Convective Heat Transfer
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Lecture – 25
Forced convection – Tutorial I

In this particular lecture we are going to do some work out examples on the forced convection. So, that we have finished the forced convection module internal and external forced convection. So, we will look at some of the sample problems mainly from Adrian Bejan and we will provide you with a kind of a worked out examples of those problems. So, the idea behind this is that you should get an idea to approach how to solve different problems which might come in handy during your exams and during your industrial calculations in whatever profession you are involved in. So, you can consider this to be a forced convection tutorial.

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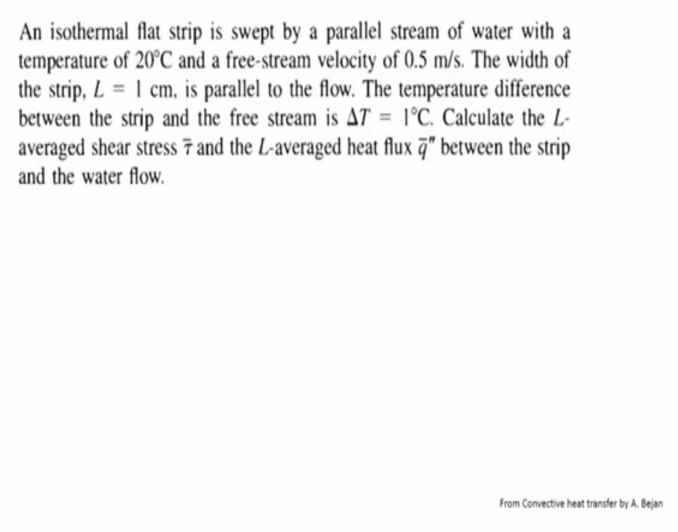
An isothermal flat strip is swept by a parallel stream of water with a temperature of 20°C and a free-stream velocity of 0.5 m/s. The width of the strip, L = 1 cm, is parallel to the flow. The temperature difference between the strip and the free stream is ΔT = 1°C. Calculate the L-averaged shear stress $\overline{\tau}$ and the L-averaged heat flux $\overline{q}''$ between the strip and the water flow.

From Convective heat transfer by A. Bejan

So, the first problem in this particular series is basically if you read the problem statement again from Adrian Bejan and isothermal flat strip is swept by a parallel stream of water with a temperature of 20 degrees Celsius and a free stream velocity of 0.5 meter per second the width of the strip is about 1 centimeter parallel to the flow the temperature difference between the strip and the free stream is delta T equal to about 1 degree Celsius. So, the things that we need to calculate is the L averaged shear stress and the L averaged; that means, the length that is the length and the width averaged shear stress and the averaged heat flux between the stream and the water flow. So, that is what the problem statement is all about.

So, let us see that how we can attack and solve this problem..

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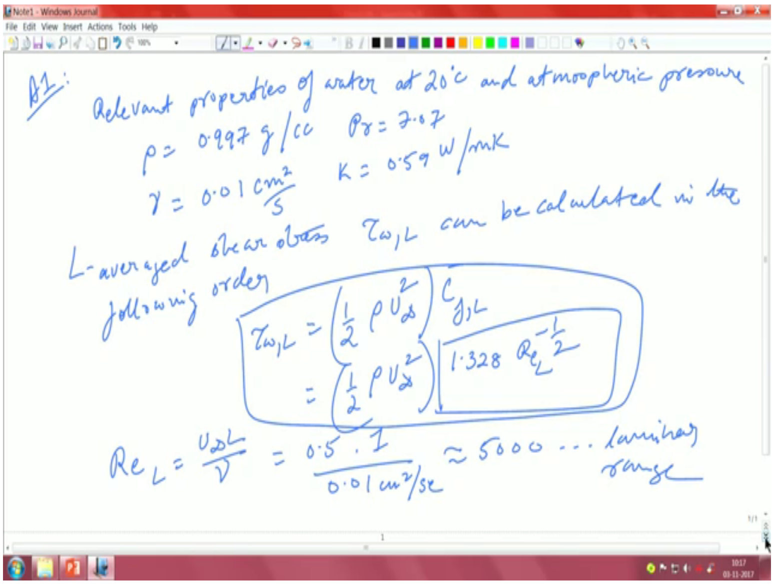

