Convective Heat Transfer
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Lecture – 24
Power law fluids

Let us look at the effect now in the last class we covered all this slug flows basically, let us look at an example in which viscous dissipation is involved and let us see that if you have to tackle problems like that how do the equations change because we have not taken viscous dissipation into consideration as of now.

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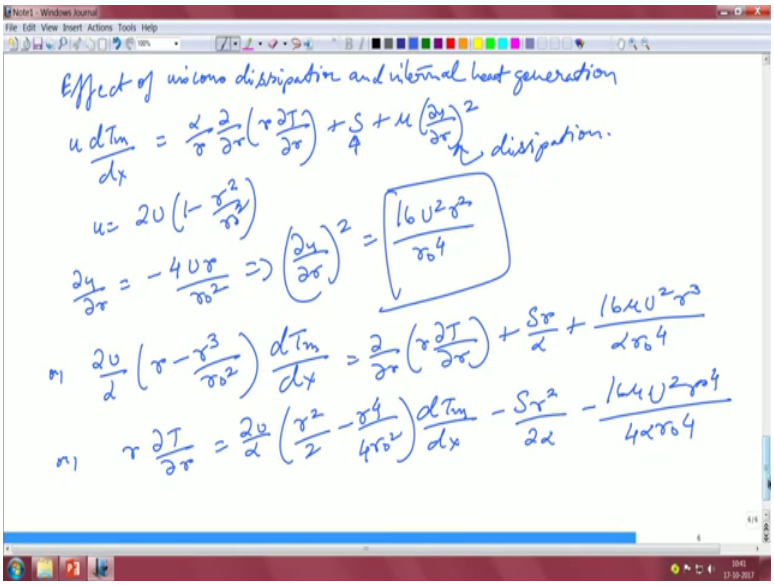

So, this particular thing will be effect of viscous dissipation and internal heat generation. So, basically you have d T m by d x I am just writing the equations and posing the problem. So, intern this is the internal heat generation term it can be constant usually it is constant I mean well it need not be constant, but for practical purposes when you have solving problems you can take that to be a constant this is the dissipation term that we all know right dissipation the dissipation term here u is; obviously, given as 2 U 1 minus r square by r naught squared basically r bar square right.

Now, based on this you d u by d r is basically 4 u r divided by r naught squared right this leads to the fact that your d u by d r square is basically given as 16 U square into r square by r naught to the power of 4 got it. So, this is basically nothing, but the viscous component in the fully developed hydro dynamically fully developed and thermally from the developed regime or in other words if we now back substitute all of these things here just I am writing through the math this is just the equation considering all the terms together now, this can be integrated out.

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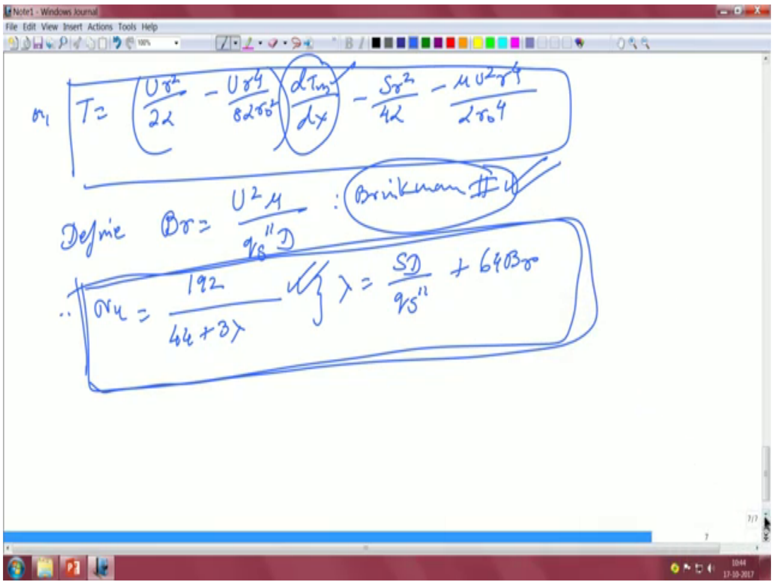

So, if we integrate the thing out T would be equal to U r square 2 alpha. So, that is the total expression now. So, this is how you solve a problem like the one that we gave and here of course, 2 or 3 numbers requires significance one is called the brinkman number which is basically U square into mu by q s double prime into D right that is a brinkman number all the brinkman number.

