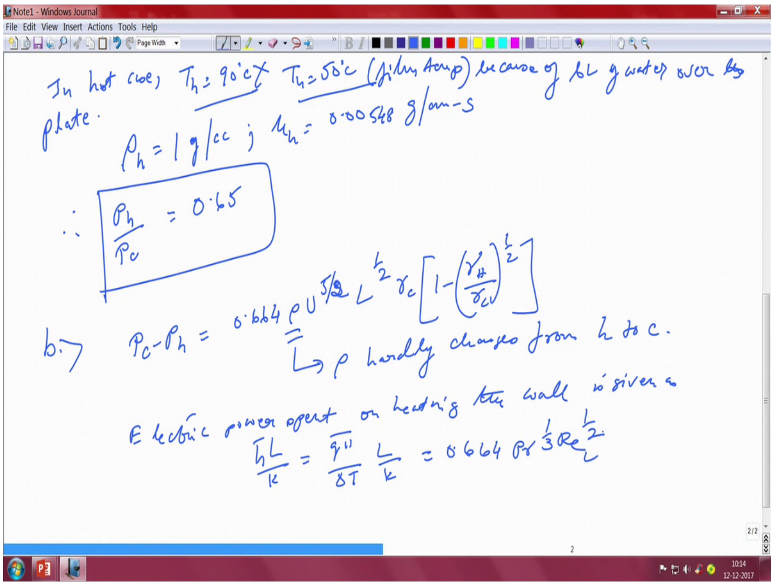

Now, in the hot case the Th, now becomes equal to 90 degree Celsius from the 10 degree Celsius wall. So, water takes place across a boundary layer where the film thickness is basically around 50. So, this is exactly. So, though the hot temperature of the fluid is about 90 degree Celsius, what we are concerned about is a film temperature, which is somewhere in between it is just a linear interpolation between the two between 10 and 90. So, that is the film temperature; that means, the temperature of the boundary layer of water on the plate.

So, this will not be 90, this will become now 50 degree Celsius which is basically the film temperature, because of the boundary layer of water over the plate. So, based on that the rho h becomes equal to 1 gram per cc and mu h becomes 0.00548 gram per centimeter second. So, therefore the power ratio Ph by Pc becomes about 0.65, which is exactly what the question actually said. So, there is a 35 percent power that you actually save just by heating the water and assuming that the film temperature is not 90, about 50 degree Celsius.

So, now question number b is that, we see one thing that water's density really does not change in this question part a, that we did we found that the waters density hardly changes in this particular regime like from 10 degree to 50 degree Celsius there is not much of a change what; however, changes is basically, because of the kinematic viscosity in essence. So, therefore, we have to find out Pc minus Ph which is 0.66 for rho U 5 by 2, sorry y by 21 to the power of half gamma c1 minus gamma h by gamma c raised to the power of half. So, that is the power, that is basically spent we have taken rho to be constant, that is because rho hardly changes from h to c, it hardly changes. So, that gives us a good indication that if rho does not change by much, it is just the kinematic viscosity change or rather the dynamic viscosity change, which is basically creating all the problem.

So, similarly the electric power spent on heating the wall, wall is given as first and foremost h bar l by k, q double prime delta T l by k, which is about 0.664 Prandtl one third Reynolds number l to the power of half. Let us go to the next page.

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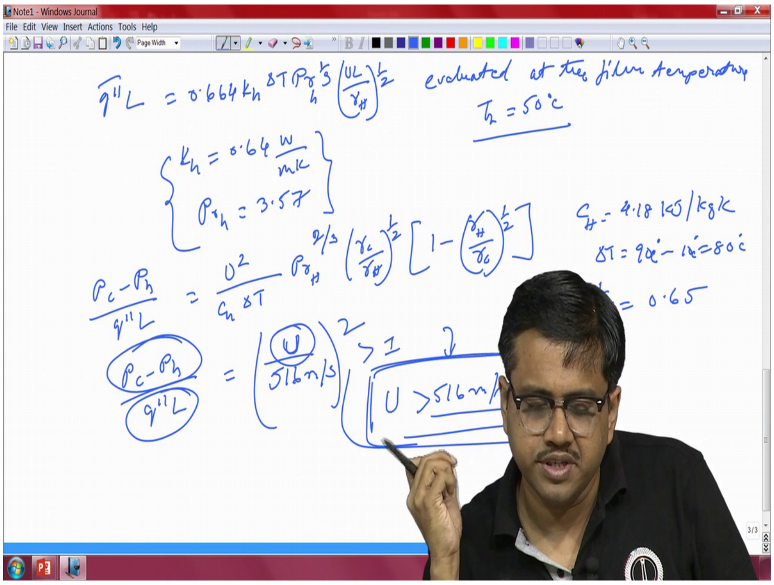