So, moving to our journal, let us call this problem A 1, this is the problem statement. So, first and foremost what we need to do is that we need to compute some relevant properties because it is given that the working fluid is water at 20 degrees Celsius and atmospheric pressure. So, some of the properties, rho for example, is coming out to be about 0.997 gram per cc, prandtl number is about 7.07, the kinematic viscosity is about 0.01 centimeter square per second, k is about 0.5 9 meter what per meter kelvin. So, that is in general the properties of water..

So, the L averaged shear stress which is tau w L or tau wall L can be calculated in the following order. So, tau L is equal to half rho U infinity square CfL skin friction coefficient right and we know what the skin friction coefficient is all about rho U infinity square 1.328 Reynolds number to the power of minus half right this is the relationship that you already know from your analysis of a flat plate all right. So, the where the Reynolds number in this particular case is given as U infinity L by a gamma which is about 0.5 remember that the velocity was about 0.5 for water that that was a velocity of water and 1 centimeter was the length. So, and it was about 0.01 centimeters square was the kinematic viscosity. So, this roughly translates to about Reynolds number of 5000 which in the laminar range for an external boundary layer or external force convection the transition to turbulence happens much later once we do the turbulence part of this particular course we will know, but as of now this is a very simple calculation where we did find out the wall shear stress as well as the Reynolds number now.

Now based on this let us move to the next one..

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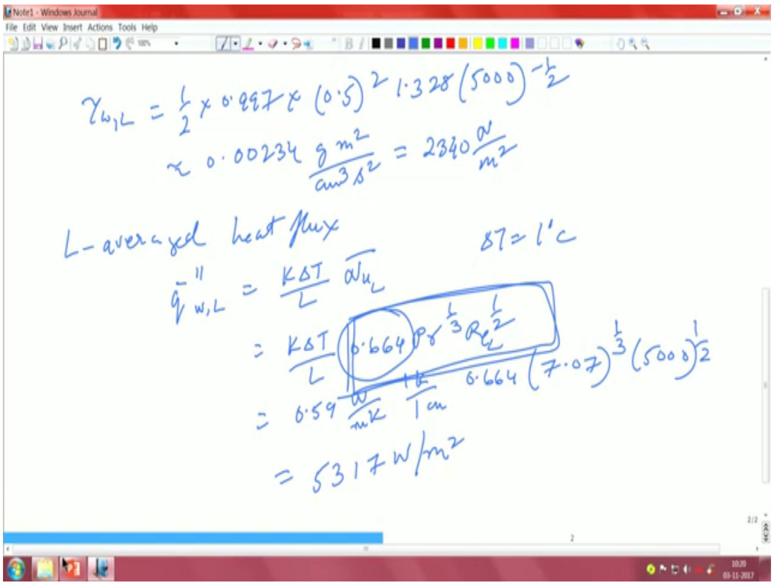

So, now you can find out tau wall you basically it is a plugged in values of all these numbers. So, basically you have half into 0.997 that is the density into and you just do the unit conversions also which I am not showing over here 1.328 into 5000 to the power of minus half something like that. So, it will come out to be approximately 0.00234 gram meter square centimeter cube second square. So, that if you convert it to the more traditional unit it is coming out to be Newton per meter square 2 3 4 0 Newton's per meter square.

Similarly the L averaged heat flux that is given as q bar double prime w comma L the K delta T by L nusselt number L bar that is given as K delta T by L 0.664 prandtl number one third Reynolds number half. So, this is the standard. So, we already are given that delta T is about 1 degree Celsius if you recall the problem statement. So, 0.59 this is what per meter kelvin it said it is a good exercise to write and the units, so, that you do not counter any problem either you convert it to the same unit or you do it at the end whatever, but then in that case you should write all the units down prandtl number is 7 we already gave that. So, this comes out to be about 5 3 1 7 watt per meter squared.

