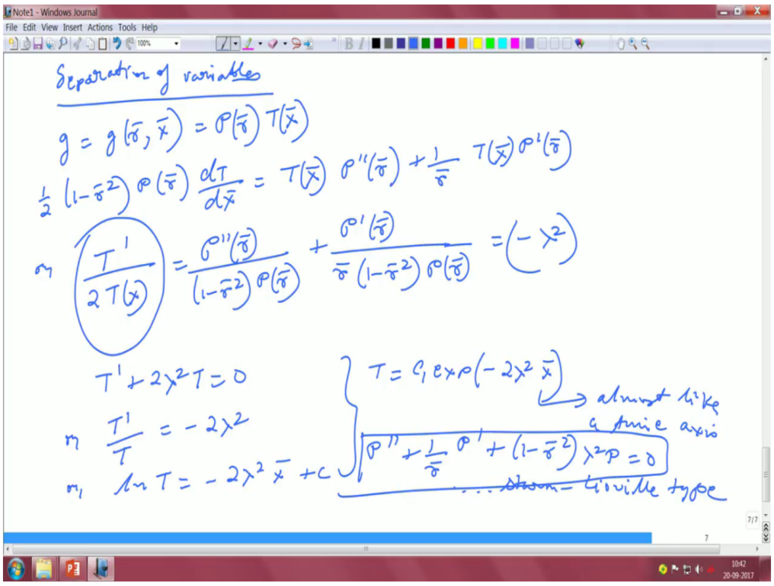
Convective Heat Transfer
Prof. Saptarshi Basu
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture – 22
Heat transfer to fully developed flow-II

So, let us do the separation of variables.

(Refer Slide Time: 00:23)



So, g as we know is a function of both r bar comma X bar. It is basically Pr bar into TX bar. That is the situation. Now if we put back substituted in the main energy equation, let us see what we get. The energy equation is 1 minus r bar square Pr bar into dT by dX bar is equal to TX P double prime r bar plus 1 over r bar TX bar P prime r bar. That is the total expression that we get.

Now, of course, here as you can see this is 2 functions. P and T which is a function of r and X only. So, that is how you could write it in the ordinary differential form. Now, do a little bit of this. As you see the left hand side and the right hand side, left hand side is only a function of X; right hand side is only a function of r. This is only possible when they are both equal to some kind of a constant, which is basically lambda square over here. We have taken minus lambda square. There is a reason behind it. Like as I said plus lambda square would have been also and exactly I mean mathematically it is correct. You can actually have a constant, which is plus lambda square.

Now, in this particular case if we put last lambda square over here, what will look at the first term right over here? That is T prime divided by 2 T X. So, if we take plus lambda square, this particular expression is actually going to blow up because it would have been T prime plus minus lambda square into 2TX. If you plot that if you solve for that it will show an exponential increase of the temperature, which is basically not a physical thing over here because temperature over here cannot actually grow exponentially. As we say that it is bounded. So, based on that we have taken minus lambda square as the limit.