So, q double prime into L becomes 0.664 Kh delta T Prandtl H to the power of one third UL by gamma H raise to the power of half. So, in which all these properties are evaluated at the film temperature T H is equal to 50 degree Celsius. Now what we do is that we have to divide the two; that means, basically let us just put some of the values this cage is now, 0.64 watt per meter kelvin and Prandtl number H is about 3.57. So, these are evaluated all at 50 degree Celsius.

So, now what we do is that Pc minus P h divided by q double prime into L is basically given as U square divided by Ch delta T Prandtl number H to the power of two-third gamma c by gamma H raised to the power of half 1 minus gamma H by gamma c raised to the power of half. Now Ch is approximately 4.18 kilo joules per kg K, delta T is of course, 90 minus 10, which is about 80 degree Celsius and gamma H by gamma c raised to the power of half, we just now found out is about 0.65. So, based on this the Pc minus Ph divided by q double prime into L basically gives you U by 516 meter per second square.

So, this ratio implies that if your U is greater than 516 meter per second, then only you are going to get a significant energy savings, because you are spending that much amount of money, in money or rather power, in basically heating the plate and this is the savings that you are having, because of the reduction in drag. Now this gives you the ratio of U by 516 now if this U; that means, the velocity is actually more than 516 meter per second then you can actually have a ratio which is greater than 1. So, this is one of the key observation that this has to be the velocity scale for it to be greater than 1, this has to be kind of satisfied; obviously, you can see this is a very high velocity 516 meter per second is very high. So, normally just by doing this you may not get that kind of a. Of course, it depends on the properties if you have a fluid whose property decreases drastically, that is basically the kinematic viscosity or the dynamic viscosity changes drastically with temperature, then perhaps this velocity scale is going to come down and you are going to get a benefit in those cases, because this has to be greater than 1, at least it has to be greater than 1 for all the effort.

So, let us go to the next problem now and this is relatively easy.

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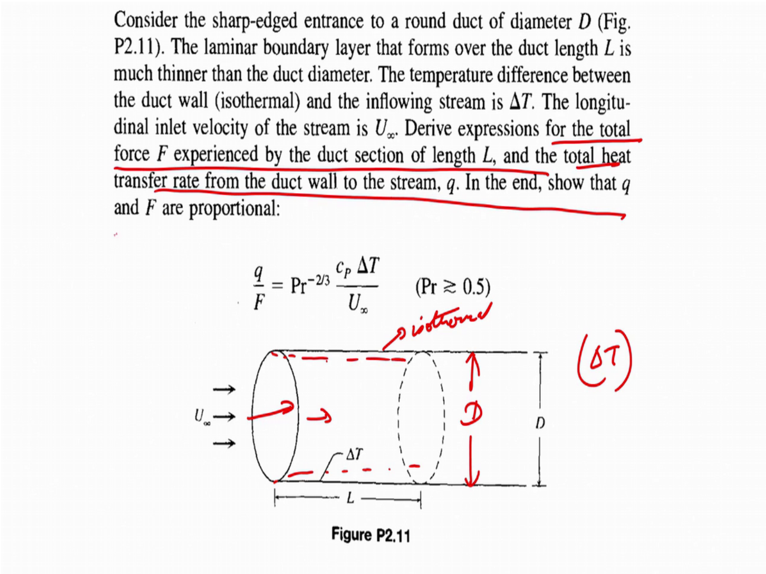

So, there is a flow sharp edged entrance to a round duct of diameter D, the laminar boundary layer that forms over the duct length is much thinner. So; that means, you basically form a very thin boundary layer forms over the duct is much thinner the temperature difference between the duct wall which is isothermal, this is isothermal wall and the in flowing stream. So, that is about delta T the longitudinal velocity is U infinity. So, derive expressions for the total force experienced by the duct section of length L and the total heat transfer rate from the duct wall to the stream in the end show that q and F are basically proportional.

