Convective Heat Transfer
Prof. Saptarshi Basu
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture – 23
Laminar slug flow

So, welcome to today is lecture. We are going to cover Laminar Slug Flow if you look at on the screen this will be the Laminar Slug Flow.

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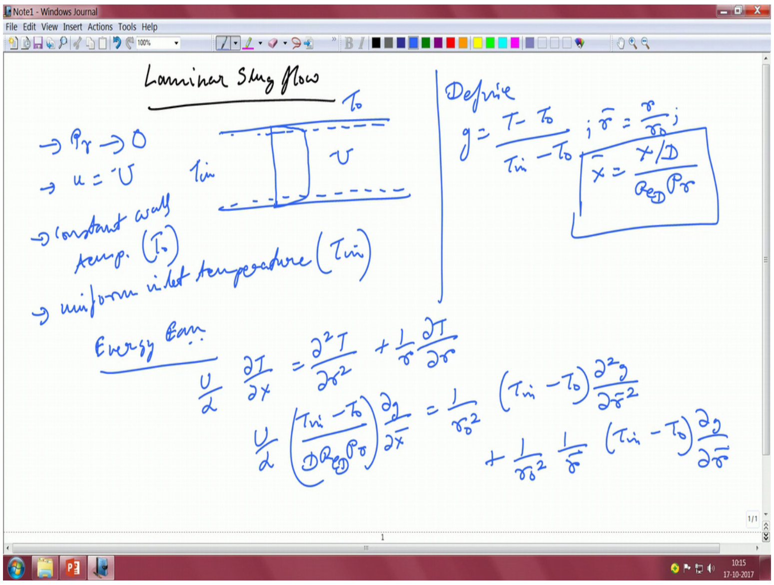

So, let us write down first the key concepts. We are going to say that prandle number almost equals to 0 so; that means, the flow is almost equal. The flow velocity at any point is almost equal and uniform to the inlet flow condition. So, this also means that the velocity boundary layer or the hydrodynamic boundary layer takes a very long time to develop. It does not get developed within the length.

So, the entrance length is basically huge. Not only that, the boundary layer is always thin for the length of the pipe or the duct that we are concerned over here. So, of course, the thermal boundary layer develops much faster than that. It is a velocity boundary layer which is very thin. It is almost like a scenario like this is the velocity boundary layer there will be a boundary layer; obviously. The velocity is kind of something like that. It is very top hat kind of a profile and it is almost equal to the velocity U for all practical purposes. It is never quite true, but since the thinness of the boundary layer is so thin that you can effectively imagine that your velocity is almost the same as you.

So, we also take that it is a constant wall temperature. The rest are usual assumptions; that means, t naught. There is a uniform inlet temperature. Which is t in. These are almost the same as what we had earlier right. So, only thing is that instead of a fully developed velocity profile, which is a function of r, here we have a constant value of the velocity which is U. Mainly because of a situation like this. So, this happens a lot in the food industry because if you are trying to push a solid food or semi liquid kind of a food through the pipes.

They are usually the boundary layers are very thin. As a result of that your velocities kind of uniform across the cross section; that means, the r dependence is not there anymore. That is what I am trying to say, but if the pipe is very long ultimately this velocity profile will develop the thinness assumption is going to be no longer valid, but for all practical purposes, for the current situation that we are dealing with, this pipe length is short enough it is still long, but short enough relatively.

So, that the velocity remains the same. As before we now define our non dimensional numbers or non dimensional variables, r bar is equal to r by r naught, X bar is equal to X by D, R e D into Pr number. If you recall from the previous lectures g is the non dimensional variable which is T minus T naught divided by T in minus T naught. T naught being the wall temperature, you notice a wall temperature; the inlet temperature is T m. Remember that we got from our entrance length analysis that X bar is equal to X by D Reynolds number prandle number.