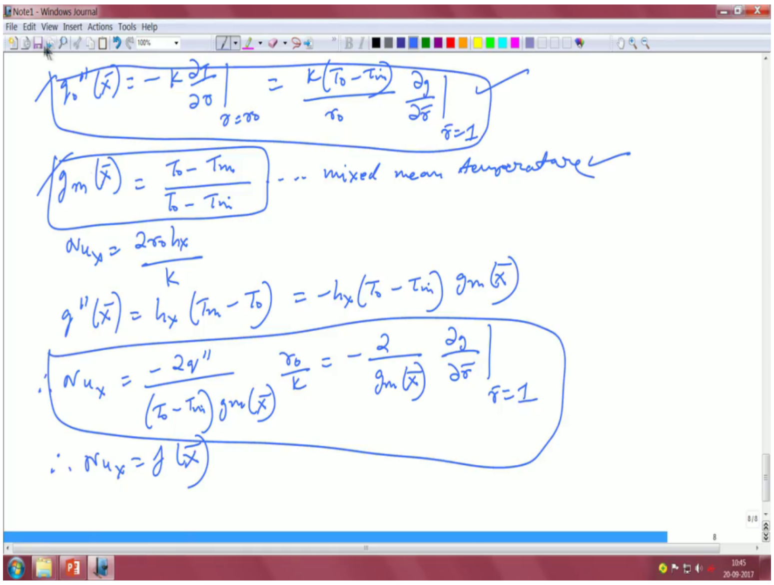
So, T prime plus 2 lambda square into T is equal to 0 or in other words T prime by T is equal to minus 2 lambda square or ln T is equal to minus 2 lambda square into X bar plus C. This actually leads to T is equal to C1 exponential minus 2 lambda square X bar. As you can see if this lambda square was chosen as positive this would have come plus over here. If you would have come plus then this temperature would have actually shown an increase with respect to x. So, that is something that is not feasible.

So, here you can see this is almost like a time axis. This is of course, what you see in conduction also that there is a decay. So, this is almost like a time axis the first solution. The second one which is basically one by r bar P prime plus 1 minus r bar square into lambda square into P is equal to 0. This particular expression is the Stern Linville type equation. This is the second part. You understood why the first part is almost like why we chose minus lambda square because otherwise it would have grown exponentially which is physically not possible. The other part is that it is almost behaves like a time axis.

So; that means, that is this x is almost like a time axis x. as x grows this temperature or this particular T. This is not temperature. This is basically the axial variation part of the temperature. You can call it that way that actually shows a decay. So, this is physically meaningful and it almost like a time axis. The year X is almost like snapshots that you are taking. As it precedes an X it is like almost it is emerging onto time. Here the X represents what we call the time axis. It is almost equivalent that you are filming a same fluid with time at a particular location. There is almost like the equivalence of that.

So, now that we have got that kind of an expression. Now that few more things that we can write down.

(Refer Slide Time: 05:55)



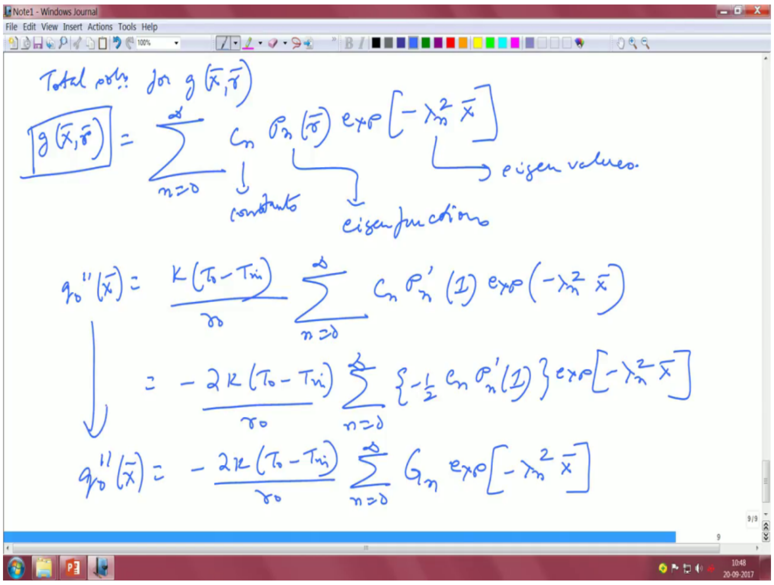
For example, your q double dot X bar equal to minus K dT by dr at r equal to r naught, which is basically K into T naught minus T in divided by r naught dg by dr bar at r bar is equal to one. You can see that this is the heat flux. Then of course, you have your g m X bar which is given as T naught minus T mean divided by T naught minus T in. This is what we call the mixed mean temperature.