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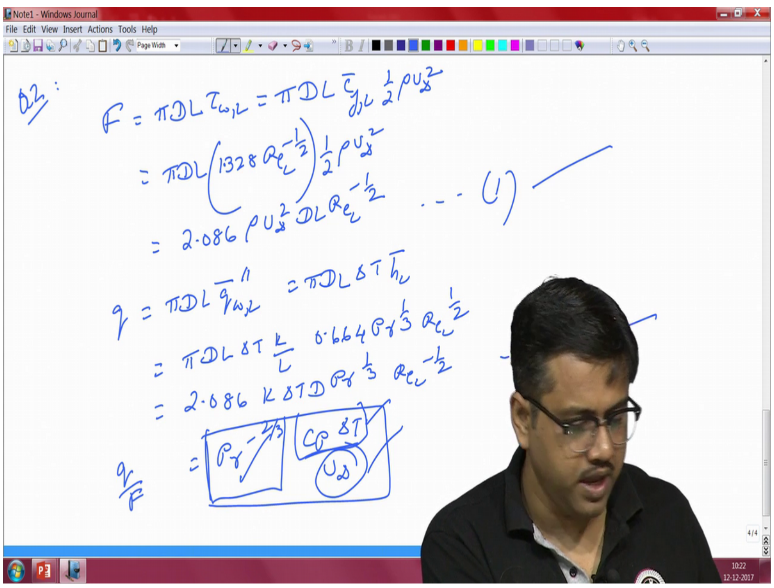

So, question 2. So, first and foremost in the boundary layer is very thinner than a pipe diameter the wall friction and the heat transfer can be estimated. So, for example the force is pi DL into tau wall that is the shear stress. So, pi DL this is CfL, half rho U infinity square. So, now, this is pi DL. So, 1.38 Reynolds number to the power of minus half rho U infinity square, this works, because this is almost like a flat plate kind of an assumption. So, we can, because it is a much thinner layer. So, it is basically does not see the other the two layers have not merged. So, if you recall our discussion on the internal convection we can say that you can almost model it like a flat plate right duct and all these varieties. So, it is 2.086 rho U infinity square DL Reynolds number to the power of minus half, then you have q which is pi DL q wall double prime which is pi DL and delta T hL bar which is basically equal to pi DL delta T K by L 0.664 Prandtl number one-third Reynolds number to the power half.

So, this becomes 2.086 K into delta T D Prandtl number one-third Reynolds number l to the power of half. So, if you this is 1 and this is 2, if you divide 1 by the other. So, you get basically q by f gives you Prandtl number two-third CP delta T by U infinity. So, that is very straightforward, we use the existing correlations or the existing formulations that we did and just we divided 1 by the other and the ratio as expected there is a Prandtl number dependence. There is a velocity dependence and then there is a temperature difference dependence. So, one is a property difference, these two are imposed condition difference right. So, this is something that you would have expected by now that this is how this flow is going to behave.

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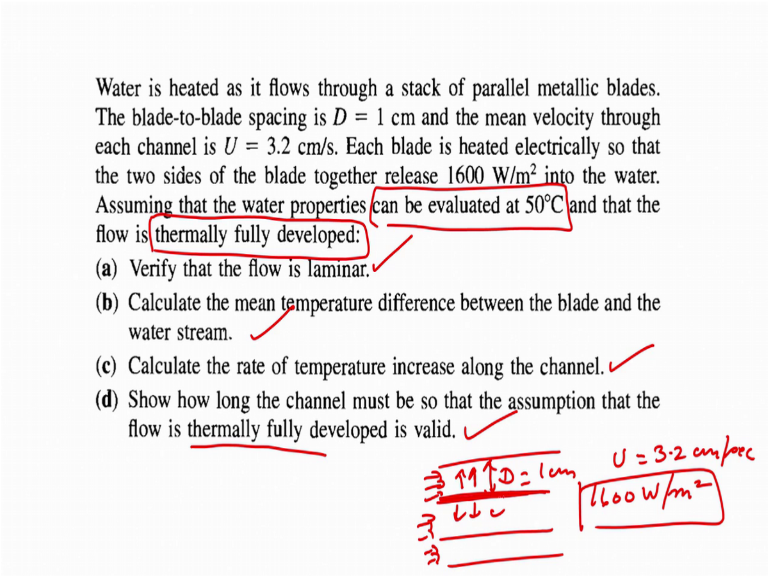

