Convective Heat Transfer
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Lecture – 20
Tube surrounded by isothermal flow

Last class we did about uniform wall temperature, now in this particular class we are going to start a little bit of a more complex problem which has got elements of both uniform wall and uniform wall temperature and uniform heat flux.

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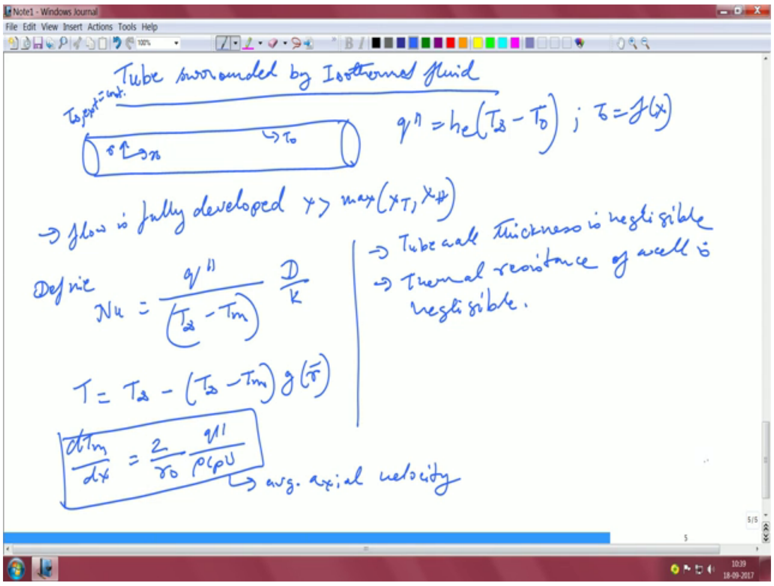

This is basically a tube surrounded by isothermal fluid. So, situation is something like this, this is once again the tube with r and x the wall temperature of the tube is T naught which is not specified the temperature of the external fluid right is specified. So, this is the temperature of the external fluid that is given as constant.

Now, imagine this to be a situation in which a tube is placed in some kind of a you know heat transfer fluid. So, it is typical heat exchanger type of a problem as an external fluid which through external convection is actually taking away the temperature. So, there is a hot fluid going inside the tube that hot fluid is transferring heat to an external fluid which is outside that you right now in the previous classes we were only concerned that there is a uniform heat flux or a uniform wall temperature, we are not concerned what happens outside the tube.

We always thought that wall temperature was kept constant by some means or there was a uniform heat flux that was supplied along the tube wall. This is more realistic in nature that this particular tube which is carrying some hot fluid is placed inside a large you know convective medium of some other external fluid, whatever that external fluid is can be the same fluid can be a different fluid also right. So, it becomes a typical heat exchanger kind of a problem.

So, it is neither a constant wall temperature neither it is a constant heat flux condition right whatever; however, the external bath is so huge right it is like a resolver large reservoir that T infinity of the external heat is kept constant so; that means, the external fluid is not varying in temperature understood. So, it is like a large ocean. So, it is what we call typically a thermal reservoir right that temperature is not changing because T infinity value is very large maybe the thermal inertia is very large, but; however, the wall and whatever is the wall temperature and the heat flux at the wall on the tube is therefore, varying because that we cannot assume to be constant.

So, similarly here q double prime is given by some kind of h e some external convection coefficient right. So, it is T infinity minus T naught where T naught; obviously, will be a function of x in this particular case right and neither is q double prime is a constant in this particular case. So, let us assume a few things that the flow is fully developed that has nothing to do with anything right the flow is fully developed; that means, x is greater than the max of x T and x H you know what x T and x H are right.

So, we are defining Nusselt number there is a little bit of a strange definition of Nusselt number T infinity minus T m into D by K right, what is T infinity, T infinity is the external fluid temperature right T m is the mean temperature of the flow inside the pipe right. So, this definition of Nusselt number is very different it is previously you had T naught minus T m or you had it in terms of the q double prime right this is very different that is because our q double prime is defined in a very different way also right.

So, therefore, let us take T is equal to T infinity minus T infinity minus T m into g r bar. So, this is the definition that we are putting forward and this is the expression that we are casting that T that is the temperature is as a function of T infinity and T m and it is this total expression is a function of r only like in our fully developed regime these are the things that we did in addition we say that the tube wall thickness is negligible so; that means, this avoids a complication that there will be a drop inside the wall got it and therefore, it is thermal resistance is also weak.