So, you can see that. So, it was a simple problem that we are able to solve just by using the correlations that were available what correlations were used for the average nusselt number we use that correlation and for the average wall shear stress we use the other correlation. So, basically you can either use the similarity solution from the similarity solution you have these numbers already at your disposal or these correlations at your disposal or you can also use the you know the integral formulation use that also because the constant is only going to vary within 10 percent right. So, this constitutes our first problem in the series.

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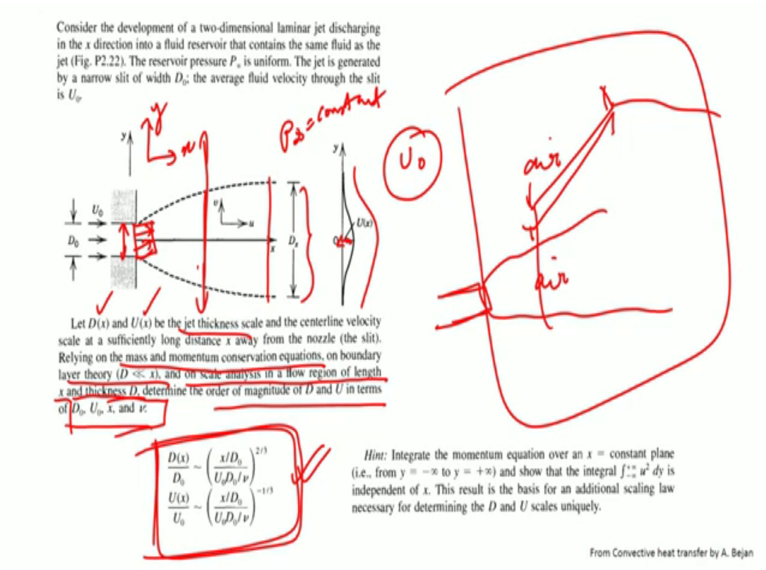

Let us do the second problem which is a little bit more complex than the first and we start with the easier problem and then we slowly see ratchet it up a little bit. So, that we see that how this problem actually works. So, in this particular case look at the problem statement very carefully. So, here what we are considering is the development of a 2 dimensional laminar jet discharging in the x direction. So, this is x and y this is x that is y into a fluid reservoir which contains the same fluid as the jet right the reservoir pressure P infinity is constant and the jet is generated by a narrow slit of width D naught this is the narrow slit the average fluid velocity through the slit is U naught .

Now, if you look at this particular problem very carefully you will find that first and foremost a few things this problem where is it for example, the smoke that is coming out of the tailpipe of your car that is very similar to this because though of course, that contains carbon monoxide in others, but here if you imagine that air is coming out into a body of air this is almost a problem like this. So, there is this and you get the flow coming out like that right. So, it is a say example this is air this is also air both can be water also it can be any other fluid, but it is the same fluid combination so; that means, one fluid is coming into another fluid where the both the fluid types are essentially the same right and this opening is very small compared to the extent of your reservoir in which it is getting flushed right.

So, this is a very typical problem of a laminar jet it is a 2 d laminar jet because it is like a slot these are like slots right. So, it is coming out through this slot basically. So, that extends to infinity obviously, whatever it is in the direction in the direction perpendicular to the board right.

Now, we have sketched something up over here to just give you an idea that how will the flow velocity actually look like we know that it is coming out to the velocity u naught which is constant. So, as the velocity as the jet actually spreads in the radial direction you can imagine that the velocity should slowly drop it will no longer here it is almost like a top hat kind of a profile right.

So, as you go here at any particular cross section you expect that this would be the kind of a profile that we have drawn over here. So, the jet will actually because the area is actually increasing as you can see by those dotted lines the area is actually increasing. So, naturally the jet will decay right and the centerline velocity is going to come down and it is going to assume some kind of a profile like that what is that profile that is an interesting question.