So, let us look at the next problem, which is basically this problem is that water is heated as it flows through a stack of parallel metallic blades. So, you have this metallic blades stack right blade to blade spacing is about D, which is about 1 centimeter and the mean velocity through these channels is U, basically equal to 3.2 centimeter per second each blade is heated electrically. So, that two sides of the blade. So, this is a blade two sides of the blade together release about 1600 watt per meter square into the water that is flowing. So, assuming the water properties can be evaluated at 50 degrees Celsius and the flow is thermally fully developed. There are a few things that you need to do, one is verify that the flow is laminar, calculate the mean temperature difference between the blade and the water stream, calculate the rate of temperature increase along the channel and show how long the channel must be.

So, that the assumption that the flow is thermally fully developed is valid. So, there are four parts to this question, one part is that the flow is laminar validate; that means, it has to be within a certain Reynolds number range. You also have to calculate the mean temperature difference and see the temperature, the rate of temperature rise and how long the channel must be for the thermally fully developed, assumption to be valid and you this blade is actually dissipating heat to both sides. So, that was the other thing that we said about 1600 watt per meter square. So, 800 basically goes to the either side of the water that is flowing through it. So, let us look at it.

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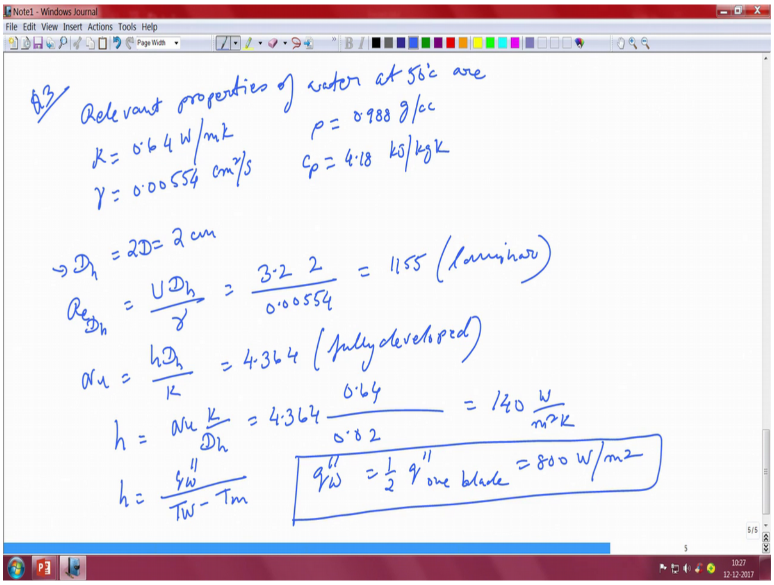

So, in this particular case question number 3, the relevant properties of water properties of water, water at 50 degree Celsius are say K equal to 0.64 watt per meter kelvin gamma is equal to 0.00554 centimeter square per second, rho is equal to 0.988 gram per cc, Cp is equal to 4.18 kilo joules per kg K. So, to begin with we have to show that the flow is laminar right. So, let us see first D h which is the hydraulic diameter is equal to 2 D, which is equal to 2 centimeter in this case. So, the Reynolds number that is based on the hydraulic diameter is what you D h by gamma which is basically 3.2 into 2 divided by 0.00554, which gives you about 1155.

So, which is very laminar around 2000, the transition starts to happen this is almost half of it. So, the Nusselt number is basically h D h by K which is 4.3644 fully developed that we already know from our discussions earlier. So, h will be Nusselt number into K by D h which is 4.364, 0.64 on the top point, you can just work on the unit this is 140 watt per meter square kelvin, h is also given as q w double prime this is Tw minus Tm. So, where this q double prime is basically half of q 1 blade which is about 800 watt per meter square.