So, the energy equation U by alpha dT by Dx. That is a standard energy equation that we had. Of course, here we are putting U as the velocity instead of the fully developed velocity profile that you had earlier. U by alpha Tn minus T naught divided by DR e D into prandle number dg by dX bar, 1 by r naught square minus T naught. After substitution let us move to the next one.

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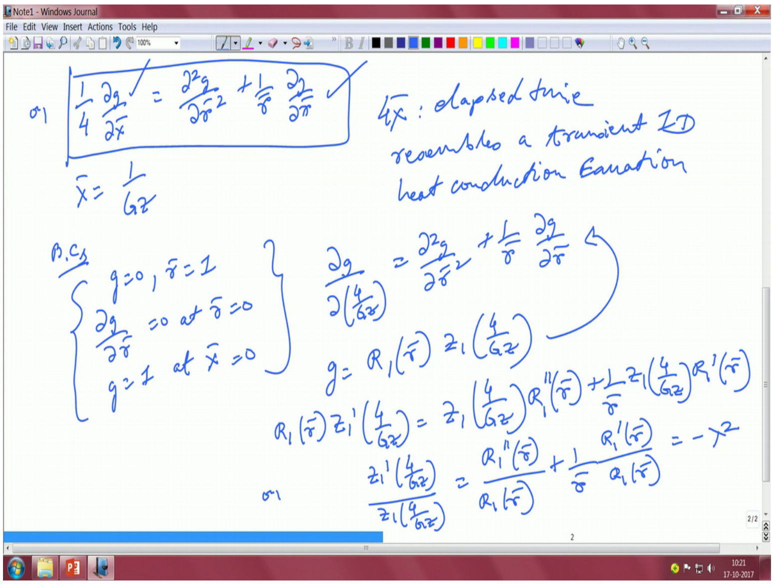

If you take the relevant parameters out or it will be 1 fourth d g by dX bar X square g by dr bar square plus 1 by r bar.

Now, as we know that one by X is sometimes written as 1 over the grads number. This is written in some of the books. In other literature you will find, but essentially this is just a reciprocal of the normalized distance. So, the boundary conditions are, g equal to 0 at r bar equal to 1, dg by d bar equal to 0 at r bar equal to 0 and g equal to 1 at x bar is equal to 0. You can easily see the veracity of this thing. At r bar equal to 1, where T is equal to T naught. So, therefore, g should be equal to 0. r about equal to 0 should have an inflection point. Therefore, dg by dr should be equal to 0 over there because of the temperature inflection point and at X equal to 0; that means X bar equal to 0 that is at the point of the inlet where T is basically equal to Tn. So, therefore, g should be equal to 1. So, these are the 3 boundary conditions that we have.

Once again as I told you earlier that this equation that you have represents a unsteady heat conduction equation. Where X bar basically doubles up adds the time scale. So, normally in a heat conduction equation or a transient heat conduction equation or 1D transient heat conduction equation, that is what it is in of course, cylindrical coordinates you would have that there will be this term will be basically your time and this will be the corresponding spatial coordinates. So, here of course, the X bar is doubling up as your time. As the actual length is basically what your time scale is all about. It is very equivalent though the problem is not the same.

So, it is basically an interesting observation nonetheless that where your 4X bar is basically like your elapsed time got it. 4 X bar is nothing, but like your elapsed time. So, it represents or resembles rather resembles transient 1D heat conduction equation got it. The solution will be kind of similar. Let us write it in a more, put the grads number. This is the most compact way of writing the same equation. Once again you use the separation of variables. Your g will be basically r 1 r bar, Z 1 2. So, one is a transient part and one is the spatial part. Correspondingly well in this case both are spatial, but one is like time.

