Convective Heat Transfer
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Lecture – 02
Governing equations I – Momentum Conservation

So, welcome to lecture 2. So, in the previous lecture we did conservation of mass, and we looked at what is convection heat transfer all about. In this particular lecture, we are going to look at conservation of momentum. Because that was the next step as I said. Now that we know what conservation of mass is now we move on and see how the conservation of momentum we will look like so conservation of momentum, right.

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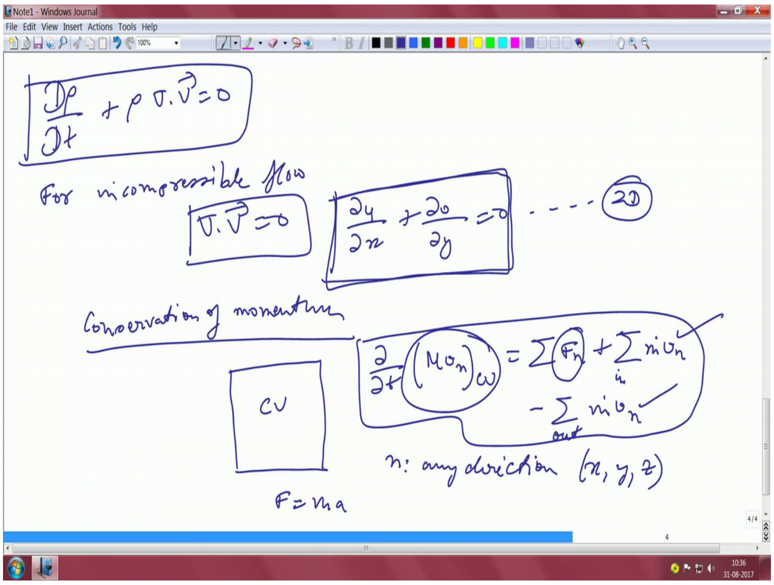

So, we look at the same control volume, there is no need to change that control volume, right. This control volume is still the same, right. So, what will be the conservation of momentum for this control volume in any particular direction? Now the direction becomes important, because mass was a scalar, this is a vector quantity. So, you have to look at actually 3 directions, if it is 2D, you have to just look at 2 directions. So, first term is essentially what we call the rate of change of momentum within the control volume, right. That will be equal to all the external forces that are acting in the control volume. Plus, whatever is the momentum in minus whatever is the momentum out, right.

Where n is basically any direction of your coordinate system; that means, it will be x y z essentially. If you deal with spherical coordinates it will be something else, right. So, in essence this particular form of this conservation equation is basically newtons second law of motion, nothing like that nothing more than that right. So, there are some external force terms, there is a rate of change of momentum within the control volume, and then there is a momentum in and momentum out. So, that is newtons second law of motion which is basically F equal to m a right. So, let us look at it from a control volume approach like what we did earlier. So, we look at the same thing.

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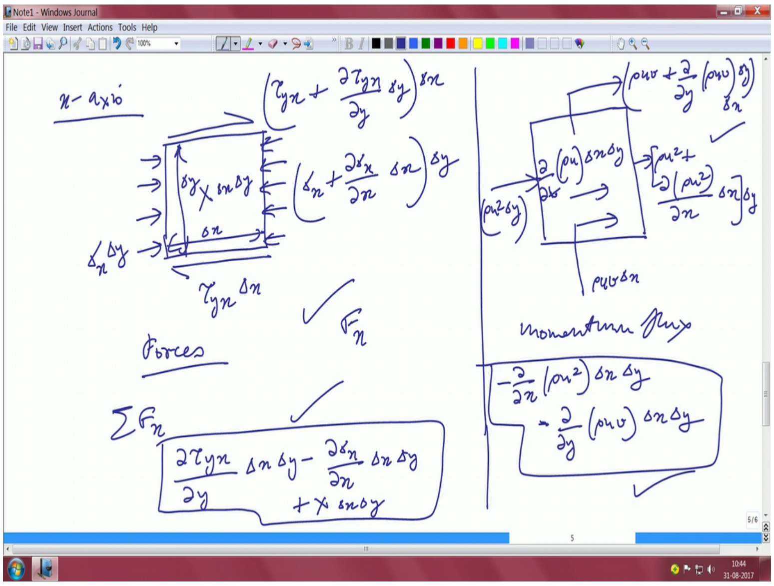

Let us look at the x axis, let us take the x momentum equation essentially. And the y and others will just fall into place. If I do one, you can do the others essentially ok.