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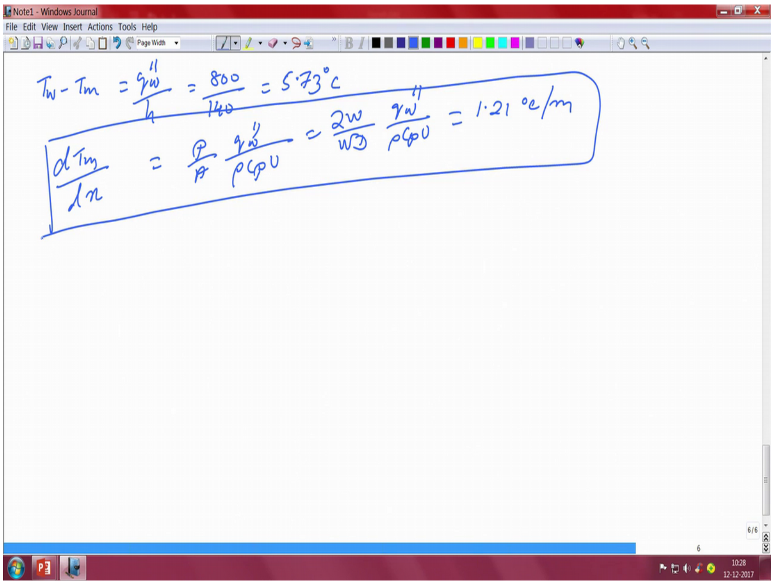

So, Tw minus Tm is equal to q w double prime by h, which is equal to 800 divided by 140 which is 5.73 degrees Celsius. So, basically your d Tm by lx, it is basically equal to P by A, q w double prime rho Cp into u which is equal to 2 w by WD u double prime, this is perimeter by area not pressure U. So, if you plug in the numbers, it will be 121 degree Celsius per meter. So, this gives you that the thermal that is the rate of temperature drop is actually given as 1.2 Celsius per meter. So, this is a simple enough problem which, we have just solved using standard correlations. These are not correlations per say, but these are basically using the standard formulations that we did earlier on in our course.

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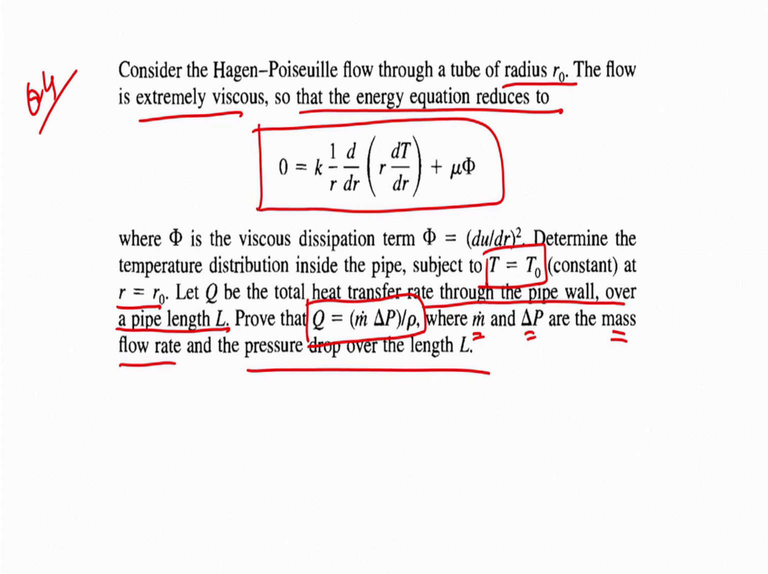

So, now the fourth problem, this question 4. So, consider a Poiseuille flow through a tube of radius r0 the flow is extremely viscous. So, that the total energy equation reduces to something like this. So, phi is the viscous dissipation term which is given as d u by dr square, determine the temperature distribution inside the pipe subject to T equal to T0 constant at r equal donors is an isothermal case. Let q be the total heat transfer rate through the pipe wall over a length L, prove that q is equal to this where m dot and delta p at the mass flow rate and pressure drop over the length L. So, that is what the equation what we have to prove in this particular situation.

