Convective Heat Transfer

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Lecture – 19
Uniform wall temperature

So, last class we did the flow through a pipe, round pipe for uniform heat flux. This time let us take the case of uniform wall temperature.

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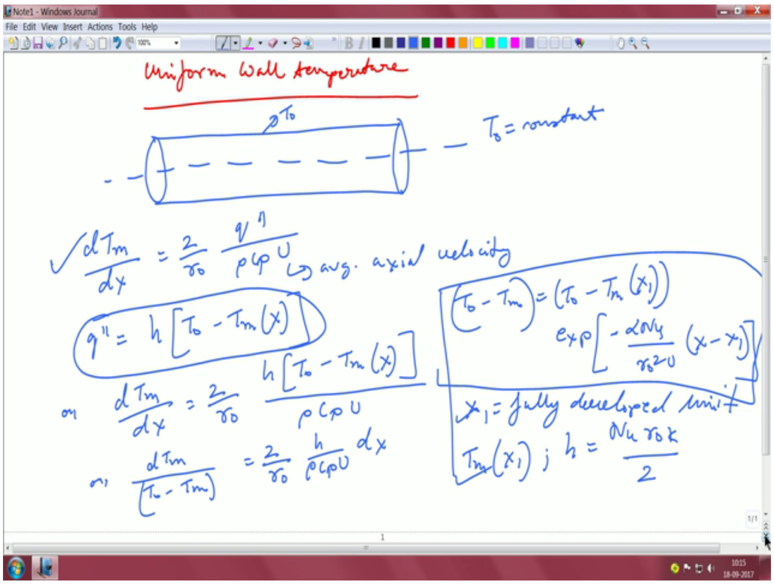

So, this is the other extreme and we will use the same methodology that we used earlier, but in this particular case only the wall temperature. So, if this is the pipe, once again; this is the central axis. So, this wall temperature is T naught, T naught now is a constant; obviously, your heat flux is not a constant anymore, because last class that is what we saw that your heat flux is a constant, and we could apply some nice little tricks and we could show that the Nusselt number was approximately 4.36. So, this time we will see that what the Nusselt number value will be, because that is one of the main purpose. Once again like in the external flow, here also the main intention is to find out what will be the value of Nusselt number? And we showed for that, you need the temperature profile.

So, similar to last time this is of course, given its 2 by r naught, this is a common thing that we wrote. Initially this, if you recall is the average axial velocity; q double prime is equal to h into T naught minus T m as a function of x, that was also kind of given or in other words d T m by d x 2 by r naught h; T naught minus T m as a function of x, divided by rho c p u or d T m by T naught minus T m equal to 2 by r naught h by rho c p u into the corresponding d x.

So, I am just using the standard definition that here of course, q, why did I do this, because my heat flux is no longer a constant, because it is not a constant heat flux problem. So, as you know your mean temperature can only able to be a function of x, because you by definition, the mean temperature takes into account the radial variation is its integrated over the radius of the pipe alright. So, it will be only a function of x.

So, therefore, once you and this is the generalized expression, the conservation of energy expression that we wrote two classes before. So, basically I have done just this much. Now let us look at; so, now once you integrate the whole thing, it offers you that you can integrate.

So, it is basically given as; so, what does this mean? x 1, means x 1 is basically the region of fully developed flow; that is the fully developed flow starts. There is a fully developed limit and T m at x 1 is basically the temperature at the fully developed limit at the starting point of the fully developed regime. And of course, h is n u r naught k by 2. So, using these two we have got this expression, this full expression.

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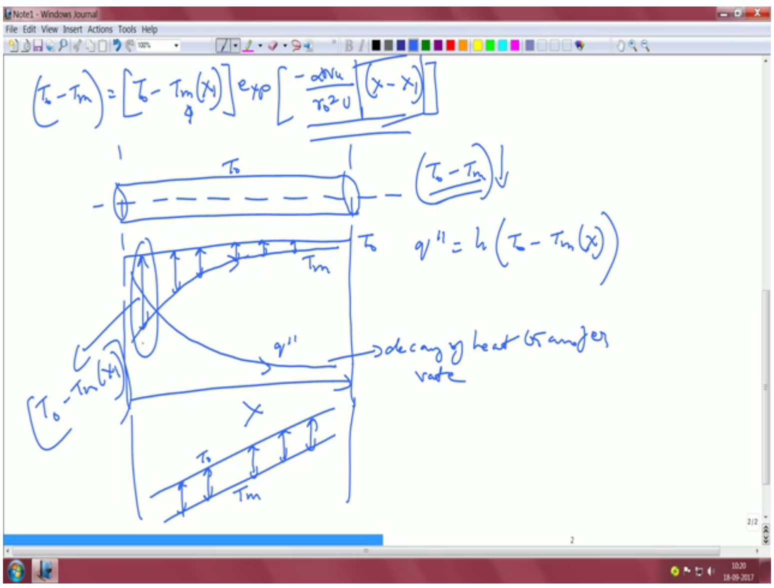