And your nusselt number if you solve this whole expression out it will be around 192, 44 plus 3 lambda where the lambda is basically S D by q S double prime plus 64 brinkman number got it. So, in the events where basically your dissipation is not important at all you recover back your original nusselt number. So, this is just to give you a pictorial case in which we have just highlighted how a problem with a constant heat generation term can be integrated out, it still requires that expression of T m right in order to solve this in a full fledged manner because we do not know what this T m is going to be. So, it involves a map it involves the same thing, but this is how the equation is actually posed right.

And the 2 key numbers that are concerned with is basically the brinkman number comes into the picture you should ideally try as a homework problem to solve for this nusselt number and show that this is the expression that you get please try it and when you are doing the course using the equations that I wrote for temperature and using the definition of brinkman number can you find out and prove that this will be the value of your nusselt number. So, this would be an interesting problem if you can solve it.

So, next what we do is that right now that I have posed the problem of brinkman number let us look briefly about the situation of power law fluids, so, the power law fluids.

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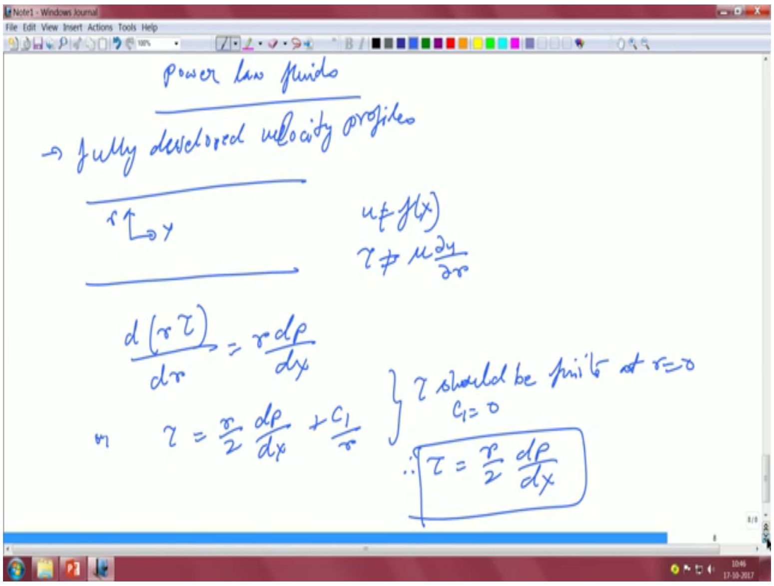

So once again the situation will be fully developed velocity profiles. So, this is the typical thing r x. So, u is not a function of x as we had earlier also as we know that tau is not a function of this anymore that is because it is not Newtonian anymore got it. So, it is not a function, but still this up to this expression will be valid tau d r equal to d r d p d x up to this point we can write because it is again a non accelerating framework the shear stress basically balances the pressure that is all there is right.

So, if you integrate this whole thing out the tau will be r by 2 d p by d x plus C 1 by r. So, this implies that tau should be finite should be finite at r equal to 0 right this implies that your C 1 has to be equal to 0 anyway tau has to be finite at r equal to 0, C 1 has to be equal to 0.

So, therefore, this brings less tau is r by 2 d p by d x that is the expression that we get after this there is no real assumption of any power law behavior the assumption will come now right, but we have not represented tau as mu d u by d r either because that would be a Newtonian approximation. So, here this is the expression that is valid regardless even for Newtonian fluids this should be valid right.

Now, we are going to write the expression for the non Newtonian kind right.

