Convective Heat Transfer
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Lecture – 26
FORCED CONVECTION – TUTORIAL II

So, welcome to the second tutorial session, in which we are going to start with internal forced convection. So, in this particular case we are going to look at the problem, which is slightly different from what we did earlier.

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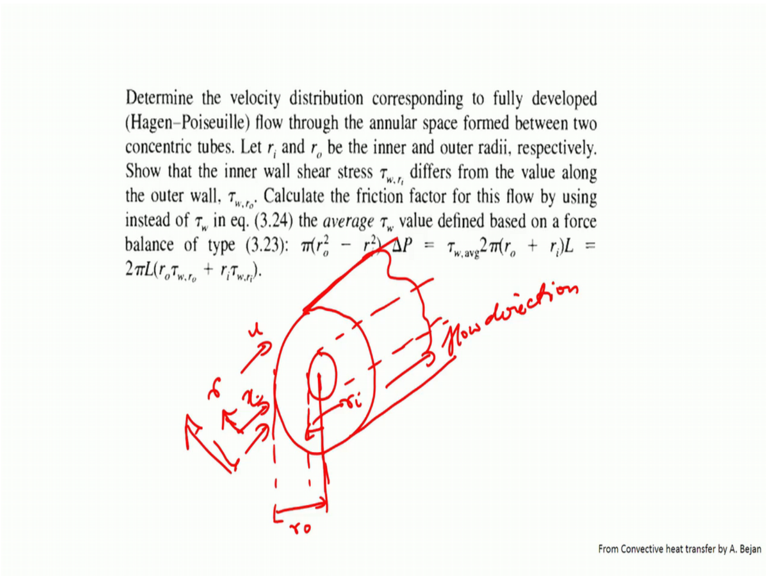

So, here the problem is that determine the velocity distribution corresponding to the fully developed Poiseuille flow through the annular space, formed between two concentric tubes of diameter or radius r1 and ri and r0. So, we have to show that the inner wall shear stress differs from the value along the outer wall, and we also have to calculate the friction factor for this flow by using a tau wall, instead of the in, and we will come to the equations and the average tau wall defined based on a force balance type. So, there are a few small things that we need to do again, it is taken from Adrian Bejan.

So, the problem essentially is something like this. So, you have this inner tube and it is enshrouded by this outer tube, something like that. The flow is actually well they are concentric. So, they are basically concentric. So, apologies for my drawing; so, this is the flow direction. So, that is the flow direction there are a few parameters over here. So, this is the center; obviously, So, and this is the corresponding three distances. So, you can say from here to here, it is basically your r0, r0 from the center from this, from the center to this, it is r0, from here to here it is ri. So, that is basically the problem, the flow in this direction it is say x, in this direction it is r x, this is r and this is the direction of the velocity is u. So, that is roughly the problem that there are basically two cylinders, one inside the other, they are not moving as such, but the flow is passed through this annular passage, very common problem in heat exchangers and other things right shell and tube heat exchangers.

You can have flow inside the inner pipe also, this is much more simpler; that means, in through that annular space you actually have a flow moving in the direction that I indicated in this particular drawing, and this is the radius r, this way is r, this way it is x right. So, from the center of the inner cylinder to the outer wall is basically your are not, whereas the center to the outer surface of the inner cylinder, it is ri. So, this is the case that we get over here. So, now let us attack this problem and see .