So, this is gm X which is basically nothing, but the mean temperature. What we have done is that we have substituted T by T m. That is the only thing. We are calling that as g m. And this is the heat flux and this is the mixed mean temperature. Similarly your Nusselt number at any X is given by 2r naught h x by K. So, that is also given. q at any X bar is also given as h x into Tm minus T naught, which is basically minus h XT naught minus T in into g m into X.

So, this is just substitution of variables therefore, your Nusselt number X is which is 2 q double prime T in g m X bar into r naught by K equal to minus 2 g m X bar dg by d r bar and r bar is equal to 1. This is theta. Therefore, you know that your Nusselt number is basically a function of your X bar and that was obvious, but we have restated it over here for convenience that this is what you have. It was not like it was any different.

So, these are some of the fundamental relationships that we have written. Particularly these expressions are all very important. What is the definition of Nusselt number? What is the definition of q flux? And what is the mixed mean temperature? These are handy things, which will come in handy in due course as we will see. The total solution

(Refer Slide Time: 08:35)

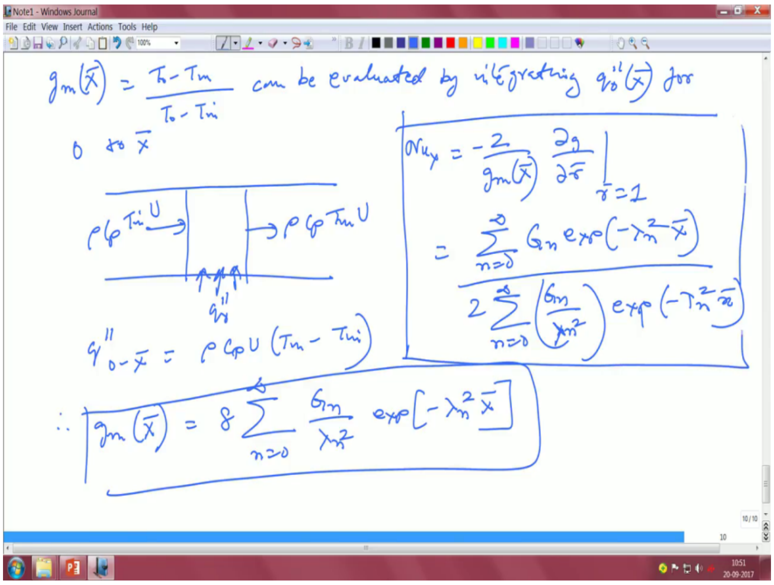


for g X bar comma r bar. So, g X bar comma r bar is given as n equal to 0 to infinity Cn Pn r bar exponential minus lambda n square into X bar. These are basically constants. This is the total solution.

These are basically your Eigen functions and these are correspondingly Eigen values, got it. That is the total expression. This is the total expression for the temperature. This is the total temperature. It is given in terms of the Eigen values and Eigen functions and as well as the constant. So, basically the main purpose then boils down that how you evaluate this constants and things like that. q double dot at X bar which is basically the heat flux is basically T naught minus T in. This has got no bearing because they are constant basically n equal to 0 to infinity Cn Pn prime 1 exponential minus lambda n square into X bar.

So, this can be further written as 2 KT naught minus T in divided by r naught n equal to 0 to infinity minus half Cn Pn 1 closed exponential minus lambda n square into X bar. In other words this can be further simplified and written. Substituting in by G n. Basically substituting that term within the third bracket and lambda n squared in to X bar. This is the total and I can write q here. From here to here we have got through these steps based on this. That is a good thing that we did. How do you evaluate gm X bar?