So, if once you substitute it back into this particular equation. What you have is basically, that is what you have. Or once again I have told you the significance of choosing minus lambda square, that is because of the reason. Otherwise you get an exponential term which increases. Here the solution has to be always bounded. You cannot have that exponential increase. Otherwise mathematically speaking plus lambda square is an equivalent solution so, but in this case from a physical point of view it cannot be the case. That is why we have written it like that. That we covered in our earlier class, when we are doing thermally developing flow. So, it is the same analogy actually works here.

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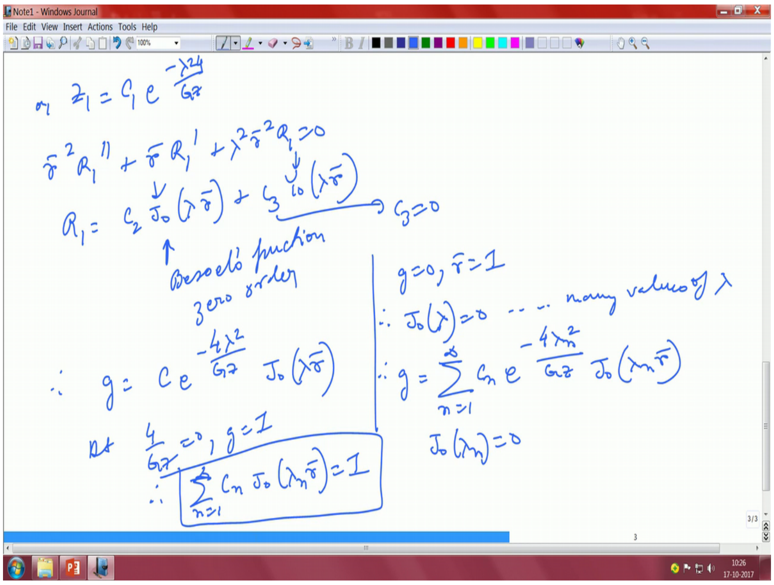

So, or Z 1 is equal to C 1 exponential lambda square by GZ into 4 alright. That is the first solution that you will have. Once again you can see that it is exponential and it is negative. So, therefore, with time or with distance the solution is bounded. On the other hand r 1 is basically C 2 and J naught lambda bar plus C 3 Y naught lambda bar that is a solution. These are basically Bessel functions of course, as you can see it is of the order 0. 0 order first kind, second kind.

So, J is the first kind Y is basically the second kind. So, that is the solution that you have, but once again one interesting thing about this particular solution is that let us look at the corresponding companion ppt.

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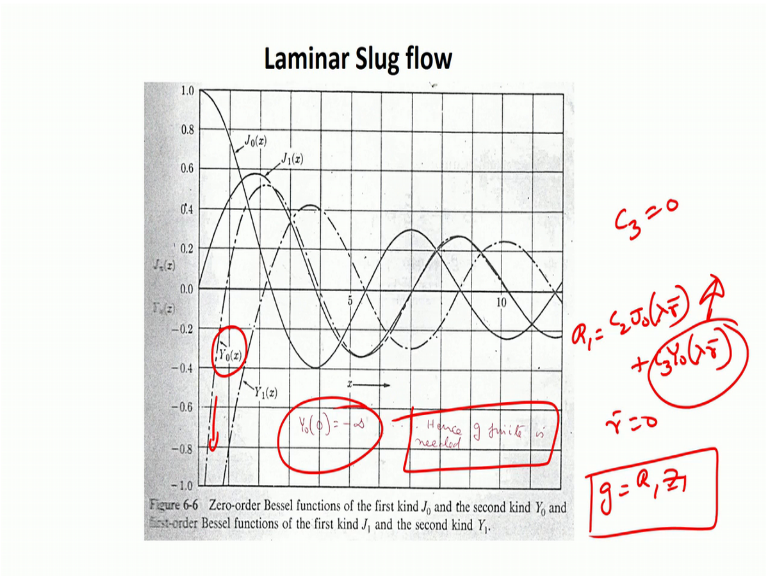

If you look at now a situation like this, where we have basically plotted the functions and I have marked a few things over here. So, as you can see that at this particular term Y is basically the second kind. At 0 it goes to minus infinity.