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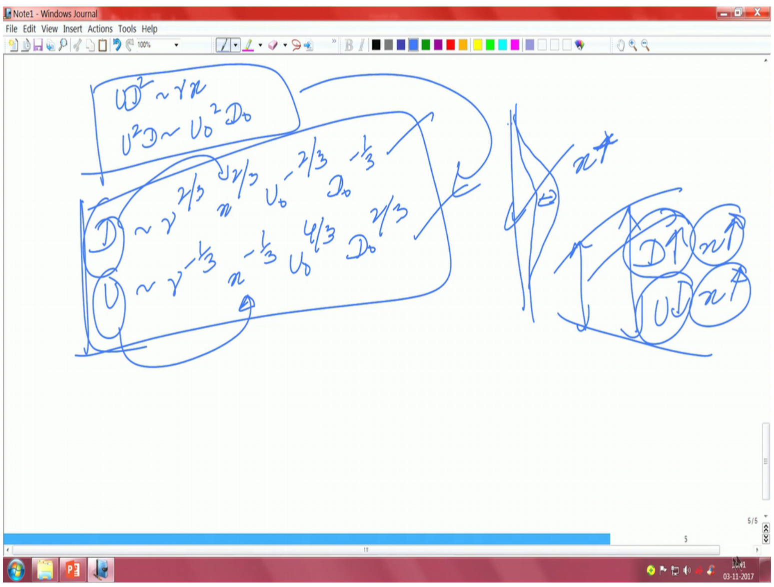

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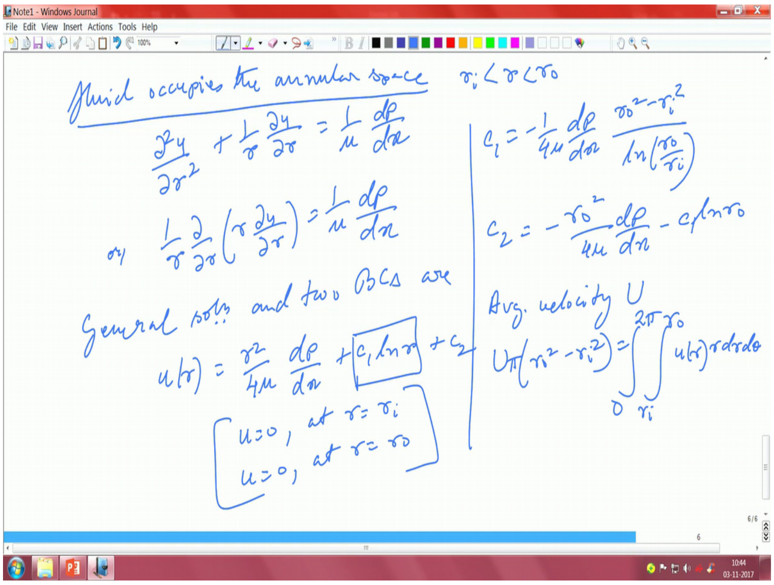

So, the fluid occupies the annular space which is ri, r is greater than ri less than r0 . So, for the fully developed appropriate equation for the fully developed flow is u square, dr square plus 1 by r du dr 1 by r dp dx or in other words 1 by r, r du dr by dp by dx. So, that is the combined equation; that is the equation that we have right and the general solution and the two boundary conditions, we already have solved equations like this, this is written in the polar coordinates.

So, general solution and the two boundary conditions are u r r square 4 mu, dp by dx plus c1ln r plus c2. So, u is equal to 0, at r equal to ri, u is equal to 0 at r equal to r0 , that is thus very simple, because the velocity will be 0 at the inner and the outer wall, it is as simple as that, because those are the two walls and the inner wall inner pipe, there is no flow. So, the flow is only there in the annular space, remember that very carefully .

So, now your c1the two constants will be 4 mu, dp by dx, r0 square minus ri square divided by ln r0 r0 by ri, c2 will be minus r0 square by 4 mu, dp by dx minus c1ln r0 . So, these are c1and c2. So, now, you can do some of these manipulations, it is going to be a more complicated expression you can see why this term comes over here, that you can pay some attention to the slightly unique nature of this. So, the average velocity u, let us call that u. So, u happening over r0 square minus ri square into pi, 0 to 2 pi, ri to r0 u r r dr d theta, it is just the integration of the new profile, over that annular space one is r, and one is the full angle; that means, you have two azimuthally integrate it across the whole angle right.

