Convective Heat Transfer

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Lecture – 21
Heat transfer to fully developed flow-I

So, let us look at the heat transfer in developing flow that is what our main aim was.

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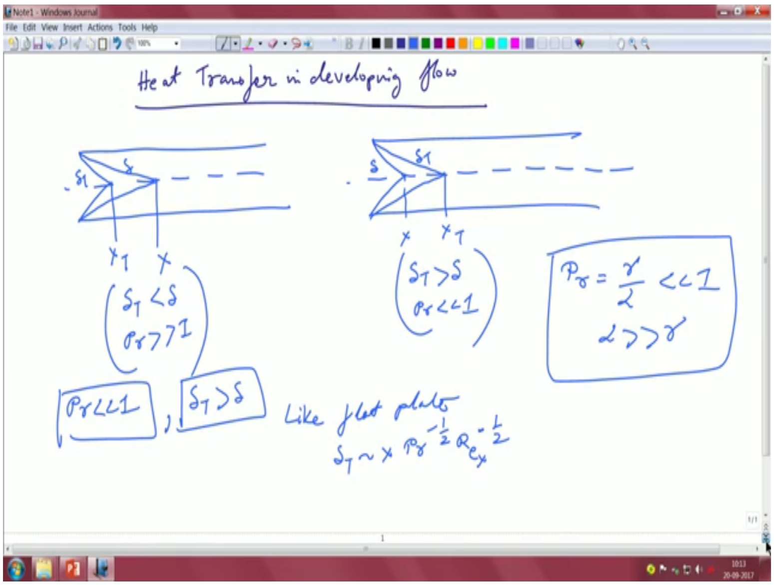

So, heat transfer in developing flow we would come to the point that where you basically are dealing with a situation that in the entrance region how does the heat transfer happen. So, far we have dealt with the fully developed limit we have established how the temperature varies fully developed temperature profile varies as a function of r only right that we established; that means, a normalized temperature profile we also saw that what is the hydro dynamically fully developed region how does it look like, but let us now look at the thermally developing part so, that we can get an idea that how that is going to vary.

So, there are situations can be 2 situations this is a very similar to what we saw in our flat plate boundary layer that how the 2 boundary layers are actually developing this is delta T. So, this is once again X this is once again X T right. So, in this particular case your delta T is less than your delta corresponds to prandtl number much greater than 1 one slot this is like delta T is much is greater than delta and prandtl number is therefore, much

much less than 1 these are the 2 limits that we are very familiar with right and we already know that what prandtl number greater than 1 means; that means, of course, that your thermal diffusivity is larger right. So, that is how you are going to carry this.

So, greater than 1 less than 1, let us take the limit of prandtl number much less than 1 which means that the delta T is actually greater than delta. So, this is the condition note that the definition of prandtl number is gamma by alpha right. So, when we say that this is much much less than 1 this essentially means that alpha is much much greater than gamma right. So, in essentially what it means is the thermal diffusivity is more than the momentum diffusivity.

So, in this case by the definition of prandtl number this is what we have. So, delta T is much greater than delta. So, like your flat plate like your flat plate delta T is proportional to X prandtl number to the power of minus half Reynolds number X to the power of minus half. So, that is from the flat plate analogy that we already did for delta T which is greater than delta do you remember, so you can just flip your notes and you can go to that particular page.

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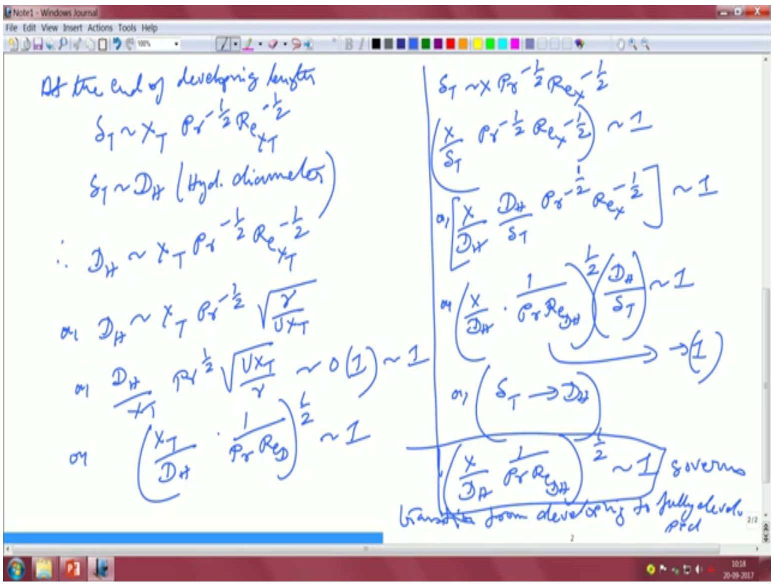