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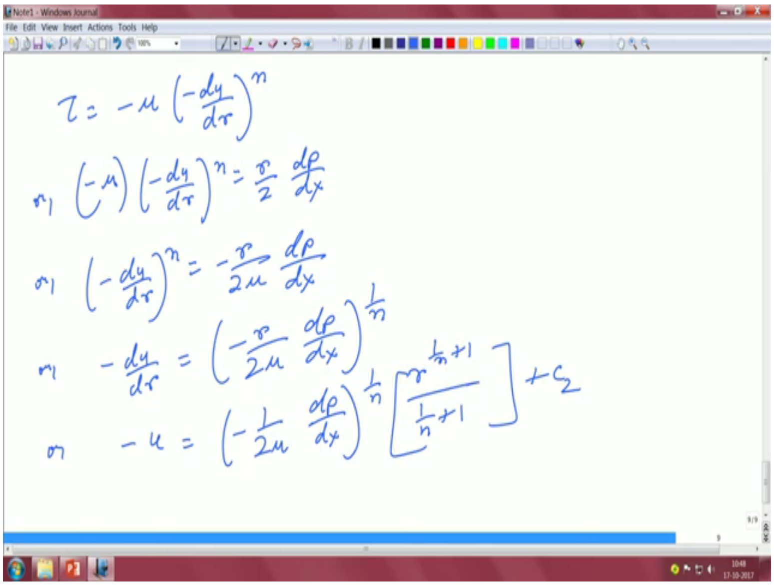

So let us write as tau actually it is tau r x though I am writing it as tau. So, mu minus d u by d r raised to the power of n where n is an exponent. So, what we do over here is now what now that if used you can substitute now you have got tau in terms of the velocity profile you can substitute it will be mu minus du by d r n r by 2 d p by d x or minus d u d r n minus r by 2 mu d p by d x or minus d u by d r is equal to minus r by 2 mu d p by d x into 1 by n or minus u will be equal to minus 2 by mu d p by d x to 1 over n or 1 plus n plus 1 divided by 1 plus n plus 1 plus C 2 right the integration that we are performing go to the next page.

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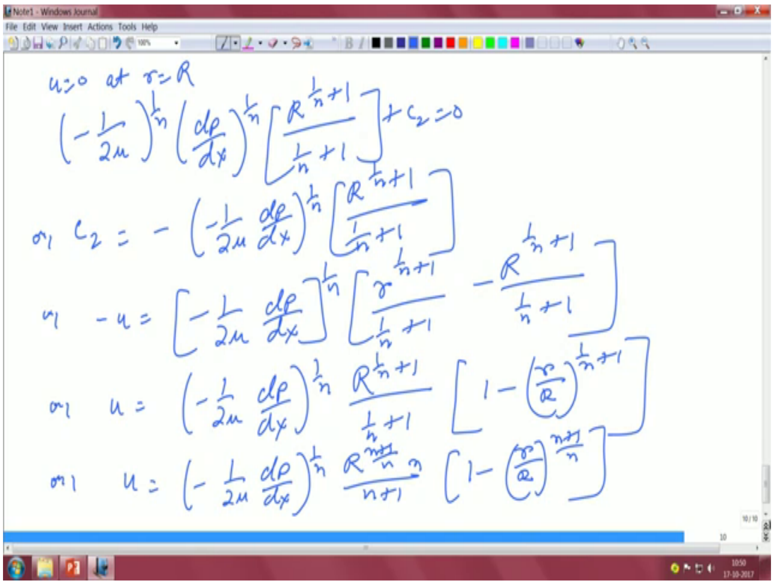

So, u should be equal to 0 and r equal to R capital R there is still the no slip condition. So, minus 1 over 2 mu 1 over n d p by d x 1 over n r 1 plus 1 plus C 2 is equal to 0 or C 2 is equal to minus d p by d x 1 over n plus r 1 over n plus 1 or minus u is equal to, but it are how do you will be this is power. So, that should be kept in mind this n into n plus 1 just long equations.