So, let us look at that what are the different types of forces that are acting on this particular control volume. So, on this face if you look at it; so, there is this normal stress which is acting on delta y face. So, once again the nomenclature remains the same. This is still delta y, and that is still delta x, delta y delta x right. So, similarly on this side of the face, what will be the once again using that same Taylor series expansion analogy, you will get this, am I right? So, that will be the same thing, I am not going through the details. It will be once again sigma at x plus delta x. When you expand it, you will get this particular form, correct.

So, the 2 faces in the x direction these are the normal stresses. Then what other form of stress do you have? You have the shear, right. The shear stress. So, that will be tau y x into delta x is acting on the x face. Similarly, on the top, you have tau y x plus d tau y x by d y, right. Into delta y. Once again that same Taylor series expansion into delta x, right. On the top same thing. So, as you go. So, it is expansion around y, because you are going across the y face, right. So, these are the surface forces that are acting. So, forces can be of 2 types one has surface forces and one are body forces.

So, similarly you can have a body force, which is represented by x, multiplied by delta x into delta y. So, that is a body force. So, most of the useful examples of body force are gravity is one example of a body force. So, we do not know the nature, we are not commenting on the nature of the body force we are just putting it as x, right. So, these are basically the forces that are acting on the control volume, right. So, that takes care of the first you, if you look at your terms, this would takes care of the sigma F n, right. These are the forces right. So, similarly now that you have done the forces, let us look at the mass now momentum in momentum out because that needs to be taken into account because that is the other part of the puzzle, right.

So, let us take the same control volume, I am not doing it in the same picture, because of the simple reason it gets very crowded and messy, all right. So, one thing is for sure, that the momentum that is coming in is rho u square delta y, is it right? On the other side, the momentum that is leaving the control volume is rho u square plus once again the Taylor series expansion comes into the picture, it is given by rho u square divided by delta x with d x into delta y, all right. You understood this part this part is very simple rho u is basically the mass, rho u into u is basically the momentum right. So, that is the momentum flux that is coming from inside the control volume.

So that means, that is momentum in this is actually momentum out, right. Is that all? It is actually not the case, because there is also mass which is entering into the control volume from the y face, right. It is there is also mass, and what is that mass that is entering into the y phase is basically rho v, right. Into delta x am I right? Now that particular mass can be carried in the x direction with a velocity u, right. So, in that particular case what will happen is, we will have u v into delta x right. So, this is the mass that is carried in the x direction, right. Similarly, there will be mass which will be exiting the control volume, but they may be washed away by the new component of the velocity.

So, that will be given by rho u v that term is common, plus once again the same Taylor series expansion of the whole thing d y rho u v into delta y entire thing is multiplied by delta x, got it. So, in the x direction when you look at the x direction momentum equation, these are the 2 principal clays, this gives you F n or in this case F x. And this gives you the m the momentum flux essentially, all right. There is coming in and out right. So now, what we do? We can assemble all these terms together and look at the individual parts of their share right.

So, for example, on the force side we will have a share on the on the momentum flux side we should have a share right. So, what will be the share on the momentum flux side? It is once again momentum in minus momentum out is a very simple thing once again. So, if you do that, then you what you will get, right? Minus so, this is the momentum basically in minus momentum out, got it? On the other hand, delta F x on this side, what that will give you if you expand the whole thing once again? I am not going through all the steps this you can work out as homework, if you want to just go through all the steps, but I have given you the basic is basically subtraction it is basically algebra.

So, sigma x y delta x delta x into delta y, plus x into delta x into delta y, all right. So, this is the F n terms that you are going to get. So, you have this F x, and then you have the momentum flux term, all right. On the 2 sides. Now you are left with only one thing, that is the change of momentum within the control volume, and what is the change of momentum within the control volume? What will be the change of momentum within the control volume, that is the thing that we are going to look at. So, remove this one. So now, that we have established that what is going to be the situation ok.

Now, the rate of change of momentum within this control volume, let us go back one slide up. So, if I draw it here, I think it will be more visible right. So, if you look at the that is the rate of change of momentum within the control volume, right. It is row u into delta x delta y, this is the x direction of course. Because once again the direction part is very important here, right. So, if you assemble all the terms now; that means, we are gathering now all the terms, right.