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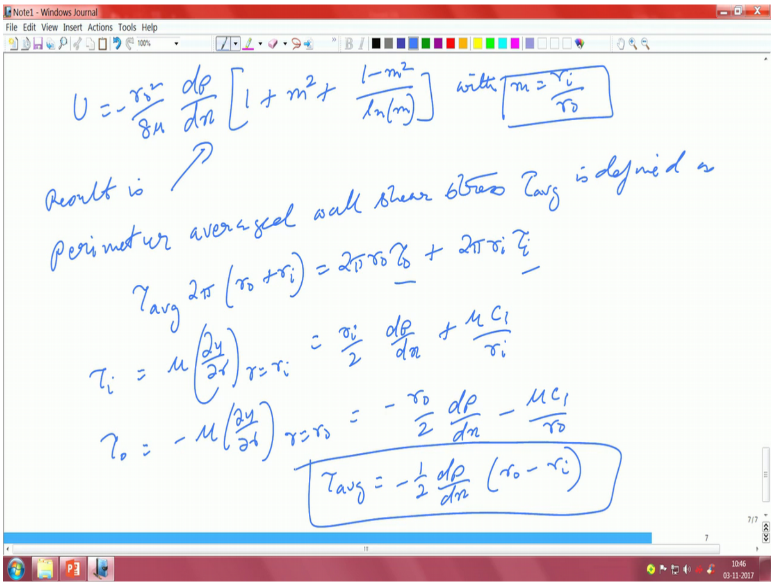

So, once we are done with that. So, u will be equal to r0square by 8 mu minus dp by dx, 1 plus m squared plus 1 minus m square into ln m with m equal to ri by r0 . So, the result is 1. So, this is the average velocity, where m is a just a ratio of the inner and the outer radius. So, the perimeter averaged, wall shear stress, tau average is defined as tau average 2 pi r0 plus ri, 2 pi r0 into tau naught, plus 2 pi ri into tau i. These are basically, this is the average and these are the individual shear stresses across the two and. So, tau i is basically du by dr at r equal to ri, it is a simple right thing, which is equal to ri by 2 dp by dx plus mu c1by ri. So, similarly tau naught is equal to minus du by dr at r equal to r0 , which is equal to minus r0 by 2 dp by dx minus mu c1 by r0 . So, in the end, the tau average is basically equal to dp by dx into r0 minus ri . So, that is the form that you get.

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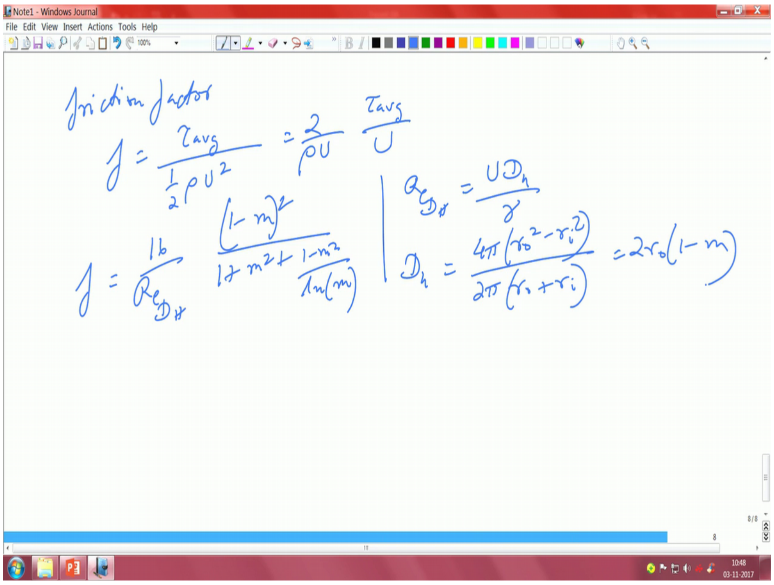