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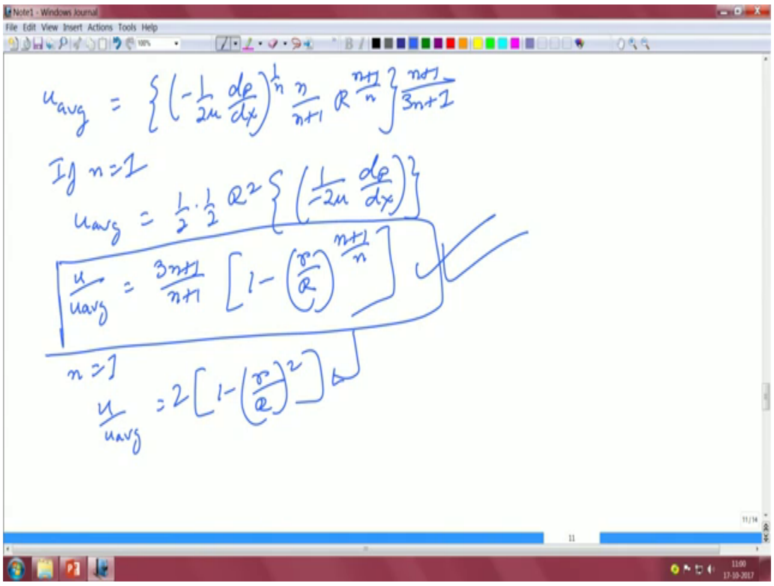

So, your u average should be equal to, that is a total thing now if n is equal to 1 which is basically nothing, but your nothing, but the situation of your Newtonian fluid right because n was the power right. So, your u average would be half into half r square. So, u by u average should be equal to 3 n plus 1 plus n plus 1 1 minus r by capital r plus 1 by n.

Once again if n is equal to 1 u by u average boils down to 1 minus r by capital r square there is a 2 in front. So, that is it that is all that is there right. So, once again we have validated that this is the most crucial expression that will have u by u average in terms of the quantities right the new quantities.

Now, let us look at the corresponding energy equation. So, this was all the hydrodynamics.

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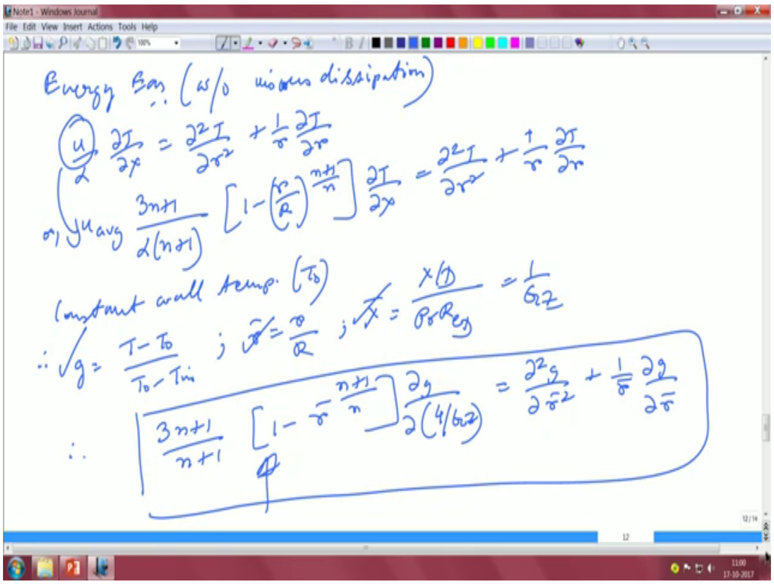

That we did let us look at the energy equation and we are looking at this without viscous dissipation got it without viscous dissipation. So, once again u by r ms d T by d x d by d r squared plus 1 by r d T by dr or u average 3 n plus 1 I will find 2 n plus 1, 1 minus r by capital R n plus 1 by n d T by d x right that is equal to d square T by d r squared by 1 over r d T by d r right.