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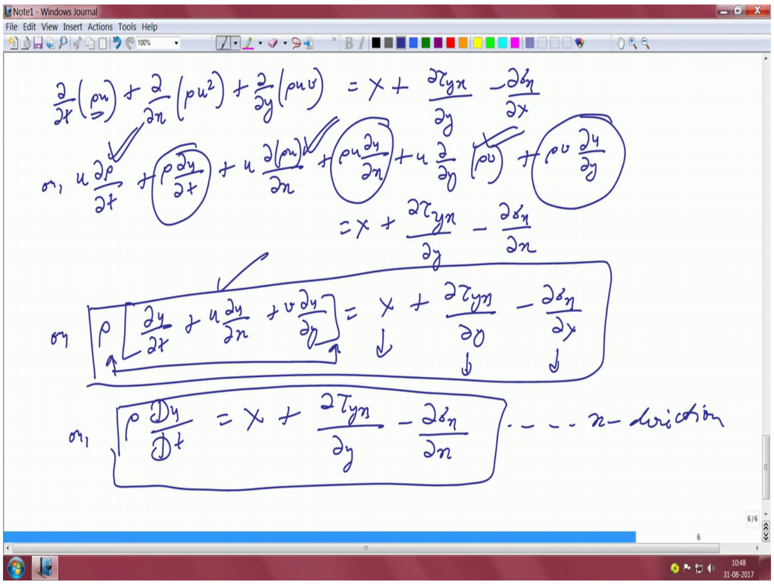

Your equation would look like, and once again I am doing it in 2D, 3D is no big deal you can just have to do a little bit more algebra than what is required. Rho u v is equal to x plus t tau y x by d y minus d sigma x by d x, right?

That is what you get. Now you can simplify things a little bit. You can I mean there are terms which are within the bracket. So, you can basically unfold them a little bit. So, what you will get? D u d rho d t plus rho into d u d t plus u d rho u d x plus rho u du d x plus, it is a long expression basically. Rho v d u d y is equal to x plus d u tau y x by d y minus d sigma x by d x. Now you just gather the terms where the rho can be taken out. D t plus u d u d x plus v d u d y. The other term why it will cancel, I will say in a second. Tau y x by d y minus d sigma x by d x, right. Now if you look at that what happened to the other term that we kind of dislodged. If you can see that the term with that really did not pay any attention to was basically already incorporated in the continuity equation.

So, that term basically goes to 0, right. So, this is the only term that actually survives. So, if you do a term by term comparison, this term survives, right. This term survives, this term survives, correct? This term will survive, what about the nature of the other, 2 terms because if you take u out of those terms this term. I am putting 2 symbols over here. This particular the not this particular term, I am sorry. So, that particular term this particular term, and this particular term will actually go to 0, if you add them that is the basically the continuity equation that you have right. So, this is the full expression that you will get. This is what we call the convective acceleration term. It is entire thing, this is the body force term, right. This is basically the surface force terms, and these 2 are basically both are surface forces in a way ok.

So, this can be also compactly written in terms of the material derivative, right. This is once again in the x direction only. Of course, y direction will be very similar to this, but y u direction is something that we are not really bothered about, right. Now because that will be essentially the same. Now looking at this particular last expression over here. And that you have, you say that this does not look anywhere like the equations that we are most familiar with right. So, what is the most common equation in fluid dynamics, right. That is given by the Navier stokes equation, right. This does not look like Navier stokes equation though, right. It does not look because the where are the forms, where is viscosity where is all these terms. Now to answer those questions this is the most generic form of the momentum conservation equation. We go to the Navier stokes equation only after we have taken into account certain constitutive relationships ok.

After we know the constitutive relationships, then only we can go and migrate to the Navier stokes equation. So, that is the most important thing to note in this particular thing; that we have to now apply certain constitutive relationships to know. For example, we do not know what is the nature of tau y x, what is the nature of sigma x; somehow you have to relate this you know, to the pressure to the velocity field. Unless you can relate them, this equation you cannot solve in that particular fashion, right. Now most of you must be must have heard things like you know Newtonian fluids non-Newtonian fluids and things like that. All it does is basically you have to cast this stress terms essentially. And the body force term also. Because body force can also have variety of forms, but that is relatively intuitive you know that gravity is the most commonly encountered body force.

So, in order to cast this in terms of the body forces, all right. Or the cast the surface forces in terms of certain known field quantities is what the purpose of the constitutive relationships will be, right. And that will actually reduce the problem to the more manageable Navier stokes form that you are most familiar with.