Now, the friction factor f is equal to tau average divided by half rho u square is equal to 2 by rho u, tau average by u. So, the friction factor, which is like 16 by ReDh the friction factor will become if you put in all the values here. So, the friction factor will become 16 by ReDh which is the Reynolds number based on the hydraulic diameter. So, one plus m square plus one minus m square into ln by m, ReDh is equal to u into Dh by gamma d h is equal to 4 pi r0square minus ri square divided by 2 pi r0 plus ri equal to 2 r0 one minus m. So, that is the final form that you get. So, by this is a very simple exercise I just needed to do a few integrations. So, there is nothing i mean great about it, but you can see that this kind of a problem can be easily attacked by from, what you already know it is a little different it is not flow through a straight channel or anything like that, but you can still if you know the equations in the polar coordinates with the appropriate boundary conditions you can solve any problems like these. So, lets go to the other problem.

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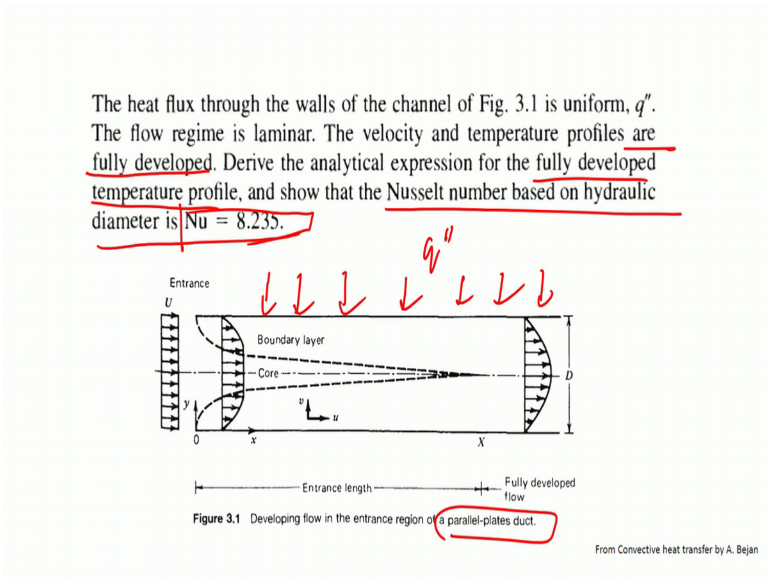

So, here and the problem is like this. So, this is like a channel, it is like a channel which is separated by a distance D and there is a uniform heat flux, that comes in through the wall q double prime, the flow is laminar the velocity and the temperature profiles are fully developed; that means, there is a hydrodynamically and thermally fully developed flow. So, derive the analytical expression for the fully developed temperature profile and show that the Nusselt number based on hydraulic diameter, and this would be hydraulic diameter because it is this channel is about 8.235. So, this is the fully developed this like a parallel plate duct again take it from Adrian Bejan. So, this is D and this is the length of the channel. So, it is very similar problem what we did earlier, except that the Nusselt number value, we are kind of expecting is 8.235, if you recall the other values were quite a bit lower than this, but other than that is a uniform heat flux condition and the flow is laminar, the temperature and the velocity profiles are fully developed. So, this should not be a much of a problem to solve we have already solved problems like this, when the flow was through a pipe right. So, that we have already done.

So, let us move on and try to see, how to attack the problem like this?

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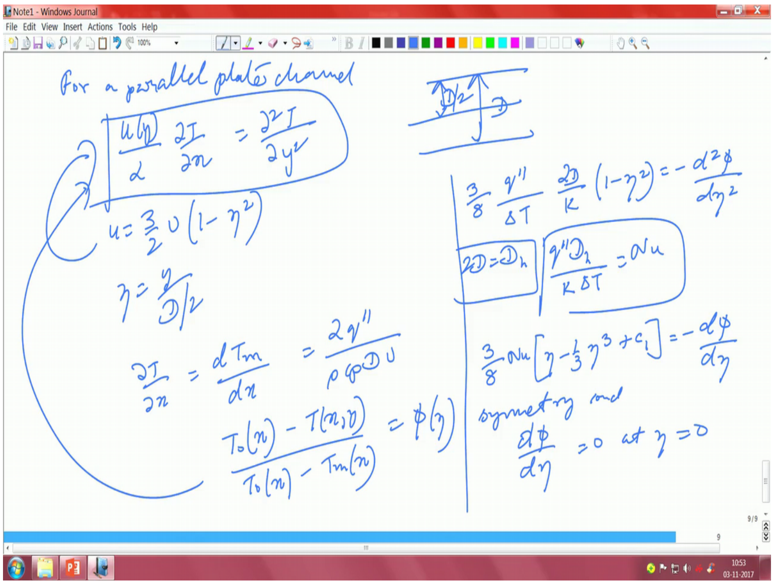