Now if we take the constant wall temperature case wall temperature T naught. So, therefore, what we will have is g T minus T naught divided by T naught minus T n r bar equal to r by capital R x bar is equal to x by D divided by prandtl number into reynolds number D which is also given as the brats number right this part is also very similar to what we did earlier right. So, if we just substitute all of these things back like what we did earlier, 1 minus r bar n plus 1 by n d g by 4 g alright. So, this expression is very similar to that unsteady, but here of course, you have this all these are terms coming into the picture and basically foiling the whole thing.

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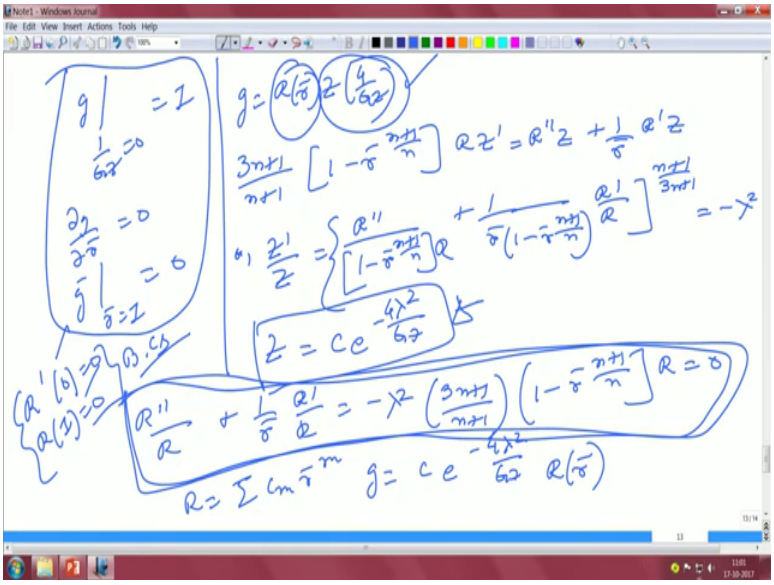

So, once again g at 1 over grads grades is equal to 0 is equal to 1 dg by d r bar is equal to 0 g bar and r bar equal to 1 is equal to 0 right. So, you have your g as r z equal to 4 by g z alright that is what you have. So, 3 n plus 1 divided by n plus 1 1 minus r bar n plus 1 over R z prime z plus 1 over r, r prime z or plus 1 over r bar 1 minus r bar n plus 1 by n R bar by R .

Something like that more very very complicated expression of course, the front part still remains the same that is your z still remains as a function of 4 lambda square by g z right whereas, on the other site your R double prime by R plus 1 by r bar R by R 2 minus lambda square 3 n plus 1 by n plus 1 into 1 minus r bar n plus 1 by n R is equal to 0. So, that is a full expression that if you get.

That is a full expression that you get and g is; obviously, equal to C minus 4 lambda square by g z into R right that is a total expression. So, this needs to be basically solved right to do that normally people do that you assume certain profiles and you try to solve the equations. So, basically you put an assumed profile on the left hand side and try to guess and if the guess is actually match then you move on.

So, but basically it is the R will be still given by a series solution of this sort c m r to the power m something like that it will be still given by that.

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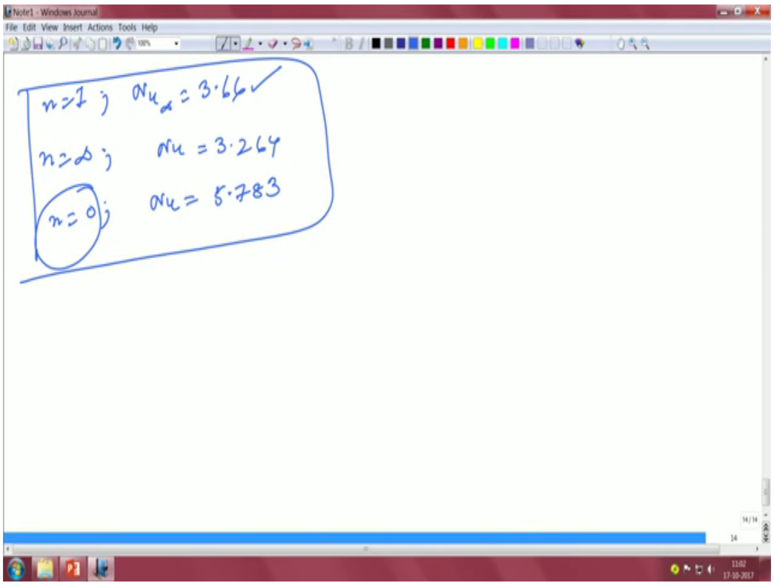