So, if D x and U x are the jet thickness right; that means, the thickness means this is the thickness and the centerline velocity which is U x it is the centerline velocity is this guy over here right at a sufficiently long distance x from the nozzle; that means, it is not in the near field. So, relying on the mass and momentum conservation equations on boundary layer theory and on scale analysis in a flow region of length x and thickness D determine the order of magnitude of D and U in terms of D naught U naught x and the kinematic viscosity and this is the solution that we need to find out..

So, there is a hint also that is given that you need to integrate the momentum equation from at any x; that means, at any particular plane you have to integrate it out in the y direction and this can result in some additional scaling laws, but these are scaling laws as you can see over here and we have to take care of these scaling laws and we have to find out that how this jet actually kind of you know behaves.

So, this is the problem that we have posed it is a little different from what you have right now..

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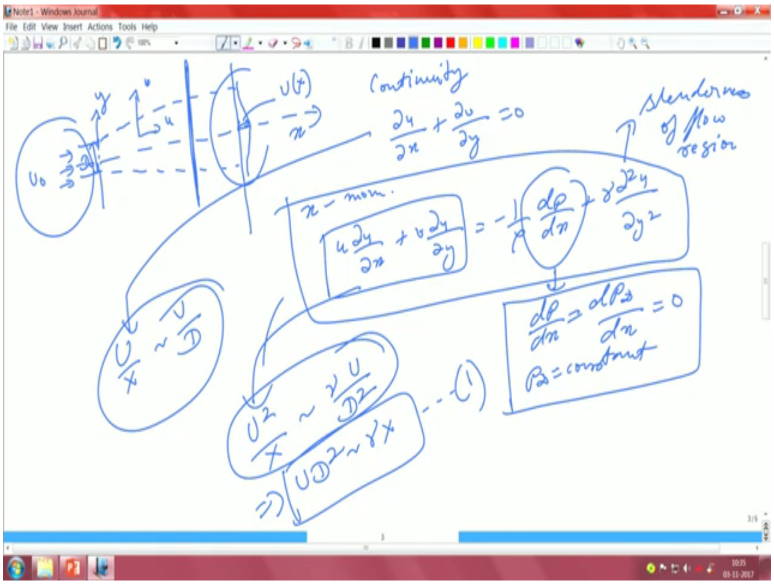

So, let us check that how this is going to be solved. So, this is a little bit more difficult problem than what we had earlier. So, let us check out the jet a little bit. So, it is U naught. So, this is coming out like this and this is at any cross section given cross section this is what the profile actually looks like this is the corresponding U centerline velocity. So, this is basically your u and v and this is the profile this is your x this is your y and this is basically your D naught which is basically the diameter of the opening of the jet..

So, for a 2 dimensional laminar jet using this as a slender flow region, we can write the continuity equal to 0 similarly the momentum and this will be the momentum that we are going to write in the in the x direction. So, the x momentum and we have applied the boundary layer approximation this comes from the slenderness, slenderness of the jet or slenderness of the flow region. So, this particular term as we know this is given as dp by dx is the same as dp infinity by dx should be equal to 0 because T infinity is equal to a constant is equal to a constant right.

So, now if we apply the normal scaling arguments, from the continuity equation you get U by X will scale as V by D right that would be the first scaling that you will have from the x momentum equation as we know that these 2 terms usually will be of the same scale. So, you can write it as U square by X right scaling as gamma U by D square. So, this gives us the equation as U D square is proportional to gamma into X right that is the entire thing we get..

So, this just this is the relation number 1 let us say first relation that we get this is nothing uncommon this is what we got in our traditional flat plate boundary layer equation also assuming this is a very similar looking equation right there is nothing very drastically different about it we have still applied the same slenderness or the boundary layer type of concept over here and of course, we have taken the dp infinity by dx to be equal to 0 because we already said that it is flashing into a reservoir where the pressure is constant .