So, at the end of developing length at the end of the developing length you have delta T proportional to X T the prandtl number to the power of minus half reynolds number X T to the power of minus half right. So, at the end of the developing length, that is what X T is that the end of that particular segment right now delta T is; obviously, at that particular point is nothing, but the hydraulic diameter right of the duct right.

So, this is the hydraulic diameter therefore, we get D H is proportional to X T prandtl number to the power of minus half reynolds number X T to the power of minus half right or in other words D H is proportional to X T remember this reynolds number is cast in the terms of the length scale, which is X not with respect to D or in other words D H by X T prandtl number to the power of half into U X T by gamma is of the order 1 right should be of the order 1 roughly scales as 1 right or in other words if you now convert this particular reynolds number 2 the corresponding diameter I mean use that as the relevant length scale. So, X T by D H into 1 over prandtl number write it properly prandtl number reynolds number D right to the power of half is proportional to 1 right. So, that is what you have over here so for this situation.

So, that is this is very important because we will use this length scale very shortly now in how we will use it we will see. Now if you are also look at this thing in a little bit more detail this is the generic expression once again say X by delta T into prandtl number to the power of minus half you know number X to the power of half it is proportional to 1 or X by D H, D H by delta T prandtl number to the power of minus half reynolds number X to the power of minus half this entire thing scales as 1 or X by D H 1 over prandtl number reynolds number D H to the power of half D H by delta T is proportional 1.

So, as D T approaches D H this particular ratio approaches 1. So, this particular ratio approaches in the limiting case it is 1 as this approaches your D H right. So, basically X by D H into 1 over prandtl number R e D H to the power of half is equal to 1 this is the 1 that governs right transition from developing to fully developed right this is the transition this is the number right as it approaches 1 the flow becomes more and more fully developed. So, that is what we have got otherwise normally your D H will be more than your delta T right. So, this particular number has to be less than if this is much much greater than 1 this has to be less than 1 for the ratio to be equal to 1 right.

So, therefore, as this approaches 1 the flow becomes more and more fully developed. So, you are going to the fully developed regime which is based on this particular criteria right. So, that is an important part that one should understand right.

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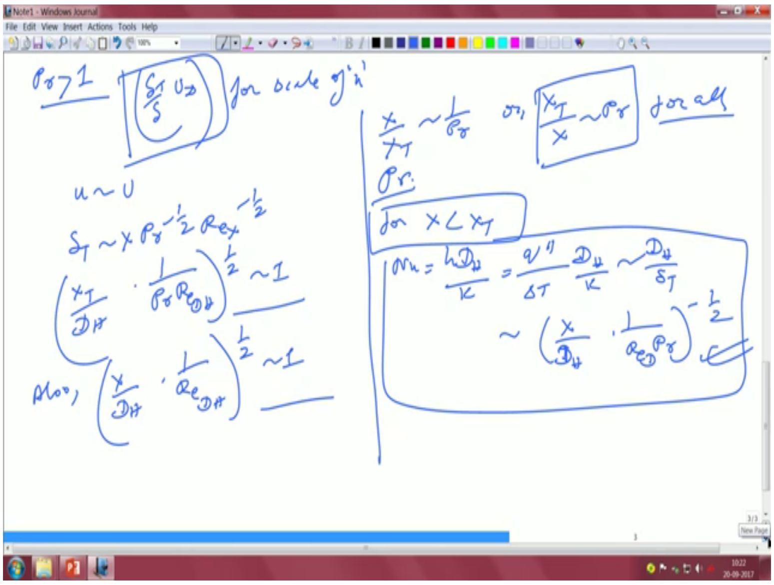

Next we look at the case of prandtl number greater than 1 right we did it for prandtl number less than 1 now here one important thing that we might want to do if you recall your flat plate boundary layer we did a scaling remember that we did something like this to establish the velocity scale this is what we did for U for scale of u; however, the basic assumption in that particular case was that delta is much much greater than delta T right that was the assumption a flat plate boundary layer.