So, if you solve this expression the results will be for n equal to 1 of course, you have your nusselt number approaching 3.66 now for n equal to infinity your nusselt number goes up to about 3.264 for n equal to 0 which is basically nothing, but you are a slug flow solution the nusselt number becomes 5.783.

So, these are the 4 very 3 2 very disparate values of the constant which you can use basically to solve this expressions right. So, the solution methodology let us talk just a little bit we do not want to rush. So, if you look at the profile with respect to R you find that this is not easily solvable expression as you can see no standard things should work like in the last case. So, you have to assume some kind of a profile for R right and try to see that by integration and then iteratively you have to solve this expression.

So, once you do that of course, your boundary conditions for R still remains the kind of like this 0 and R 1 is equal to 0. So, these are your boundary conditions. So, that is the boundary conditions that you should apply for this. So, the power law fluid as you can see and this is a very simple power law fluid where it is given as the nth power of the velocity gradient right and we have seen that of course, our normal Newtonian fluid is just a special case of this power aw fluid.

But in any case what we have seen out of this is basically you follow a very similar methodology all you need to do, you need to solve the hydrodynamics first as we did earlier right now the hydrodynamics will come become a little complex that is what we showed here let us move on. So, this is the hydrodynamics right this is a special case of course, when n equal to 1. So, it is a more complex function essentially then what we do is that once we get this function.

Then we move on to the energy equation what do we do in the energy equation only this u part is the one that we have substituted right rest of the things more or less remains the same your exact assumption for g remains the same, r remains the same, x bar remains the same, x bar is a grads number equivalent to the grads number right. So, what do you do you substitute this expression here for u and basically solve it in a very similar way right.

So, what you do once again you do the separation of variable there is a time term and then there is the corresponding special right and these are the corresponding boundary conditions. Then what we did we went through the normal process we separated out the variables the time term or basically the x bar term is exactly the same what we had earlier it is like an exponential variation only the R term which is basically the spatial term is very complex because your velocity profile at that independence. So, because of that we had a profile something like this how we got around this problem we basically have to iteratively solve this guy right applying the following boundary conditions.

Once that part is done then you can solve it in different limits limiting conditions for the case of one which is basically the Newtonian you recover the 3.66 which is the case for a constant wall temperature for a constant wall heat flux it will be a little higher and for n equal to infinity you get a much lower value which is 3 point well not much lower, but some lower value and for n equal to 0, we essentially recover back the old slug flow solution right in this particular case.

So, this pretty much completes our forced internal convection part of the module. So, we what we have done is that we have done the external forced convection we did the scaling arguments then we did the internal forced convection right which is basically all these things that we have covered till now.

Now, in the next class what we are going to do is that we are going to start on the internal convection. So, first once again we will do external conviction and then internal convection; that means external natural convection and internal natural convection. So, natural convection is where there is no forcing right it happens due to the density gradient. So, we have to first establish once again the equations look at certain approximations like (Refer Time: 25:00) type of approximation and then try to see that how this problem can be actually applied to certain cases once again format will remain the same will still do scaling arguments.

Then we will move on to the to the other stuff right we will find out canonical solutions to the problems and try to see what are the scaling laws over there you will find some interesting numbers coming out like Rayleigh number and grashof number and things like that. So, that we intend to start from the next class before we move on to turbulent convection in the last few lectures.

So, we will see you next class where we will exactly look at this natural convection part that what is natural convection, how it happens, what are the principle equations for the same right and how would the equations change the energy equations and the velocity equations how would they actually change. So, it happens due to subtle density differences and it is buoyancy driven essentially. So, that is what we are going to see here.

Thanks.