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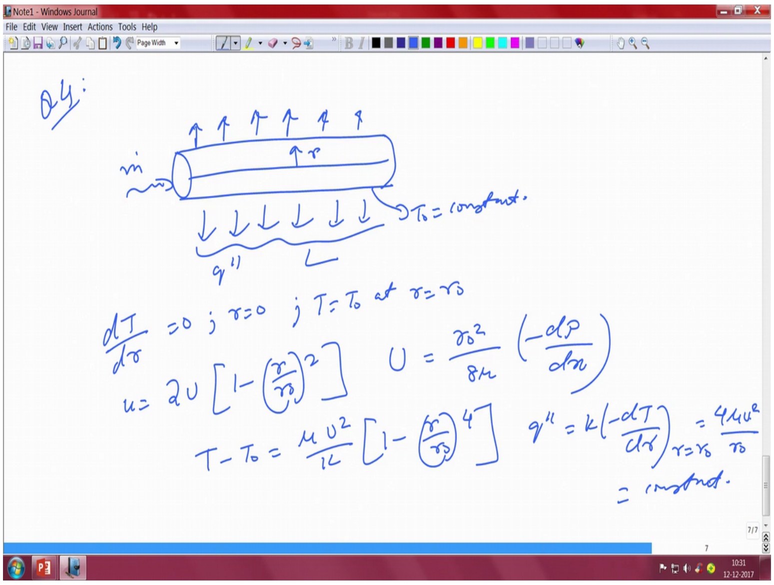

So, let us see. So, this is the pipe. So, m dot is going through the pipe, this is the radius r. So, T0 is basically equal to constant. So, and q prime is; obviously, the total heat through this entire length L, these are subjected to the boundary condition dT by d r is equal to 0 at r equal to 0 and T equal to T0 and r equal to r0. These are the two things that are given, U is of course, equal to 2 U into 1 minus r by r0 square and U is equal to r0 squared by 8 mu divided by dp by dx. This is all given comes from the. So, if you solve this particular expression, the equations is already given if you solve the equation now, you will get that T minus T0 will be equal to mu U square by k into 1 minus r by r0 raised to the power of 4 and out of that q double prime is equal to K minus dT by d r and r equal to r0 equal to 4 mu U square by r0, this is also equal to constant that is what you are getting.

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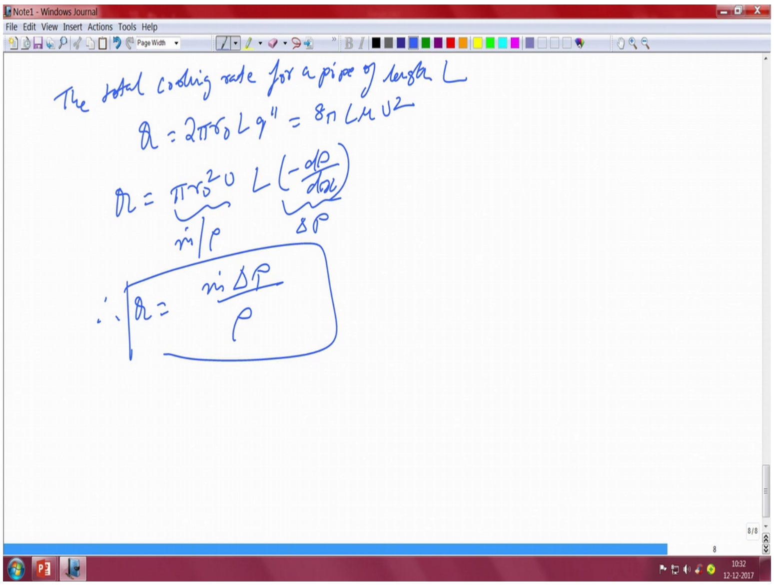

So, the total cooling rate, cooling rate for a pipe of length L, 2 pi r0 L q double prime which is 8 pi L mu U square. So, this can be written as further pi r0 square into U, which is basically m dot by rho and that L minus dp by dx, which is basically equal to delta p the pressure drop. So, in conclusion we can write q is equal to m dot into delta p by rho. So, this is how you can actually solve a problem of this sort. So, basically we end our problem solving sessions. So, we have spent some time in solving problems for both for forced convection. We will solve similar problems for natural convections convection as well. So, you can practice similar problems like this as homework, you can look into the Bejan case and Crawford and similar such books and you can solve the problems and try to see whether the answer kind of makes sense or not? We have given you some examples of some problems, typical problems which can be attacked and solved just by using your intuition and existing relationships.

Thank you.