So, some interesting points to note about this particular expression is, let me write it once again clearly, so that you guys can kind of appreciate, where x 1 is the fully developed limit alright; so, that is the expression. Now using this expression you can see that this is the start of that, this is the maximum variation of temperature.

This is the starting point of the fully developed regime; after that you can see that the T naught minus T m term actually starts to go down exponentially. If you can look at, because it is exponential minus; so, T naught minus T m this particular expression starts to come down exponentially; that is what is given by this form of the equation. So, if once again show it on a scale here, say this is the pipe. These are the two axis is the center line, this is T naught.

So, T naught is; obviously, a constant, has to be a constant. So, what it implies is that your T m starts to do this. This is the profile and this is of course, your x. So, this is actually your T m. So, T m T naught minus T m is basically coming down. Now if you look at that what was the expression for q double prime; that is the heat flux, it was h into T naught minus T m at any x. So, naturally what it means is that q is given by this particular difference between the two temperatures. So, what should be the nature of q, q should also come down in an exponential fashion. So, in other words this comes down like this correct. So, there is a d k of heat transfer rate, and there is a d k in T naught minus T m.

So, basically what happens is that the mean temperature increases, increases, increases till it kind of approaches the wall temperature. Of course, it is never quite true, because its an exponential relationship. So, it never quite becomes equal, but the difference slowly and steadily goes down with x; that is what you can see from this particular expression, because its an exponential term involving x.

So, as x increases this term should come down compared to whatever is the initial value. So, initial value is this one say. So, let us say this one you can call it as T naught minus T m at x 1. It slowly comes down from that particular value at the initial condition from the initial conditions. So, that gives that is an interesting thing. So, compared to compare this to the constant heat flux case, whereas we saw that T naught and T m both were increasing, but the slopes were the same.

So, their difference was always the same; that is bound to happen, because your q double prime is happens to be the same, it happens to be a constant. So, compare this to your constant heat flux case if you recall; so, that was your T m T naught and T m there is this constant difference always that was maintained; that is, because of the reason that q double prime is a constant.

And both of them increased in a constant fashion, both of them increase, they were like parallel lines essentially, because their slopes were the same and actually the slopes were a constant; that is why we got the linear variation. So, here of course, the variation of T m is necessarily exponential in nature, its exponential in nature, it increases exponential now based on this. So, let us look at the full expression now, if we have to do this.

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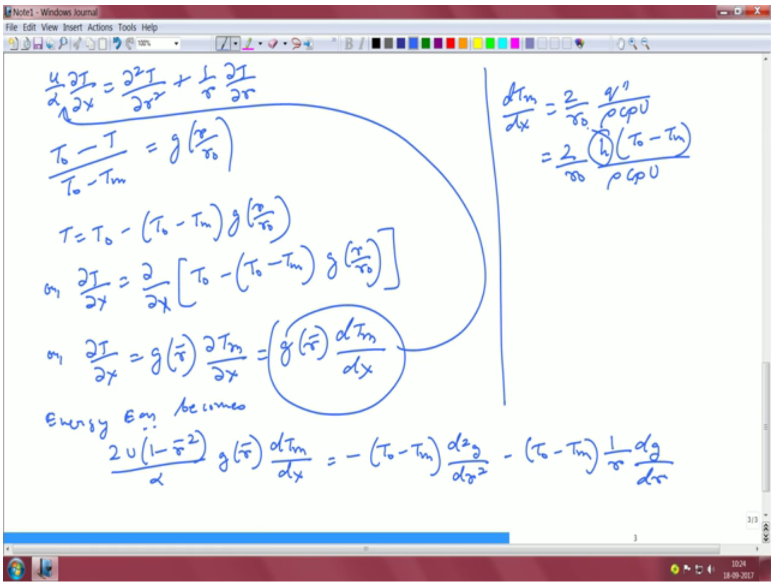

So, u by alpha d T by d x; that is essentially the expression that we have, that is the energy equation we have done it 20 number of times. So, T naught minus T by T naught minus T m, recall your definition of the fully developed temperature, its g r by r naught, remember the Nusselt number is still a constant, if you recall though the modes the equations.