So, for a parallel plate channel for a parallel plate channel your u y by alpha dt by dx, remember this equation we always wrote right, but here of course, your u now becomes 3 by 2 u into 1 minus nita square, where nita is basically y by D by 2 right it is like in the normalized coordinate system, y divided by half the distance between the two plates this is D and this half is D by 2 .

Now, also we know that dT by dx is equal to dT m by dx equal to 2 q double prime rho cp d into u, this we already know from your class on the exercises, that we did all right and T0 which is now a function of x T0 is a wall temperature, which is a function of x minus d x y divided by T0 x minus D mean which is also a function of x, now should be a function of phi that is the fully developed temperature profile this is nothing new, this is all that we have done earlier right the all these things have been done earlier so.

Now, what we do is that we just do a substitution of these into equation one. So, equation one is our parent equation. So, there this will go and this will go, both of these two things needs to be substituted there right. So, what we will get is if you substitute it there you will get 3 by 8 q double prime by delta T 2 d by k one minus nita square delta minus d square phi by d square. So, where 2 D is equal to the D h, the hydraulic diameter is basically half the thing and q double prime D h divided by k, k delta T is basically your Nusslet number. This is also known, this is known this is an introduction that the hydraulic diameter is twice the diameter between the two between the spacing between the two duct. So, naturally what do you get 3 by 8 Nusselt number nita minus one third nita cube, plus c1equal to minus d phi by d nita. Once we integrate it once and the constant of integration is already there. So, what the condition the symmetry condition it is still a symmetry condition will be valid; that means, d phi by d nita will be equal to zero at nita equal to zero that is the inflection point at the center line , at the center line the temperature profile has to show that link right. So, that is of course, valid this will lead to c1 to be equal to zero also . So, now what we do we integrate this equation one more time because c1 is now gone it is a goner.

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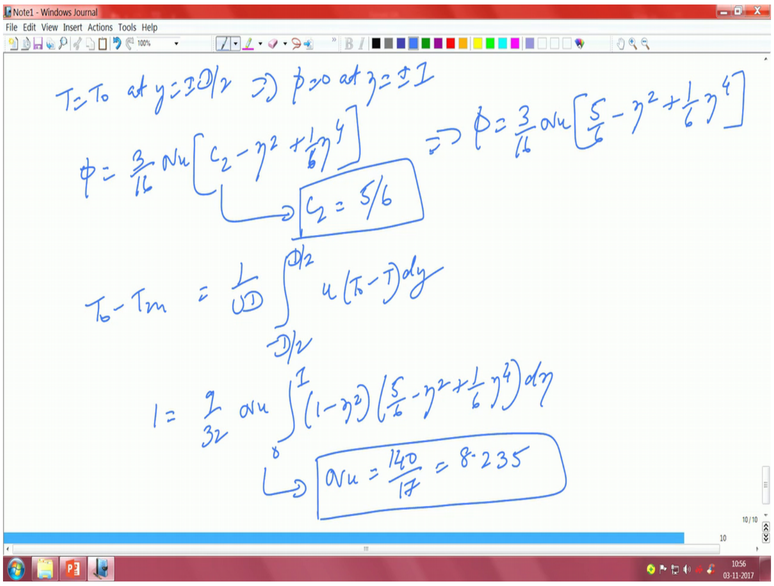