(Refer Slide Time: 11:25)



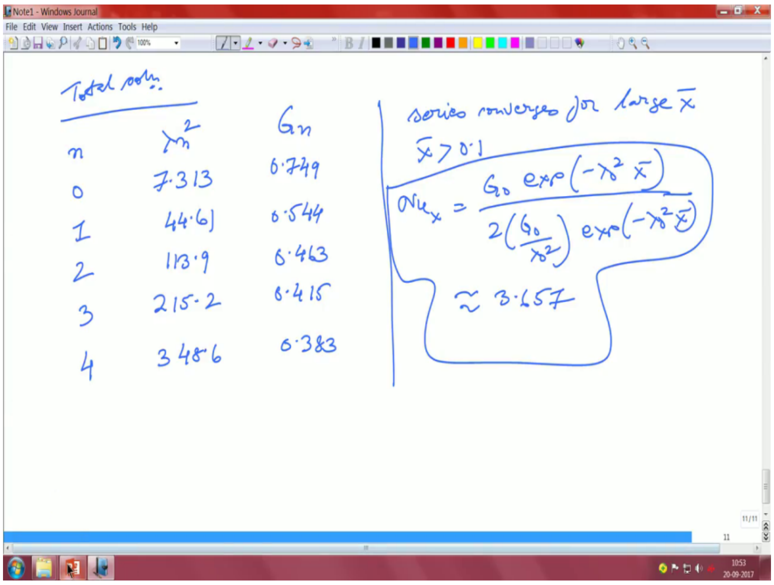
That is basically once again if you recall T naught minus T in.

So, at every step we give this expression. So, that you do not lose track this can be evaluated by integrating. q naught double X bar for 0 to X bar. So, this is almost boils down to something like this. This enters with a temperature of T in into U. This exits with a temperature of T m into U. Like this and there is of course, q naught that has been added to this control volume. That is what we have done.

So, q double prime from 0 to X bar. So, this is not local. It is averaged over whatever distance. That is equal to rho C p tm minus T in. T in is basically a constant as we know. So, therefore, g m X is basically given as n equal to 0 to infinity g n by lambda n square exponential and then square into X bar not going through the math details, but this is how you actually evaluate it.

So, this is the mixed mean temperature. Then of course, you have still the Nusselt number. The Nusselt number is basically given by 2 by g m X bar dg by dr bar at r bar equal to 1. This is of course, given as n equal to 0 to infinity capital g n exponential minus lambda n square into X bar divided by 2 n equal to 0 to infinity g n by lambda n square is lambda and square exponential minus lambda n squared into X bar. So, that is the solution for this. Now similarly therefore, we can. Nusselt number we know what will be the final expression. g m we know what will be the final expression.

(Refer Slide Time: 14:08)

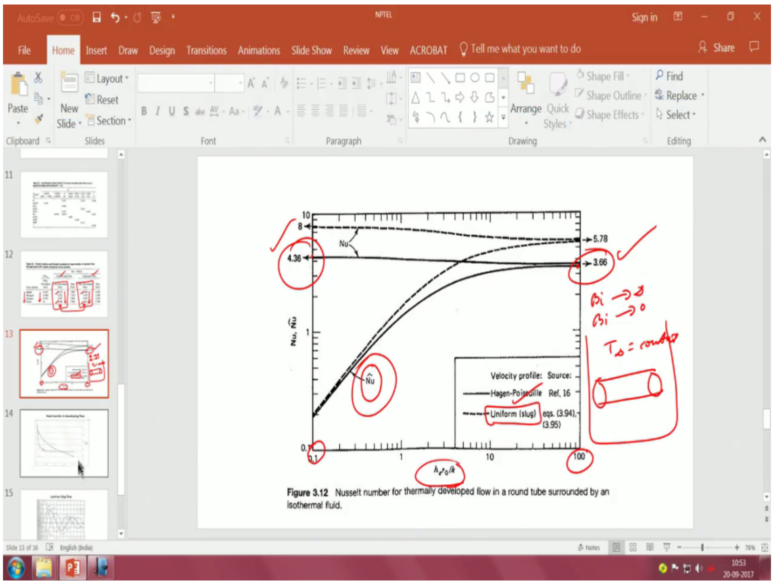


So, the total solution the total solution is n and 0, 1, 2, 3, 4. Then there is lambda n square which is 7.313, 44. 61 and 113.9, 215.2, 348.6. The capital G n values which we already established here are these the series converges actually pretty fast.