But you know that your g because the r is basically a combination. If you recall what we did just now was basically that you had r, you had r 1. If we write it here then it is equal to J naught lambda r bar plus C 3 Y naught lambda r bar correct. These are the 2 things that we have just now. Of course, as you can see at r equal to 0 therefore, or r bar equal to 0. This particular term basically blows up. That is because it goes to infinity from what you can see over here there is a infinity.

So, in order to assure if I write it as g, because what is g is basically nothing, but R 1 into Z 1 that is g. R 1 into Z 1 is g. In order to keep the g finiteness of g, what we can technically assure in order to do that is basically to make C 3 equal to 0. So, C 3 has to be equal to 0. Because otherwise this entire term will blow up at r bar equal to 0. So, that is the logic that we will apply over here and make that C 3 therefore, should be equal to 0 correct.

So, moving on going back to our journal therefore, your r here in this case C 3 must be equal to 0. Therefore, your total g will be now C exponential 4 lambda square by G Z into j naught into lambda r bar. Combining the 2 constants and assuring that C 3 is equal to 0. Now of course, you have g equal to 0 at r bar equal to 1. So, this of course, implies that J naught at lambda is equal to 0 as many values of lambda.

So, therefore, your g should be equal to n equal to 1 to infinity C n exponential minus 4 lambda n square G z J naught lambda n into r bar got it. And J naught into lambda n is equal to 0 got it. So, multi values of lambda n. Also at 4 by G Z is equal to 0 and g is equal to 1 that also we know. Therefore, n equal to 1 to infinity C n J naught lambda n r bar is equal to one got it.

Now, taking this forward therefore,

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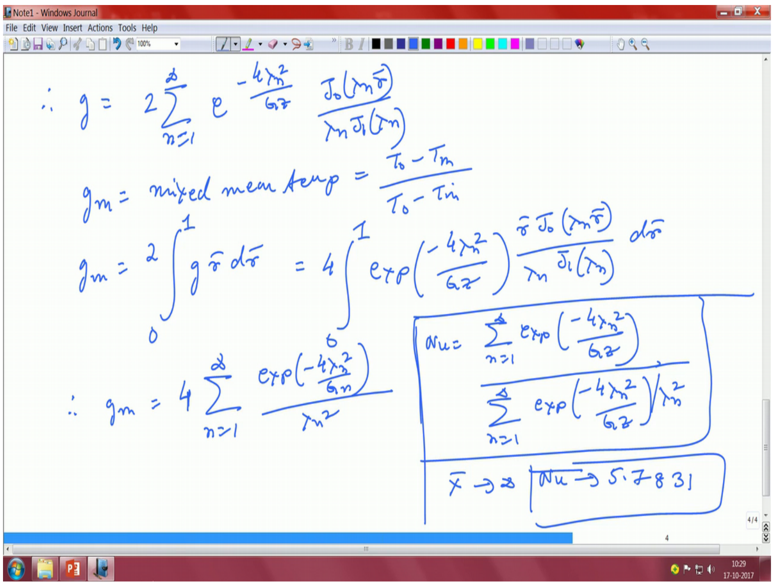

g is equal to 2 into n equal to 1 lambda e 4 lambda n square into G z J naught lambda in r bar divided by lambda n J 1 lambda n. This is of the first order Bessel function of the first kind. So, g m which if you recall is the mean temperature or the mixed mean temperature. That is given as T naught minus T m divided by T naught minus T n. That is the mixed mean temperature that we have alright got it.

So, g m in order to evaluate that you already know how to do it, it is integral 0 to 1 and g r d r, that is what it is. 4 0 to 1 exponential minus 4 lambda n square by G z r bar J naught lambda n r bar lambda n J1 lambda n dr bar therefore, g m was would be equal to 4. That is g m for you got it. So, that will be g m. That is the mixed mean temperature what we defined similarly your nusselt number which will be one of the most important parameter over here will be see all the series of converges pretty fast. You do not need to consider a whole lot of terms to get convergence.