So, now, we integrate this equation one more time and invoke and the wall condition; that means, T equal to T0 at y equal to plus minus d by 2 which means also that phi is equal to zero at nita equal to plus minus one, because of the definition of T0 right, because it is in the numerator T0 minus T. So, if T becomes equal to T0 the numerator of phi becomes equal to zero, so that means, phi becomes equal to zero. So, this condition yields that your c2; that means, the second constant of integration. So, before that let us write the second constant of integration. So, phi will be equal to 3 by 16 Nusselt number c2 minus nita square plus one by 6 nita 4. So, once you substitute it over here this will yield that c2 is equal to 5 by 6. So, c2 will be equal to 5 by 6.

So, now that we have got the things. So, this will lead to now further that phi will be equal to 3 by 6 Nusselt number 5 by 6 minus nita squared plus 1 by 6 nita to the power of

4. So, the final step for us is to determine Nusselt number from the temperature, the mean temperature difference right. So, T0 minus Tm, we already know is one over u by d integrated minus d by 2 to d by 2 u T0 minus T dy, now you know the traditional, what we did earlier now you can combine the all the equation this we did repeatedly. So, it will give you 19 by 3 by 2 Nusselt number from zero to one, one minus nita square 5 by 6 minus nita square plus one by sixth nita 4 d nita. This actually further yields that Nusselt number is 140 by 17 which is 8.235. So, this particular thing solves this particular aspect of the problem. So, you can see that, this was easy enough solution except that, you had to take into account a few things, but the procedure was essentially the same, what we did earlier, there is no deviation in that particular way.

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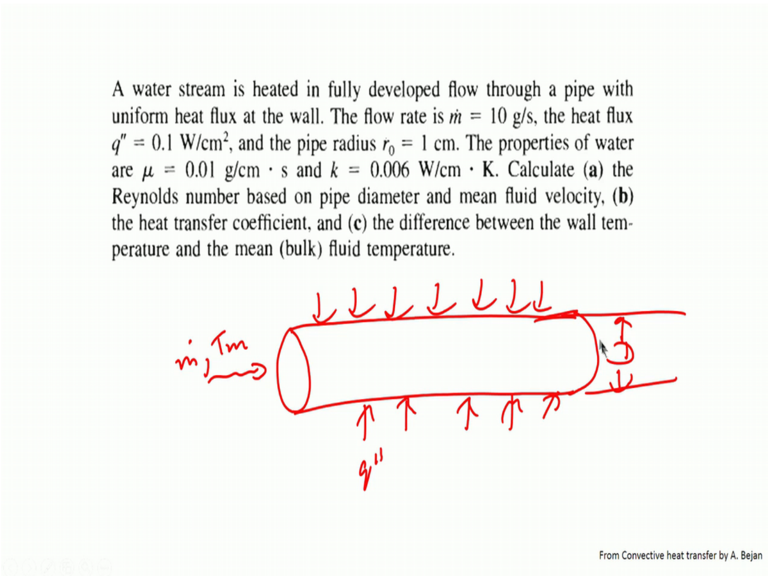

So, let us look at the final problem of our test set, which is basically problem involving a water stream is heated in a fully developed flow through a pipe with a uniform heat flux at the wall, the flow rate is some m dot equal to 10 gram per second, the heat flux is about 0.1 watt per centimeter square is a uniform heat flux and the pipe radius is about 1 centimeter the properties are given we are to calculate the Reynolds number based on pipe diameter and mean flow velocity and the heat transfer coefficient and the difference between the wall temperature and the mean fluid temperature. So, it is something like this. So, this is how this m dot is coming with some mean temperature, you are applying a uniform heat flux all over which is q double prime and this is the diameter of the pipe .

So, we have to solve this particular problem, it is an easy problem. So, should not be very difficult for us to quickly go through the motion.