So, there is no thermal resistance of the wall a wall is negligible this just because if it is a thick wall you have to also take into account the thick wall effect. So, we are discounting all that also your d T m by d x is given by 2 r naught q double prime rho c p u this is the average axial velocity right these are average actual velocity once again this definition is once again the same as what we had earlier right that whatever we had in our previous class.

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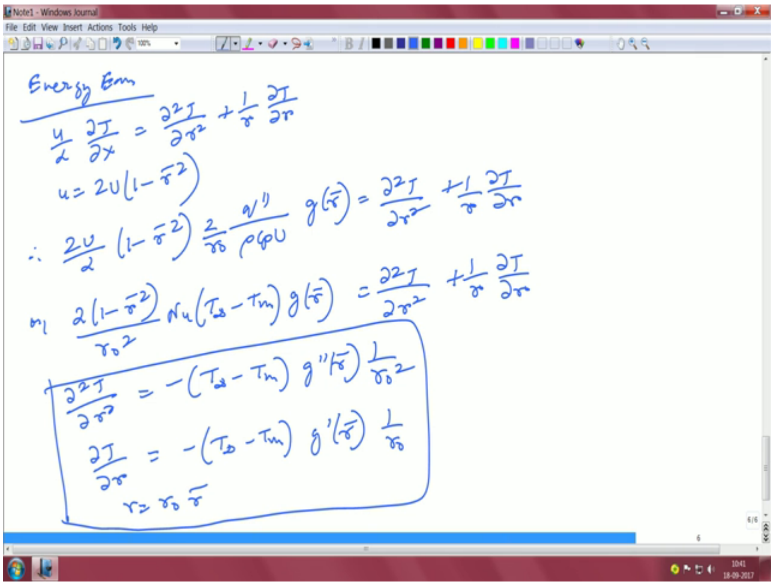

Now, let us cast this problem; that means, we first write the energy equation once again I make it a habit that you write the energy equation even though this has been written countless number of times just that you guys get a feel of writing the same equation again and again. So, once again this is the same flow field right because U is accurately solvable. So, therefore, you get this or right just by using our own definition. Now individually d square T by d r squared is given as T infinity T minus T m g double prime r bar 1 minus g naught r square similarly the first derivative and r is a; obviously, r naught into r bar these few things are given right given as a.

So, I have not done anything funny over here except my definition of temperature and my definition of q double prime I have tweaked them according to the problem; that means, I have involved the external temperature field into the picture because that is the one that is constant right and the Nusselt number definition has changed a little bit.

So, based on that I have started working on this particular problem, in other words if you cancel a few terms then you will get g double prime bar right we need several boundary conditions.

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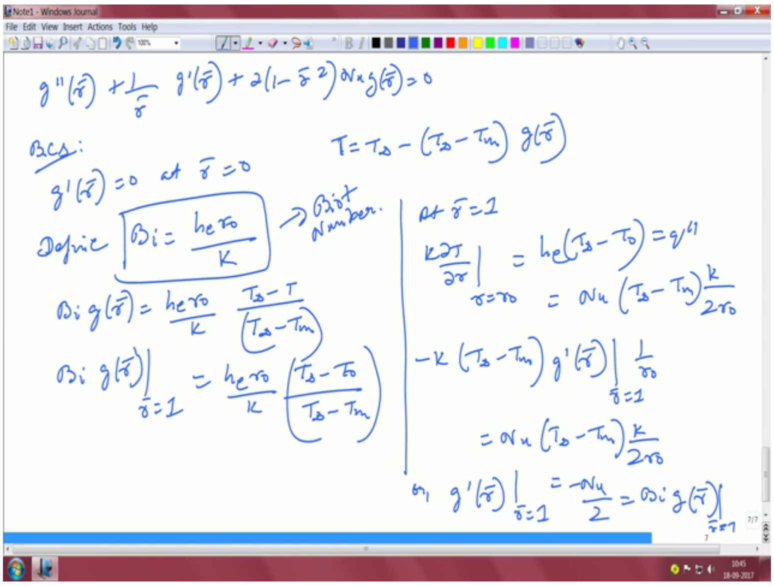

So, g prime bar is equal to 0 at r bar is equal to 0 why that will be the case let us revisit the situation that what was our definition of g; that means, your T was equal to T infinity minus T infinity minus T m into g r right. So, why g bar r will be or g prime r will be equal to 0 at r equal to 0 that is given by the axis symmetry of the temperature profile right.