So, this is the value of nusselts number at any point; obviously, as X bar approaches infinity, what happens is that your nusselt number this particular form approaches 5 point 7831. It is a little higher than your other cases. That is because you are dealing with a uniform flow field right now. So, this is the key value and of course, we have in the PPT.

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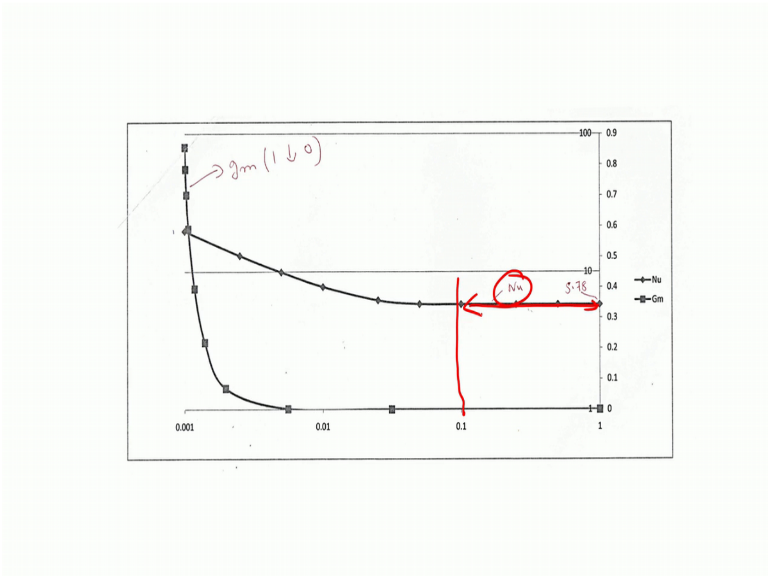

We can show that this is how the entire thing actually works. As you can see the, this is a mixed mean temperature and this is the corresponding value of nusselt number. Once again it starts off pretty high at around X bar is equal to 0.1. It slowly asymptotes to the value of about 5.7831 and after; that it is constant. Because that is the thermally developed regime and the only thing is that here u is equal to capital U. So, that is the only caveat that we have in this particular case.

So, that is interesting. You should note the difference between this and the corresponding situation, where the velocity profile was not like this. I would post a problem because of the constant wall heat flux for the same situation.

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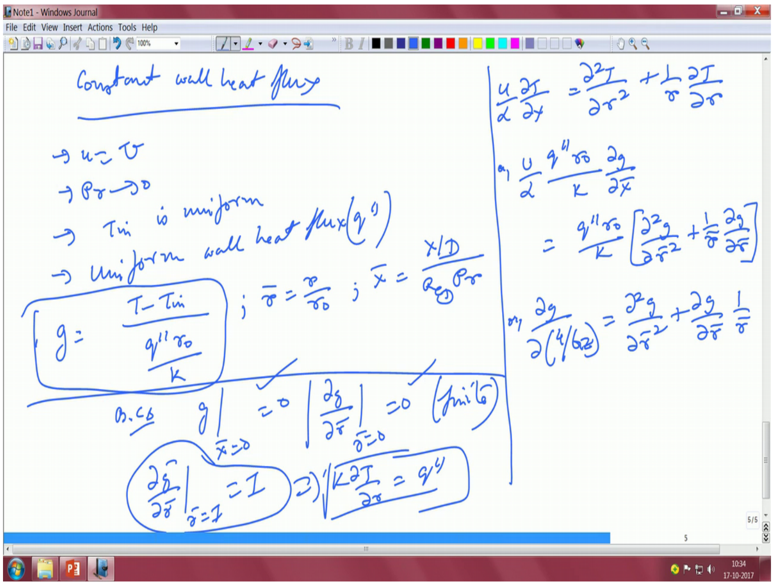

But, you can solve it on your own. Once again the flow is a slug flow. The prandle number goes to infinity goes to 0. Inlet temperature is uniform. This is uniform. There is a uniform wall heat flux. That is only difference which is q double prime. That is all. There is a uniform heat flux which is given by q double prime.