So, about 4 or 5 terms are actually what is needed. The series converges for large X bar that is. Basically X bar greater than about 0.1. It kind of converges. Your Nusselt number based on this becomes G naught exponential minus lambda naught square into X bar divided by 2 G naught by lambda naught square exponential minus lambda naught square into X bar. Give you in the fully developed regime some value of approximately 3.657 which agrees very well with our estimate of a constant wall temperature that we had earlier. So, that would be very consistent with this value at the end after the series converges.

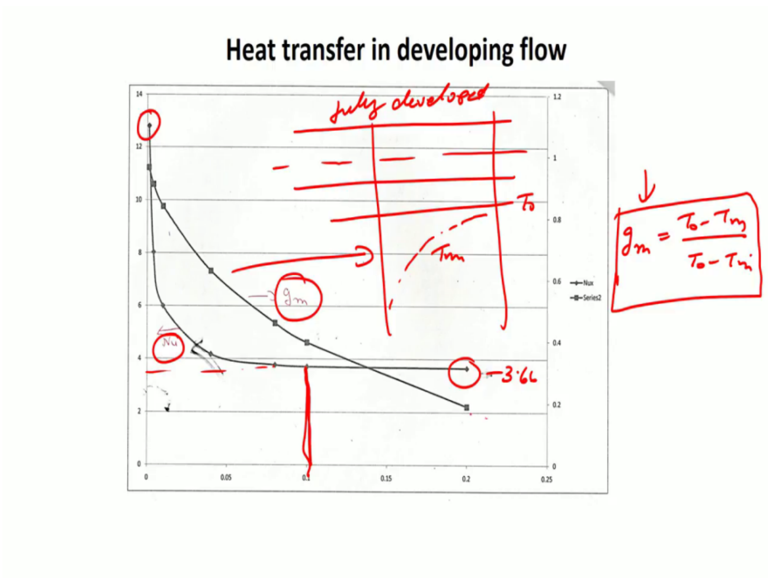
So, in between if you want to see that, what is the nature of these curves.

(Refer Slide Time: 16:05)



Take a look at this particular plot over here that we have.

(Refer Slide Time: 16:12)



Now here this is Nusselt number. This is basically your G m. So, as you can see the Nusselt number starts off very high and right around 0.1 is the 0.1 axis. It converges and comes down to a value which is 3.66. This is the 3 point we should look at this axis this is plotted in the primary axis whereas, this is plotted on the secondary axis g m that is the mixed mean temperature. So, that is plotted on the secondary axis. As you can see the mixed mean temperature actually comes down as we increase the as we go along X. That is because once again g m if you look at it is basically T naught minus T mean divided by T naught minus T in.

So, as we go down as we increase in X. We can see that this particular value of your temperature profile becomes smaller and smaller. Because as the mixed mean temperature approaches the T naught limit that is you have seen that in your laminar duct flow results also if you recall that what we saw in your laminar duct flow result. What does it show? It kind of curves up and approaches the T infinity so, it always tries to approach that T naught. If you recall the last time I mean I can show you that particular plot once again if you have forgotton about it.

If you look at the plot for your fully developed regime for fully developed if you recall what we did, we saw that if this was the temperature profile, this was T naught and if you recall that your T m continuously approaches. That is the reason why this number becomes smaller and smaller and smaller. It continuously will fall into the approach. It will start very high which is natural because the mean temperature at that point of time will be much lower, but then it approaches that T naught with time, but this particular denominator; obviously, remains the same that is the initial temperature difference.

The mean temperature slowly becomes closer and closer to the wall temperature. That is the reason because it is a uniform wall temperature problem or isothermal wall problem. Therefore, it continuously goes and kind of meets that continuously should decay even after you go to the fully developed. After this point you are basically entering into the full later of that regime.