So, therefore, this scaling hell was valid, but here it basically spreads over the whole diameter of the pipe. So, therefore, this scaling is not correct over here the scale of U is still capital U that is still the scale because there is no there was no imposed scale right. So, and that layer of delta T was much much thinner compared to the delta. So, therefore, U could get away with this kind of a scaling.

Now, so, therefore, in this particular case also you apply the same thing which makes it more X by D H right. So, this is what you got earlier from your hydrodynamics this is from the flat plate instead of writing or let us put also over here. So, therefore, X by X T is 1 over prandtl number or X T by X is equal to prandtl number for all prandtl number now right because your scale has not changed, there is no difference got it. So, therefore, for X less than X T the nusselt number is given by D q double prime into delta K into D H by K correct that is the nusselt number value right.

So, in this developing region got it. So, remember 2 things one is that this is that for all prandtl number this is different from a flat plate where we had that prandtl number one third and half kind of a discrepancy right, here actually that does not exist and also the nusselt number definition is given by this. So, these 2 are 2 important points that you should always remember.

Now, this sets the stage now to do a detailed analysis of the thermally developing flow which is basically the poiseuille flow. So, we would take that the hydrodynamics is fully developed; that means, the flow is hydro dynamically fully develop, but it is still thermally developing right. So, that is the case that we are going to take over here.

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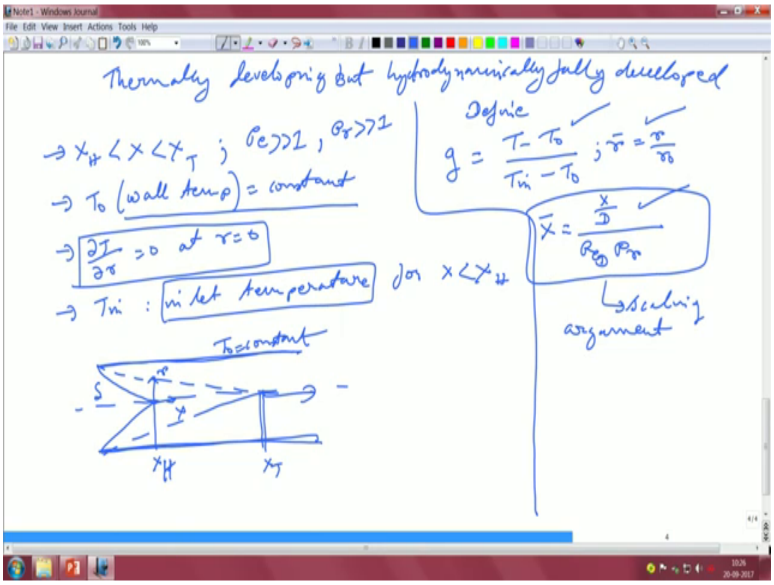

Thermally developing, but hydrodynamically fully developed. So, as you can see the first thing is your X lies in between these 2 limits; that means, X is greater than the hydrodynamic entrance length right, but it is lower than X T which is the thermal boundary length peclet number is much greater than 1 and prandtl number is much much greater than 1 right. So, these are the 2 things that we have kind of put over here. T naught which is the wall temperature is equal to constant and of course, you have d T by d r is equal to 0 still at r equal to 0 because of the symmetry and we put a temperature T in which is basically the inlet temperature in the temperature for X greater less than X H.

So, you can fix the origin with whatever. So, we can fix the origin at X equal to X H in that case this will be less than less than 0. So, it depends on where you are fixing the whole thing right. So, this looks like this. So, basically your hydrodynamic boundary layer has fully developed here as delta. So, you this is your basically your coordinate this is your region of interest. So, to say and this is T naught is equal to constant. So, this is your T naught equal to constant this part and this is your hydro dynamically fully developed regime got it.