So, now you definition of g is T minus T m divided by q double prime r naught by K. r bar equal to r by r naught, X bar is equal to X by D divided by R e D into prandle number. So, those are the things that we have. Then you have your u by alpha dT by dX is equal to d square T by d r square plus 1 by r dT by dr. The same thing is valid now over here except now your u becomes once again capital U. q double prime now comes over here because of your definition. This dg by dX bar is equal to q double prime r naught by K r bar square plus 1 by r bar dg by dr bar.

Now, if you take a few terms off from both sides. Once again your equation remains exactly the same. Of course, the boundary conditions will be different as you know r bar square plus dg by dr bar into 1 by r bar. Only thing is that. So, nothing has changed as far as the equations are concerned. They still are the same. Except now the boundary conditions are a little bit different. So, g at X bar equal to 0 is equal to 0. That is obvious once again because your t in basically equal to t in. That particular case dg by dr bar at r bar equal to 0 is 0. Because of the finiteness of the whole thing and dg by dr bar at r bar equal to 1 is equal to 1. Now that is because your heat flux because you have a constant wall heat flux.

Now, this is because your K dt by dr is equal to q double prime correct. That is because of that. So, the equations as I said the only boundary condition that has changed is this guy over here and your definition of g also has undergone a change right. Based on these 2 factors that your g has undergone a change and your dg by dr you have to write the flux conditions like that if the rest of the problem can be worked out exactly in the same way as before because this part of the equation does not change at all. It is just the boundary conditions. Now that you have to apply and then you have to see what kind of answer that you get.

So, that is what is left as a task, but I have posed the problem and we shown how the problem actually works nothing more changes except please note that now this is the way that we have written it. So, here what we do is that in this particular lecture we have covered the uniform slug flow.

In which we have shown that though this is an unrealistic situation, but for all practical purposes in the boundary layer remains small throughout the entire span of the pipe length. Then perhaps this is not a bad approximation. It might over predict the nusselt number by a little bit, but that is so long because your analysis is very simple. And it almost represents like a transient heat conduction equation as we saw where the conduction term is axial conduction term is or the time varying conduction from is basically the axial conduction term.

So, based on this we have formulated we have shown that the limiting case of nusselt number. At least in the case of constant wall temperature is 5.78 which is higher than the nusselt number. For both the other cases; that means, for uniform heat flux and uniform wall temperature for the fully developed thermally and hydro dynamically fully developed flows that we did much earlier. The solution is a little bit on errors in the sense that it involves separation of very and series solutions essentially, but as we know that they are though they are at infinite number of terms. The series converges usually pretty fast; that means, the terms do not change progressively gets smaller. So, that you do not have much of a change as we include more higher and higher order terms. Within 4, 5 terms, 6 terms you basically have convergence.

So, in the next class what we are going to do is that we are going to look at the viscous dissipation part just a little bit and we are going to post a problem of power law fluids; that means, fluids which may have, which is not a Newtonian fluid by nature right. So, they have got a power law behavior. This is not a rheological flow course; that means, it is not deals with Rheology or complex fluids. What we are going to do is a very straightforward.

There is a lot people can spend an entire course on rheological flows. How the rheological flows? Actually you know the convective heat transfer mode of such flows where shear is not as such a simple function of viscosity, but in this particular case we are just going to give a simple example and see how that those problems can be tackled and move on, if you want a more fully fledged course, you should consult you know convective heat transfer or heat transfer in the complex fluids.

Thank you.