So, this boundary condition you can easily make out, but what about the other boundary condition because at the wall you do not have a boundary condition properly now right because of your definition because you have T infinity now you cannot say when r is r bar is equal to one you will have g going to some value you cannot say that.

So, let us recast it in a certain way define something called a Biot number h e r naught by k right h e is the external heat transfer coefficient right h e r naught by k. So, that is a perfectly valid definition of the Biot number right. So, B i into g bar is given as T infinity minus T divided by T infinity minus T infinity mean got it. So, similarly Biot number and g r bar at r bar equal to 1 right that is given as h e r naught by K T infinity minus T naught divided by T infinity minus T m . So, that is what we have.

Similarly, at r bar equal to 1 K d T by d r at r equal to r naught is exactly given as h e T infinity minus T naught right that is nothing, but your q double prime right that is also equal to your Nusselt number T infinity minus T m into K by 2 r naught right. So, if I now do a little bit of manipulations on this or right which is nothing, but if you are not r bar is equal to 1 right. So, you got your definitions right of that what will be your g prime at r bar equal to 1 you also got the definition of what is going to be your g bar right by introducing this basic definition of Biot number this is Biot number right by using this basic definition of Biot number in this particular case.

So, now let us define something else also let us make it a little interesting.

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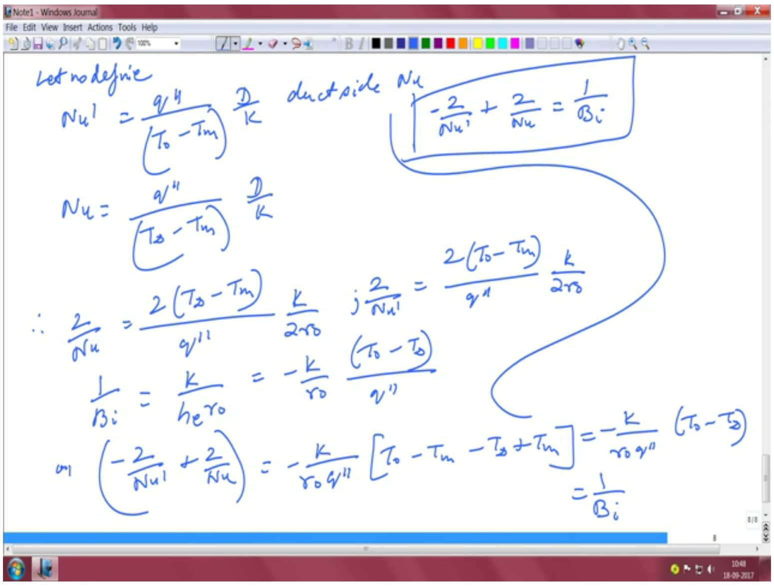

Let us define Nusselt number prime something called a Nusselt number prime which is basically q double prime T naught minus T m into d by K this is basically what we call the duct side Nusselt number right because this uses the conventional definition of T naught minus T m this is the definition that you are most familiar with right recall that your Nusslet number is basically T infinity minus T m into d by K correct. So, therefore, 2 by Nusselt number is equal to 2 T infinity by T m divided by q double prime K by 2 r naught 2 by Nusselt number prime is basically 2 T naught minus T m divided by q double prime K by 2 r naught right.

On the other hand your Biot number 1 over Biot number is basically K by h e into r naught which is basically K by r naught T naught minus T infinity by q double prime right or in other words 2 by a Nu prime plus 2 by a Nu you just add the 2 together right it will come out as K r naught double prime T naught minus T m minus T infinity plus T m right or in other words it will come out to be minus K r naught into q double prime T naught minus T infinity which is nothing, but 1 over the Biot number which is nothing, but 1 over the Biot number got it.

In other words what we can write over here is that 2 by Nusselt number prime plus 2 by Nusselt number is basically equal to 1 over the Biot number in this case got it.