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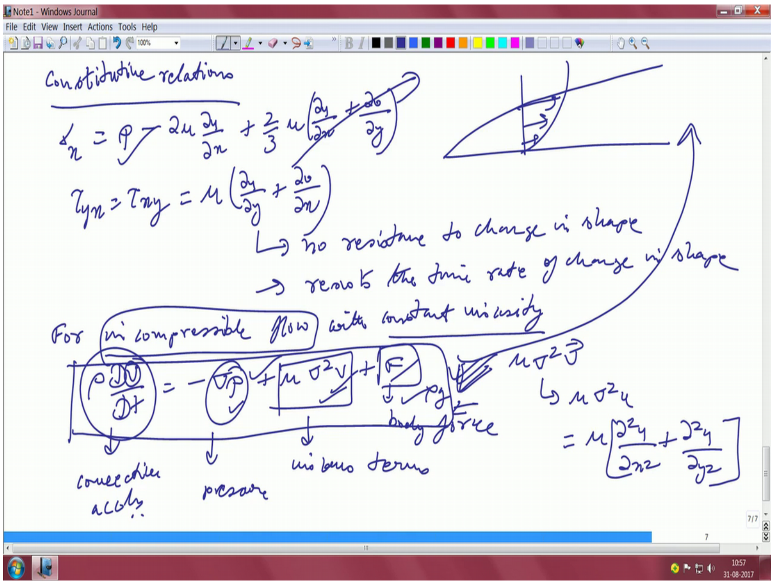

So, let us look at the constitutive relationships relations. So, the first constitutive relation is sigma x, right. That is the normal stress is given by the pressure minus 2 mu d u d x plus 2 third mu d u d x plus d v d y, got it? That is the first term. Then tau y x is the same as tau x y, all right. This is the stress the shear stress tensor.

Now, you guys are all familiar with tensor notations perhaps right. So, that is a tensor. So, that is given by mu d u d y plus d v d x. So, these are the cross derivatives of the velocity. There is a change of u velocity in the y direction, change of y velocity in the x direction. That is what it is right. So, this is basically what is the Newtonian fluid concept or newtons law of viscosity, the second relationship that we have put forward over here. So, this essentially means that there is no resistance to change in shape, all right. And it, but it resists the time rate of change of shape, the time rate of change in shape, got it? These are the 2 important things to note. No resistance to change in shape, but resists the time rate of change of shape, time rate of change of shape.

Now, for incompressible flow, once again now these are the special assumptions for incompressible flow with constant viscosity; that means, the viscosity does not change with space. There can be fluids where the viscosity is can change actually spatially or with the flow field. So, there what we get is basically d v d t is given by is basically the body force. So, we have reduced this using the constitutive relationships, and for constant viscosity, this is the term that you are most familiar with this is the Laplace of the velocity field; which is basically the viscous terms, right. These are the viscous terms which is viscous stress which is basically the shear stress. This is basically the pressure term, where does it come from? It comes from here, all right.

And this is basically your convective acceleration, all right. If you look when you are using incompressible flow, this particular term actually goes to 0. That particular term actually goes to 0, got it? So, if you do the math after that, you will get that this is the most compact form of this equation, right. Now mu v, right which is basically the viscous term, right. Now for the x direction it can be written as u, right. Which will be mu d square u d x square, right. Plus, d square u d y square right. So, that is basically the Laplace of the velocity field, right. And this is the body force term most of the time it will be like rho g; which is basically the body force, if the z direction and the x direction actually matches. Otherwise it will come in the y component of the velocity, all right.

So, this is of course, the total equation the total equation. Form now it v will become equal to u or small v depending on which direction you are applying the momentum equation with. So, this looks very familiar to you guys, right. So, all we have done is that we have taken a control volume infinitesimal control volume, we have applied the conservation of momentum. And by using the conservation of momentum, what we have achieved? We have achieved that the rate of change of momentum is equal to the momentum in minus momentum out, and plus the whatever there are the body forces, whatever are the body and the surface forces, right. So, the surface forces we already saw they were the normal stresses and the shear stresses. Body force was basically the gravity and similar stuff.

So, after we have derived that, we have gone through the constitutive relationships, and one of the constitutive relationships is newtons law of viscosity, right. So, for Newtonian fluid and other things, we have been able to prove, right. That this will be the form of the equation that you will get. For incompressible flow constant viscosity Newtonian fluid using the constitutive relationships that we just laid down. You would be able to show, that this is the entire relationship, which has got a pressure term a viscous term remember this is not just a pressure this is the pressure gradient which is important, right. And then you have the viscous term, you have the body force term, and you have the term which is related to the convective acceleration. So, this particular equation takes care of the flow field. So, therefore, when you have a flow field over a flat plate.

Using this equation over there, right technically you will be able to solve the velocity profile. And the velocity profile in this case will look something like this; that we will come a little later that what the velocity profile should look like. Now this equation should suffice, should you be able to apply this you should be able to get the flow field information naturally coming out of this right. So, why I said this was important? Because you need to solve the momentum equation before you go to the energy equation. Energy equation is nothing but conservation of energy this is the same equation, that you saw in your thermodynamics course, right. But before that you need to know what is the flow field in nature, as you saw the flow field here it is for example, v is the flow field, right. The v is the velocity, depends on so many interplay of so many terms. There is a viscous term, there is a pressure term, there is a forcefield term.