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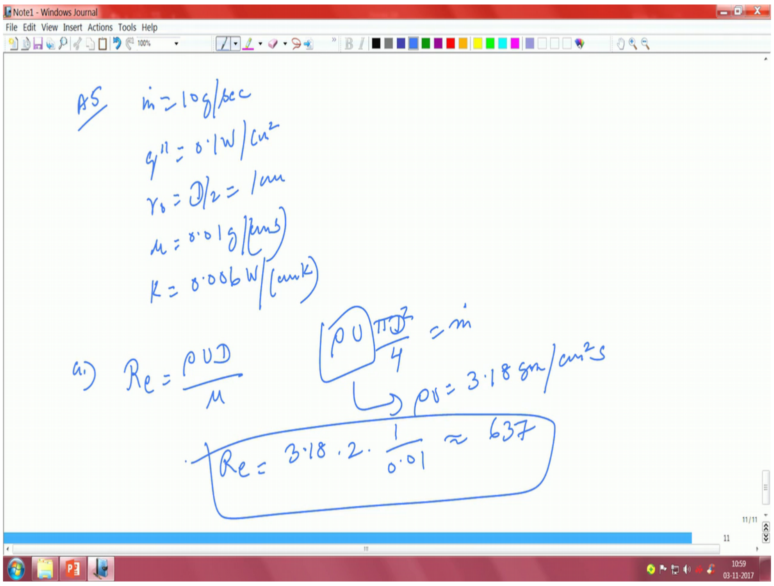

So, in this problem which is A5 m dot is equal to 10 gram per second, q double prime is equal to 0.1 watt per centimeter squared, r0which is d by 2 is equal to one centimeter and then mu is equal to 0.1 gram per centimeter second and K is equal to 0.006 watt per centimeter Kelvin. So, the Reynolds number which is rho uD by mu. So, that you need to calculate. So, we already know that rho u into pi d square by 4 that is basically your m dot right it is a simple enough thing.

So, therefore, rho u if you calculate it from here it will be something like 3.18 gram per centimeter square second right that should be what it is now your therefore, your Reynolds number therefore, should be 3.18 into 2, I am not writing the unit that you can try and 0.01. So, this will roughly give you about 637, which is in a laminar regime you know up to about 2000 is the laminar after that the transition starts around 3000 to 4000, it transitions fully into turbulent flow. So, the Reynolds number is very easy to compute. So, this was a very easy exercise.

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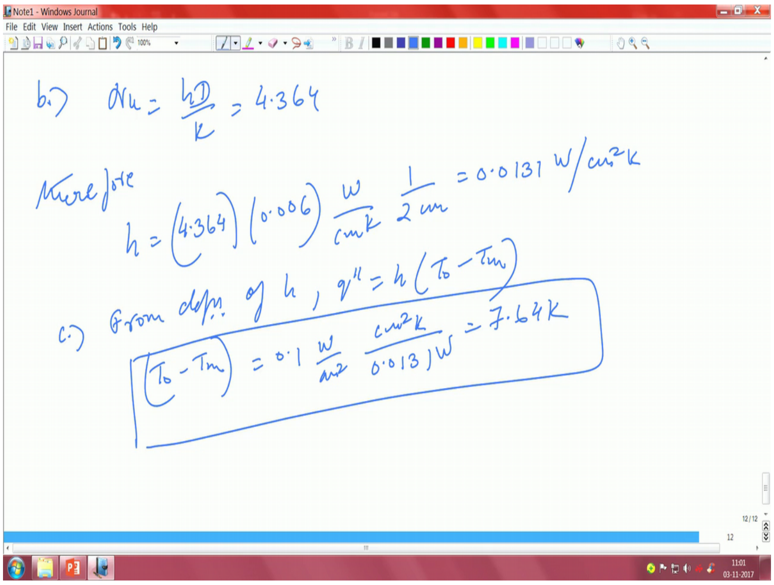

So, now the heat transfer coefficient calculation, so, from the definition of Nusselt number, we know that it is hd by K should be about 4.364, this is the fully developed Nusselt number limit that we already calculated. So, therefore, your h should be equal to 4.364 into 0.006, if you have to calculate the heat transfer coefficient that is one by 2 centimeter which is equal to 0.0131 watt per centimeter Kelvin, now c from definition of h the definition of h we know that q double prime is h, T0 minus Tm. So, we can say that T0 minus Tm will be equal to 0.1 watt per centimeter square, this is centimeter square Kelvin divided by 0.0131 one watt which is about 7.64 Kelvin very simply we have been able to solve this particular problem.