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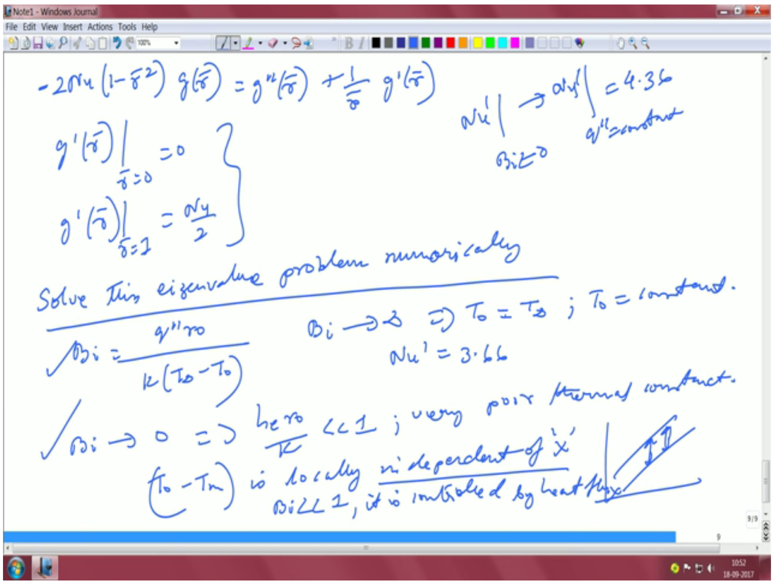

The final equation that needs to be solved in all these things is basically minus 2 Nusselt number 1 minus r bar square in g r bar it is no different than the expressions that we had earlier right with subject to the boundary conditions g bar at r bar equal to 0 is equal to 0 g bar at r bar and r bar equal to 1 is equal to Nusselt number by 2, 2 particular definitions we have.

Now, solve this problem this basically is an eigenvalue problem. So, solve this eigenvalue problem numerically to get the answer now there are 2 limiting conditions that are strictly possible for this particular case right, what are those limiting conditions. So, let us before we spend a lot of time discussing it let us look at that if your Biot number is basically let us write down the most common definition of Biot number this is this right. So, in the limit that Biot number goes to infinity that would imply they should lead to the T naught must be equal to T infinity right. So, this basically means that T naught is equal to constant right. So, in this particular case the Nusselt number a Nu prime right which is the basically the duct side Nusselt number that value should be equal to 3 point 6 6 in the limit that Biot number approaches infinity right.

Let us take the condition when Biot number approaches 0 right which implies that h e r naught by K is much less than 1 correct. So, this also implies very poor thermal contact right. So, in other words T naught minus T m in that particular case is locally independent of 'X' right, it is locally independent of X and T naught minus T m is locally independent of X corresponds to a uniform heat flux case right because if you recall in your uniform heat flux you had this lines which were parallel this difference was always constant regardless of X at any X the difference will be the same.

So, when they are locally independent of X; that means, when Biot number is much less than 1 it is controlled by heat flux or in other words this becomes a uniform heat flux case; that means, the Nusselt number at Biot number approaching 0 right will approach then this is Nusslet number trying basically Nusselt number of q double prime equal to constant right; that means, will be equal to 4.36. So, that will be the value of Nusselt number that you are going to get when once you have this kind of a mediation understood.

So, we can see that these are the 2 limiting conditions and in between you will have any value these are the 2 bounds right 4.36 is one bound and 3.66 is the other bound right 0 and a very high number right. So, in between all other Nusselt number values will be packed in between. So, we can see that when we actually did the uniform heat flux and the uniform temperature we basically covered these 2 extreme ends the actual situation is like us pipe in an isothermal fluid right. So, that in between Nusselt number whatever may be your configuration will lie somewhere in between right. So, that is an important case to note over here right.

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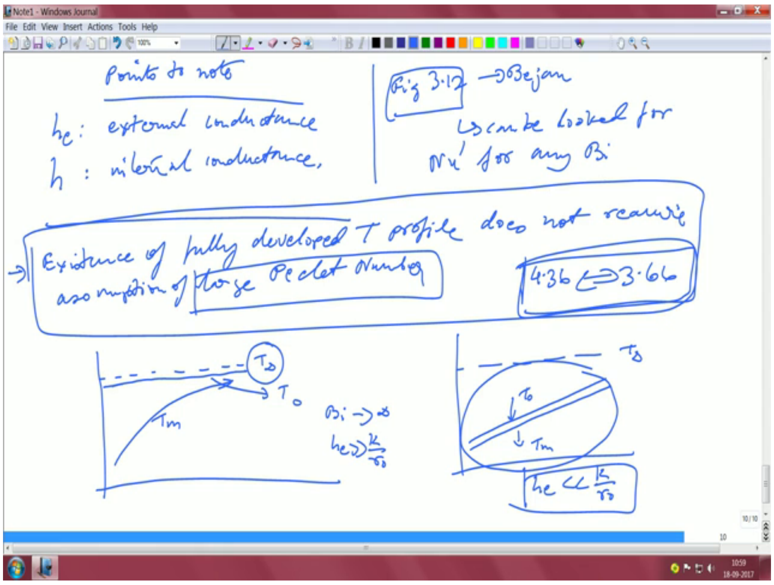

Points to note in here, h e what we said over here is the external conductance, do not confuse it with h, h is basically the internal conductance, got it is the simple error that you try to make that is internal and external right. So, external conductance and internal conductance are very important just keep that thing in mind because sometimes things might get a little jumbled up.