So, all these play a role in resulting in a spatiotemporal variation of the velocity field. Spatiotemporal velocity field means, that v is a function of x y z and t, all right. So, that velocity field we should be able to identify through this, got it? So, what we will do immediately now is basically we are going to look at the third player in this which will be the energy equation.

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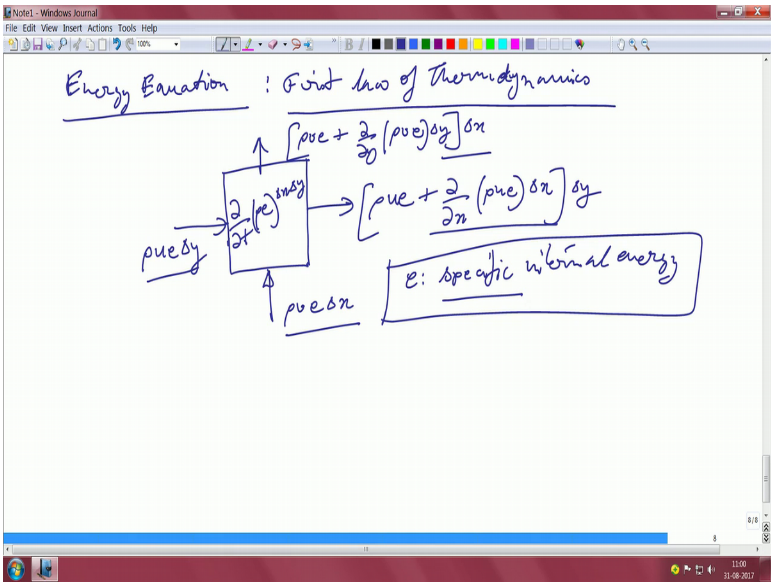

So, energy equation why the energy equation is vital, because of the reason that we said that energy equation is nothing but the first law of thermodynamics. First law of thermodynamics; that means, this is the first law of thermodynamics. So, what does first law of thermodynamics? Says basically says energy is conserved, right. It is nothing more than that, energy is conserved right. So, and that energy conservation we have to go in to write it in a slightly different form in the in the in in a field form essentially, right.

So, the energy so, what we will do? We will take that same control volume, and we are going to work out the math on that. So, we are going to first look at say if this is the control volume, once again there will be some energy that will be coming in there will be some energy that will be leaving out, right. So, the energy coming in say it is rho u e; that means, the flux in the x direction is carrying this much amount of energy into the and e is basically the internal energy, right. And then there is a delta y. On the other side, if you just do the Taylor series expansion; for that, you will get d by d x rho u e into delta x 2 delta y, right. So, that is the energy that is going out.

Similarly, there will be energy that will be brought in by this vertical stream, all right. Which will be rho v e into delta x and what will be coming out will be rho v e plus d by d y rho v e into delta y into delta x right. So, this is the energy that is coming in this is the energy that is going out. This is the energy that is coming in the x direction. And it is being taken out though energy is like a mass right. So, essentially whatever the momentum is whatever that mass is bringing with it a certain amount of energy associated with it, right. There is a blob of mass that is flowing in, it carries with a with it certain amount of internal energy. That is what we are actually budgeting for here.

Similarly, there will be a change of internal energy within the control volume; which is rho e into delta x into delta y, correct? That is the rate of change of internal energy within the control volume, because of all this flux terms is essentially the same concept, energy in minus energy out is equal to the rate of change of energy within the control volume that is what you are most familiar with. E is basically the specific internal energy, right. We do not know what that is so, but from thermodynamics you are very familiar with what is called specific internal energy. So, this particular expression basically lays down like mass. It is m e basically mass into whatever. So, specific that is why it is specific it is an intensive property right.

So, you multiply it by the mass you get the total energy that is flowing in minus that is whatever is going out. So, in the next lecture what we are going to do we are going to take this premise that we have already laid down, we very similar to your mass conservation. And we are going to see that how you can convert this to a temperature field. Because that is what you need. That is the quantity that you can measure internal energy is not something that you can measure, right. So, in order to cast it in terms of the temperature field, because that will lay down now the foundation that you know the velocity field now if you know the temperature field, you also know the continuity, you would be in a position to solve all the problems later on using these equations. So, that is what we are going to do next class.