On the other hand Nusselt number as we can from common sense we can see that the delta T is very small. So, the Nusselt number value will be very very high. Now, Nusselt number starts off very high, but with time with distance and it just equivalent to time what happens is that the Nusselt number slowly approaches this asymptotic limit which is 3.66 which we just proved that, that is indeed the case.

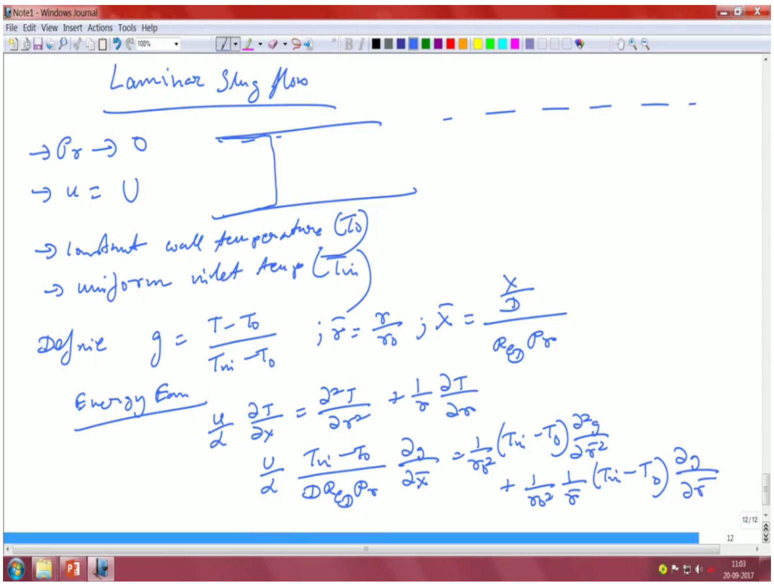
So, from 0.1 onwards X bar greater than about 0.1. Your Nusselt number starts to show a very constant value which was exactly what we did in our laminar flow simulation. You know a laminar flow analysis in the fully developed regime. So, you can understand. You have used a simple separation of variables. We have got these answers and this answers makes sense. As we can see that these answers makes perfect sense, when you actually look at the problem, but at end we have shown through graph that what is the nature of the variation of this and we also validated that the end of the boundary layer that and at the end of the developing flow, we still get the same value as what we would have got in your fully developed regime. So, similar things can be done for constant heat flux as well that is also possible to do and we can do that as an exercise that is one other thing.

Let us go back to the journal article part once again. So, this is the solution you do not have to memorize anything. It is basically this is the solution. If you do it properly this is what is available and most of it is are available in handbooks and stem (Refer Time 20:49) type of equation you must have solved in your math courses as well. So, this is nothing new.

But the results that key definitions of g m, Nusselt number, heat flux, how we approach the problem those things are very important. Remember one key thing here is a hydro dynamically fully developed flow with a uniform temperature inlet and a uniform wall temperature. These are the major assumptions that has gone into this.

Now, we can look into a variation of this similar problem which we call as the Laminar Slug Flow we will introduce the problem here.

(Refer Slide Time: 21:28)



So, Laminar Slug Flow will take up the discussion a little later. A Laminar Slug Flow is normally happens when prandle number is much low and your U is basically a constant. Now that is not physical because when you say U is not a pipe flow profile anymore that is just a constant; that means, that does not it is not physically intuitive and it is not, it does not make physical sense either.