Some general points to note is that the existence of fully developed T, T profile, does not require assumption of large Peclet number note it down properly it does not require the definition that the Peclet number has to be has to be very large right, but; however,. So, this is one of the key assumption it does not require the assumption that the of the large Peclet number; however, Nusselt number equal to constant do come from that large Peclet number assumption they do come from that large Peclet number assumption.

So, based on this let us we can also draw a couple of profiles over here say for example, in the case of hide Biot number case. So, this is your T infinity look at it carefully this is almost like what your T wall looks like right and your T mean kind of curls up in this particular way. So, this is when the Biot number is approaches infinity or in other words h e is much greater than K over r naught right; that means, the external conductance is very high right that is what you have; that means, there is no gradient between the external fluid in the external fluid because if you recall your example that when you say that you have this typical problem like a sphere immersed suddenly in a large thermal reservoir what we say that when we do not have to take the thermal gradient within the sphere we tell that when Biot number is much less than 1 this is the opposite to that if Biot number is very high; that means, there is no thermal gradient on the surface of the sphere with the external fluid right.

It is almost like that is that heat transfer is like kind of instantaneous, but there will be substantial gradient that will be created inside the sphere right. So, that is essentially if you look at it this essentially means that there is no real temperature gradient between T infinity and T naught both are basically the same in a way, but therefore, there is a high temperature gradient within the fluid which basically corresponds to the uniform wall temperature case.

Similarly, you have the other situation this is T infinity this is actually your T naught this is actually your T m they are. So, here h e is less than K over r naught. So, whatever happens in the external fluid we do not really care because our internal conductance is very high right. So, it is always controlled it is not controlled by X it is basically controlled by the heat flux and that is exactly what we have drawn over here.

So, that is I think the 2 grand definition and if you look at any other fluid flow you can refer to figure 3.1 2 of Bejan convective heat transfer there you will find that one would give you the flow; that means, the Nusselt number variation with Biot number here we have drawn the 2 extremes cases there they will show you for the for a Poiseuille flow which is basically the flow that we are most concerned with for that kind of a flow that what will be the value of your Nusselt number right. So, figured 3.1 2 of Bejan can be looked for Nusselt number values for any order here it is Nusslet number prime for any Biot number.

Remember Bejan in the Bejans book that the Nusselt number is cast in a different way. So, you should once you do it you should keep in mind that what is the actual definition of Nusselt number right how he has defined it and how we have defined it as a part of this particular course right. So, in other words Bejans 3.1 2 figure 3.1 2 should be looked at very very careful right. So, and we will also share that particular figure with you in the next class we will start it there and we will also show you the Nusselt numbers with the for the different configurations; that means, the cross sections those are mainly for handbook purpose because actual solving of this equation requires a little bit of effort right, but you can still take a quick look, but all we can say that it varies from 4.36 to 3.66. So, that is the variation that is the limit of variation of your actual Nusselt number that is the duct side Nusselt number it always remains within these 2 bounds.

So, that is, but also you would like to pose a question that why is the Nusselt number for uniform heat flux higher consistently higher for any cross sections as we saw than the corresponding Nusselt number for a uniform wall temperature why is that the case this is an open question that we are throwing open in this particular class and it will give the answer a little later, but the students starts to think about that why is that the case why this is consistently higher and we saw all other Nusselt number seems to stack up in between these 2 limits that is 4.36 and 3.66. So, what is the reasoning behind it if there is any reasoning that is.

So, I think spend some time looking into this in the next class we are going to start looking at heat transfer to developing flow. So, far we have looked at heat transfer in the in the fully developed regime and we have also looked at the hydro dynamically fully developed and you know what are the situations that are possible let us look at the developing flow regime and try to see what we can extract out of that many of the equations are basically not solvable in their full I mean not solvable in a class, but you can always try you can have numerical methods you can have other sophisticated mathematical techniques by which you can solve it, but this is just to give you an essence that what is the key physics behind this kind of flow configurations.