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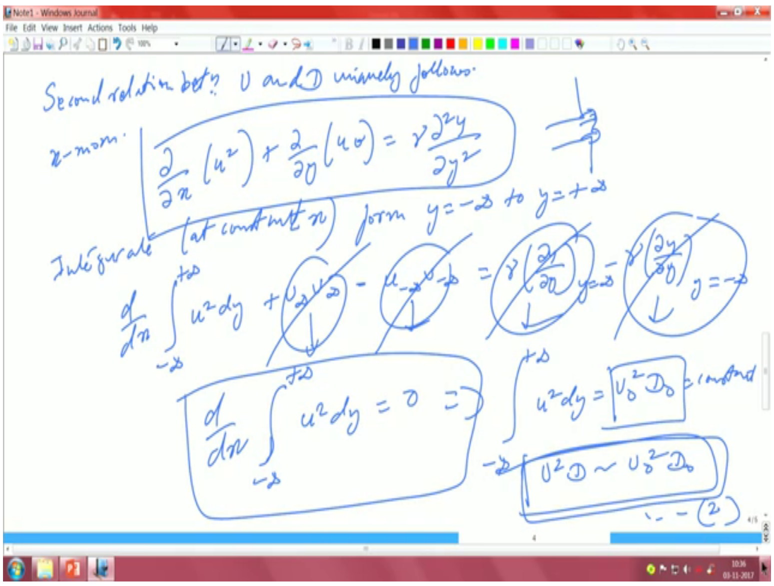

Now the second relationship between you need a second relation between U and D that is because you have one relation there are 2 unknowns you are supposed to find out 2 scales for U and D one for U one for D and you need another equation period right. So, uniquely follows the second relationship uniquely follows that is the hint that we gave. So, let us take the x momentum equation and let us do it like this. So, integrate at constant x at constant x from y equal to minus infinity to y equal to plus infinity we are integrating the whole thing. So, you have d by dx minus infinity to plus infinity u square dy plus U infinity V infinity right that is the first term minus u in U minus infinity V minus infinity right that is equal to gamma d u by dy y equal to infinity evaluated minus d u by dy at y equal to minus infinity right.

So, basically we have taken a section which you can see over here and we have basically integrating out across this particular direction that is minus infinity to plus infinity all right. So, based on that you can see that most of these terms are actually equal to 0 this is 0 this is 0 this is 0 this is 0 because of the far field everything vanishes right it is a far field. So, at y infinity is a reservoir which was initially questioned right. So, everything should vanish at the far field. So, similarly things have vanished in the far field this has vanished this everything kind of vanishes in the far field..

So, all your left with is d by dx minus infinity to plus infinity u square dy is equal to 0 this actually leads to minus infinity to plus infinity u square dy well U naught square into D naught that is equal to a constant must be because why this is so this is equal to zero; that means, this must be a constant and what can that constant be the constant should be the initial momentum that you are flushing out and what is the initial momentum initial momentum is U squared D naught right. So, that is the initial momentum that is the initial momentum of the jet as it comes out of this nozzle right. So, because this is integer this is act like a constant momentum source the jet is like a constant momentum source say any integral formulation that has to conserve whatever initial momentum that you have flushed in right.

So, when you apply the scaling law to this. So, it will be U squared into D should scale as U naught square into D naught that is your second relationship that you have got using this additional scaling argument that the jet is a constant momentum source right..