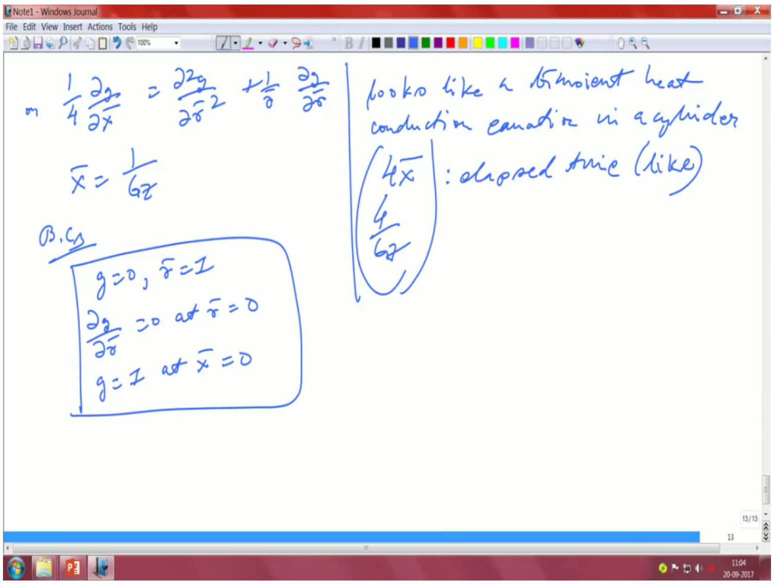
But; however, if you look at the problem say for example, you are pushing some kind of a semi solid food you know through a pipe. That you can call something, that the boundary layers are so thin. That is almost like a uniform profile; that means the flow that does not have. That is why we say that the Prandtl number is very low; that means, the hydrodynamic boundary layer takes an enormous amount of time to develop. So, it develops with a very thin boundary layer by the time the pipe is over.

The hydrodynamic boundary layer has barely developed. But, you see that it is physically impossible because of the viscous effects. There will be as no slip condition, but for all practical purposes this boundary layer is so thin. Like I mean it is something like this. This boundary layer takes an enormous amount of time to develop. That is why I say that Prandtl number goes to 0 kind of a limit; that means, the thermal diffusivity is very high. This is very very small.

So, this is the context of the problem. It is not like that. It is physically meaningful, but all it means is that your hydrodynamic boundary layer barely develops for whatever may be the reason. So, then there is a constant. We assume that is a constant wall temperature. Which is T naught and uniform inlet temperature. Inlet temperature which is T in. These 2 assumptions are basically still the same.

You can define now g equal to T minus T naught by T in minus T naught. Again the similar type of definition, r is equal to r bar naught and again X bar is X by D divided by R e D into P r. So, it still we are in that developing region, but the flow here shows a very interesting characteristics; that means, it is a constant flow. Again I say it is physically not possible. This is just for assumption purpose. So, the energy equation P r squared plus 1 over r dt by d r. This part of the equation is still the same whatever you do. This becomes right. So, that is the equation that you get.

(Refer Slide Time: 25:17)



So, now or it is a little simpler looking question from what we have because that 1 minus r square is not there. In some books this X bar will be written as 1 over grades number one over G z, but it is just how the definition works. Once again this looks very similar that there is seems to be like a time type of a term on the left hand side and then there is the radial variation.

I will write down the boundary conditions that g is equal to 0 at r bar is equal to 1 d g by d r bar is equal to 0 at r bar is equal to 0 and g is equal to 1 at X bar is equal to 0. These are the 3 sets of boundary conditions that we normally have. So, this is an interesting observation. This looks exactly like a transient heat conduction equation. It looks like a transient heat conduction equation. Looks like a time marching in a cylinder; that means, in cylindrical coordinate systems.

So, 4 into X bar is something like an elapsed time like an elapsed time. It is exactly like a heat conduction type of an equation. With that kind of an elapsed time or whether you write it in terms of X or grades number or 4 by grades number, whatever it is, it is exactly the same. Both of these 2 terms exactly represents the same thing. So, in the next class what we will do? We will pick up from here and we will try to finish this Slug Flow analysis. There are a few other things that we need to solve in the case of a developing flow and in the pipe flow regime once we are done that we are ready to move into the natural convection.

Thank you.