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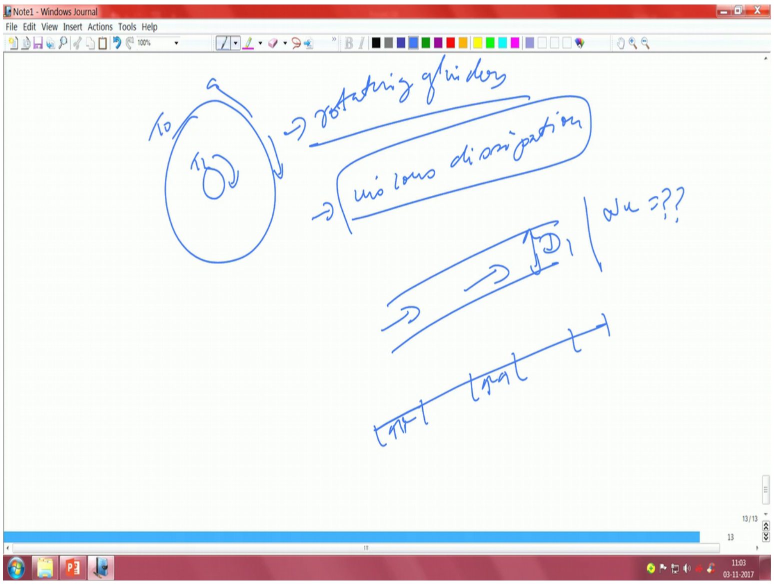

So, the problem set is up to this much, but what I can suggest to you, you can try a few more things, say for example, when you have two cylinders, you rotate one of the cylinder and you can calculate what the flow field looks like, you can rotate the inner and the outer together, you can rotate them in the opposite direction to each other, you can rotate one keeping the other fixed. So, there are varieties of problems that you can solve in which the flow is brought about by the rotation, in that particular case you also have to take into account, you have to also take into account the boundary conditions, essentially you start with the same equation now you have to put in in place the boundary condition. So, this is one exercise that you can do rotating cylinders. A problem which is very common in the fluid mechanics community, you can also have the wall temperature T0 and T1 and then solve for the Nusselt number and the heat transfer coefficient.

So, that is one interesting problem that you might want to attempt apart which we are, we have not done during this there is this time period . So, rotating cylinder is important and it is applicable in many of the cases that we have looked into you can also look into problems, which involves viscous dissipation. So, viscous dissipation is very important we just did a little bit of that when we brought about the Brinkman number if you recall the last lectures in a forced convection. So, viscous dissipation how to incorporate viscous dissipation in the equations and how if we say that there are two slots and there is a flow going through it and the distance is so small that there is viscous dissipation and it is important how would you find out what will be the Nusselt number and other things.

We did a little bit of an example, but you can attack problems like this, once again Bejan and there are many other books from where you can try this kind of problem. So, viscous dissipation rotating cylinders then plates with different types of heating, we also did that then there is like patch hitting. So, here heat is supplied here, there is no heat again heat is supplied and things like that we did what we call a cumulative a superposition type of a problem using this. So, you can try problems in that way also just by putting in numbers we did it in a more generic way, you can just put in the numbers and you would be good to go.

So, I think try problems in this area, we will also give a little bit of a problem and answer set at the end of the course, which you can take a look which you are not going to go through in the class, but this will be like handouts, where you can see more problems like this and practice problems from Adrian Bejan and any other book on convective heat transfer.

Thank you.