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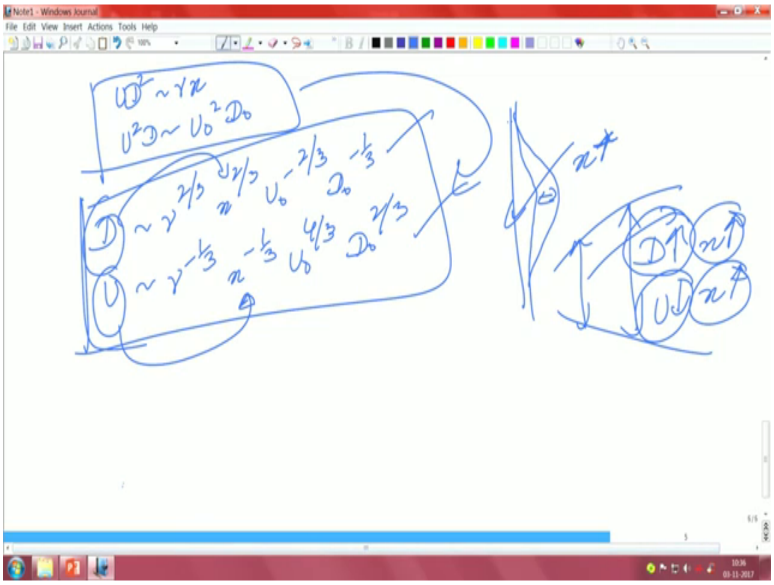

So, now, you can combine the 2 relationships one is U D square scaling as gamma x the other one is U square D scaling as U naught squared D naught. So, we have 2 expressions we have 2 unknowns, it is very easy actually to do. So, D scales as gamma 2 third x 2 third U naught minus 2 by 3 D naught minus 1 third right and similarly U scaling as gamma 1 third x minus 1 third U naught 4 by 3 D naught 2 by 3.

So, you get the expression now you can do the algebra and you can get the forms that we actually mentioned over there D by X and other things that we mentioned if you look at what the question was you can cast it D X by D naught in that kind of a way. So, that is left as an exercise. So, we have derived the main things from this right. So, it was a very simple thing first we did the scaling a simple scaling argument, next what we did was that we used the fact that the jet is actually bleeding out into a constant it is like a quotient reserver as a result far field quantities vanishes. So, the only thing that is remaining is that the initial momentum of the jet is actually is a constant momentum source. So, equating those 2 we related U and D, U naught and D naught and we found our crucial second expression.

So, now you can take this problem further you can also apply it to a temperature field you can see several things for example, that the U of the jet shows x to the power of minus 1 third decay which is kind of logical right; that means, with x this should actually decay the centerline velocity should actually decay right. So, that is one important expression that we get and that jet diameter increases by x to the power of 2 third that is also an interesting observation right so; that means, the jet core expands and the jet centerline velocity decays.

So, the jet core expands as a 2 third of x; that means, as you march on to x the core of the jet expands in that 2 third way whereas, the velocity of the jet the centerline velocity of the jet decays as 1 third minus 1 third of x; that means, it is as x increases this actually decreases right. So, that is kind of very logical because we know that the centerline velocity at different sections if I just plot it like this and it will be like this then it will spread out; obviously, it will look like that. So, there is a decay and this decay in this direction x actually is going up right.

Similarly, the core diameter of the jet as you saw over here the diameter is different from that it is almost like that boundary layer growth right. So, d decays increases with x increase whereas, our U decreases with x increase which is kind of ideological from the diagram that we drew in that in the previous statement. So, if you just to recap. So, this was the way that we solved the problem that this was a constant momentum source that was the argument that we put forward this was the kind of the profile of the jet very simple continuity we get this from the momentum you get this we needed one more equation connecting U and D because there are 2 unknowns..

So, naturally what we did was that we took the we applied an integral formulation to this particular equation the conservative form of the momentum equation and then we canceled most of the terms because of the far field nature of the thing we came across to this expression and from there we got the key relationship the second relationship which is basically says that the jet is a constant momentum source and based on this we have basically found out these 2 crucial relationships which also implies that D increases with x which is logical and U decreases with x which is also logical and they decrease and increase in that 2 third and 1 third fashion.

So, these are some of the crucial things that we did in this particular lecture we will now look at a slightly different problem these were both on external forced convection. So, we in the next one which we are going to do it will be on internal conviction because internal forced convection that is what we are going to do in the next class.

Thank you.