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Table 3.3 Friction factors and Nusselt numbers for heat transfer to laminar flow through ducts with regular polygonal cross sections

	$f\,Re_{D_h}$	$Nu = hD_h/k$			
		Uniform Heat Flux		Isothermal Wall	
Cross Section	Fully Developed Flow	Fully Developed Flow	Slug Flow	Fully Developed Flow	Slug Flow
Square	14.167	3.614	7.083	2.980	4.926
Hexagon	15.065	4.021	7.533	3.353	5.380
Octagon	15.381	4.207	7.690	3.467	5.526
Circle	16	4.364	7.962	3.66	5.769

As discussed in the last class we decided that we will show that how the Nusselt number and the friction factor for ah laminar flow with regular polygonal cross sections should vary. So, this is what you have. So, for there are 2 limits one is uniform heat flux, one is the isothermal wall as we can see for the fully developed flow forget about the slug flow portion we are going to come to that a little later for the fully developed flow as you can see for uniform heat flux and for isothermal wall these are the corresponding values of the Nusselt number moving from square to circle.

So, as you can see as we increase the number of sites; that means we move closer to a circle, circle is almost like an infinite polygon right. So, as we move closer to a circle as you can see the Nusselt number increase that is the heat transfer coefficient also increases for both isothermal wall as well as for uniform heat flux case, from 3.6 all the way up to 4.3 we move and here from 2.9 8 all the way up to 3.6 6 this 2 as you we did the math and we showed that 3.6 6 and 4.3 6 4 where the 2 limits.

Similarly, you can also look at the friction factor which is basically the from comes from the hydrodynamics there also as you can see that the friction factor actually increases a little bit as we move towards from a square to a circle.

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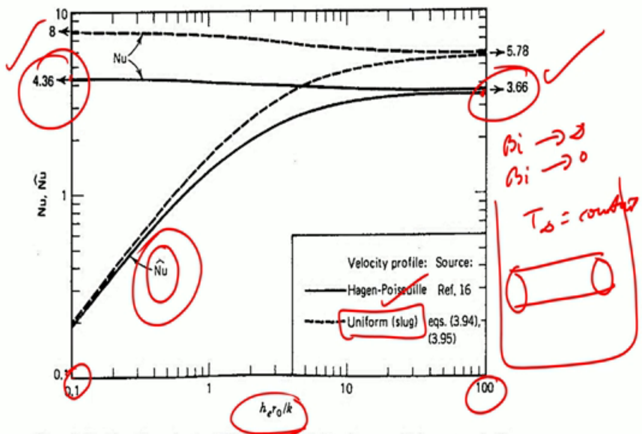

Figure 3.12 Nusselt number for thermally developed flow in a round tube surrounded by an isothermal fluid.

So, that is one and if you recall in the last class we say it that when you actually have a cylinder placed in an isothermal reservoir right with T infinity was equal to constant if you recall. So, we did that and we kind of showed that for that particular thing there are 2 limits which exist if you recall Biot number approaching infinity Biot number approaching 0 right and we said that those 2 number values lies between 4.3 6 and 3.6 6 right. So, this graph effectively shows that particular thing all right, this is basically what we call the duct side Nusselt number.

Recall this graph is taken from Bejan. So, the nomenclature is a little different what Bejan uses as a Nu prime I have used it as N u. So, there is a little bit of what we call a little bit of ambiguity between these 2 the Nusslet number, but the Nusslet number this Nu is basically the duct side Nusslet number in this particular graph I have used a Nu prime and I have used a instead of a Nu hat what they have used I have a Nu in my derivation. So, just better be a little careful when you actually look at and read the graphs. So, as you can see here very clearly it goes from 4.3 6 to 3.6 6 and you should look only into the Poiseuille flow profile not the slug.

What is slug will come a little later, but as of now this particular graph you can see that these are the 2 limits for a wide variety of Biot number this is the Biot number that we have as we can see Biot number 100 is a very high that is it mimics the Biot number approaching infinity and 0.1 is actually a low enough Biot number. So, you can take that to be the Biot number to be equal to a very low value.

So, based on these 2 things you have these 2 limits that we have established in the last class where we have shown that this is the way the Nusselt number the duct side Nusslet number should vary this is of course, the external Nusselt number which was based on a Nu that we wrote in our previous lectures. So, that varies in this particular way as you can see from this particular graph.

So, this basically completes our discussion the pending discussions that we had and we have now given graphically as well as in the form of a table that how these constants will vary what will be the nature of these profiles and remember all these equations were basically solved in a numerical fashion. So, now, we will go back to our thermally developing flow and try to understand that how thermally developing flow actually varies, we go to that particular part.