So, based on this let us start defining define g is equal to T minus T naught divided by T in minus T naught right T in is the inlet temperature I write r bar is equal to r by r naught this you are already familiar with for X bar that is for dimension non dimensional if you have to non dimensionalized X then it is X by D and divided by R e D into prandtl number remember this scaling now becomes important which we established just a few minutes ago right. So, this comes essentially from your scaling argument and this scaling argument evolved very naturally right because we know that when this was equal to 1 we got to the hydro dynamically fully developed to the thermally fully developed regime right that was what we actually did right.

If we recall all the assumptions that we had over here what does the assumptions actually say that first and foremost you are within. So, if this is basically your X T somewhere there. So, you are only interested in this intermediate region because beyond this we have already done our analysis for constant wall temperature and constant heat flux right.

So, here we are taking the case of wall temperature is still constant. So, it is we are not varying that the profile of temperature by intuition you can say that it actually should have an inflection point at r equal to 0 that is at the centerline and the inlet temperature is taken to be the inlet whatever is the inlet temperature, but we are considering our axis starts basically from your X equal to H right.

So, inlet temperature actually is X less than 0 to say and there are 3 parameters we are defining 1 is g which is basically nothing, but a non dimensional temperature is different from how we define it earlier now you have a T in coming into the picture previously we did not have that then of course, you have the r bar and this particular term is very important which one could only established through the scaling argument that we did earlier. So, this is just a brief recap or brief emphasis on what needs to be done to approach the problem.

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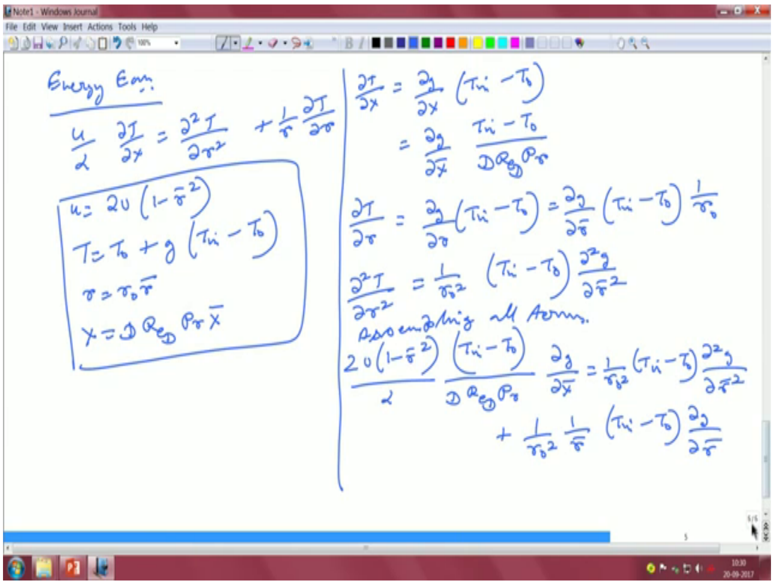

So, next is the energy equation; obviously, energy equation right that is the full fledged energy equation of course,, you still have the peclet number much much greater than 1 limit. So, u is basically 2 U into 1 minus r bar square T equal to T naught plus g T in minus T naught r is equal to r naught into r bar X is basically equal to D R e D prandtl number X bar right. So, these are the parameters which we are going to substitute now in the expression and try to see what does the math how does the math actually evolve alright.

So, let us take term by term that is easier for us to do because that would give us that flexibility and then we can assemble all the terms together. So, d T by d X is equal to d g by d X T in minus T naught which is basically equal to dg by d X bar T in minus T naught divided by D reynolds number D into prandtl number alright. So, we have just used the scale for X in this particular expression.

So, then the of course, is d T by d r and that is given as d g by d r T in minus T naught d g by d r bar T in minus T naught into 1 by r naught. So, that is the second term that you get, similarly d T square by d r square. So, we are writing term by term as we said 1 by r naught square T in minus T naught d square g by d r bar square right that is what we have done. So, these are the 3 terms.

Now, the basic task is to basically assemble all these terms together so, that we can get a nice expression. So, 2 U 1 minus r bar square by alpha T in minus T naught by D into R e

D into prandtl number and d g by d X bar right that is the first term. 1 over r naught square T in minus T naught d square g by d r bar square plus 1 over r naught square 1 over r bar T in minus T naught and d g by d r bar right is the full fledged expression assembling all the terms. So, this is basically assembling all terms right, therefore, moving on r.

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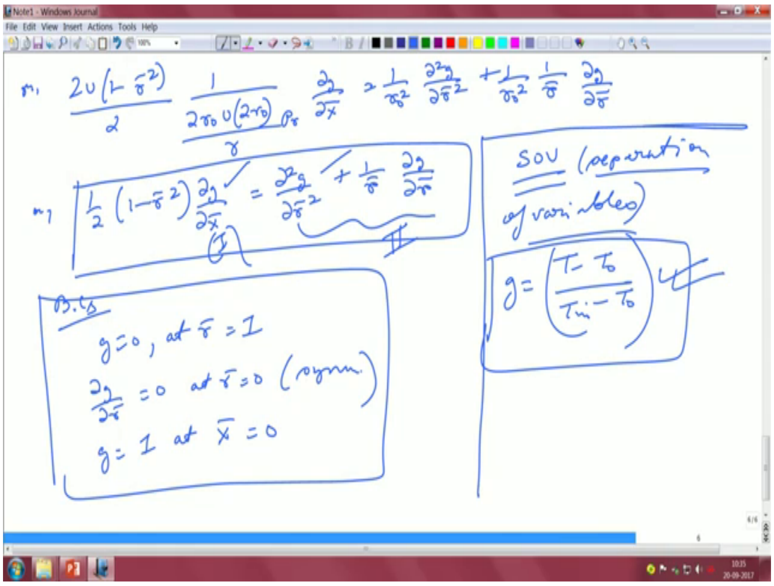

So, a little bit more mathematical manipulations or in other words half plus by r bar d g by d r bar right. So, that is the final form of the expression that you have, as you can see there are 2 variables almost impossible to solve because g is no longer a function of only r it is a function of both X and r right. So, the boundary conditions are g equal to 0 at r bar equal to 1 which is given right d g by d r bar is equal to 0 at r bar equal to 0 this is symmetry and g is equal to 1 at X bar is equal to 0 right because that is the point where we have the same temperature, the temperature is the same as the inlet temperature right.

So, based on these expressions, to say we move forward got it. So, this equation actually presents it can be only solved we will try to solve it analytically. So, there are methodologies by which you can actually solve it analytically. So, one of those methodologies will be the separation of variables or we. So, separation of variables you can read up a little bit on separation of variables in your math from your math book. So, separation of variables is something that you have encountered quite a bit in your conduction courses there is many of the conduction equations actually has that kind of a thing.

Now, if you look at the nature of this equation once. So, you can see one thing very prominently if you just look at this term this term I mean this term and these 2 are basically the same so, these 2 terms right, you can see that it is now a function of 2 variables where your X bar kind of is like a time whereas, r is more like the spatial distance. So, though, we will see while we are doing this. So, what is our expectation our expectation is that the temperature would show a certain amount of behavior certain kind of behavior with X bar right because as X bar increases something is going to happen to the temperature profile right that is what is expected is going to happen.

Now, what is the general expectation see when we actually propose that what is going to be your g let us write the g here one more time. So, that you guys get an idea T minus T naught into T in minus T naught. So, what happens to g as we go on increasing X bar does g skyrocket or it does not right so; obviously, T is something that is bounded it cannot go beyond a certain value it cannot go beyond T naught right. So, based on that whatever is a solution that we will get out of solving this equation has to be physically meaningful also; that means, you cannot have a temperature profile which skyrockets which increases all the time.

Similarly you cannot have a temperature profile which is cannot have a temperature profile which shows an unphysical nature right even if it is mathematically correct right. So, based on some of those limitations that we have we are going to do the separation of variable keeping in mind that this is our main variable which is g which is a function of both r and X right. So, separation of variables we are going to attempt in the next class and we are going to see that how this separation of variable actually works and what it actually leads to what are the expressions that we get ultimately out of it.

Thank you.