How we cast? It is very different, but Nusselt number is still a constant, there is no that Nusselt number limit that we got through the scaling argument that Nusselt number should be of the order 1, and it will be a constant; that is still valid here. So, whatever we found about this, about this T naught minus T divided by T naught minus T m equal to g as a function of r only that it still holds over here.

So, T in equal to T naught minus T naught minus T m into g r by r naught or d T by d x or this is g r bar; that is the expression that you get. Now what is the standard thing to use this particular expression back here in the first term of the series.

So, the first term; obviously, is going to be u, u is given by the hydrodynamically fully developed profile. So, the energy equation becomes this is the. Ok now also we know d T m by d x is given by 2 by r naught q rho c p into u; that is also given that is a standard expression regardless of whatever. So, 2 by r naught into h T naught minus T m divided by rho c p u correct and h if you recall is nothing, but Nusselt number has to be a constant from our scaling argument, because that part we know that it has to be a constant, has to be of the order 1.

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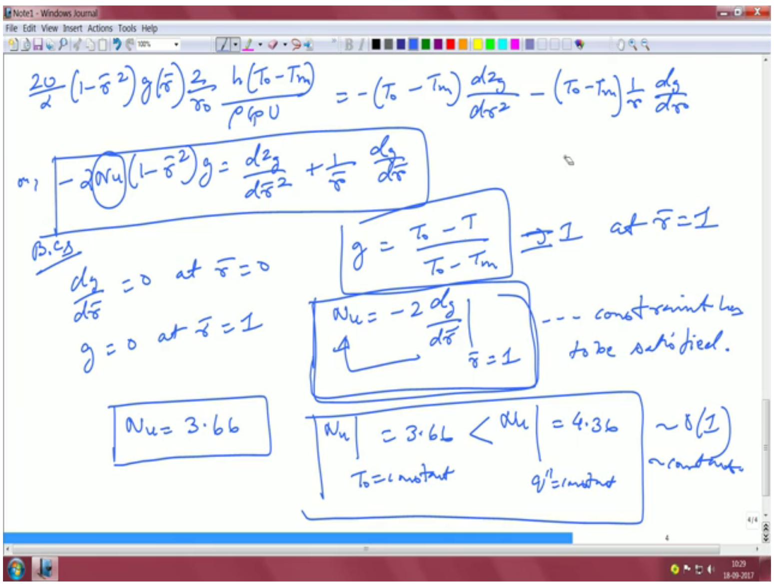

So, using all these expressions; now, let us substitute it back in the energy equation. So, the energy equation becomes 2 u alpha or cast it in terms of Nusselt number ok. So, that is what you will get. Now of course, to solve this equation you need the boundary conditions. So, the boundary conditions will be d g by d r bar is equal to 0 at r bar equal to 0, at r bar is equal to 0, and g is equal to 0 at r bar equal to 1, why g should be equal to 0 at r bar equal to 1, because what is the definition of g, g is given by T naught minus T divided by T naught minus T m. So, at r bar equal to 1; that means, at the wall what will be the value of temperature, it will be the same as the wall temperature. So, naturally this will approach 1 or this will be equal to 1 at r bar equal to 1. And similarly the temperature profile would have a general inflection. So, that d g by d r at r equal to 0 will be equal to 0.

So, these two boundary conditions are fine. At the same time one more boundary condition, one more thing that we have to be consistent over here, is that there is a Nusselt number; that is involved in this expression to solve for this Nusselt number, you have to have a physical and meaningful definition of Nusselt number. You cannot have an arbitrary solution with any arbitrary value of Nusselt number.

So, Nusselt number by definition is given by 2 g r bar at r bar equal to 1; that is the generalized definition of Nusselt number. So, whatever we do, this particular constraint has to be satisfied, to be satisfied; that means, whatever is a profile for that g. It has to obey that Nusselt number definition; that means, the Nusselt number has to be equal to this, this particular definition has to be obeyed. Now there is no way of solving this equation in a normal way. So, you can do numerical schemes and other things ok.

So, if you solve for all those things your Nusselt number value will come out to be 3.66, 3.66 will be the value of your Nusselt number, if you do all the manipulations that I showed over here. So, this actually shows that the Nusselt number from the uniform wall temperature is actually a little lower than the uniform heat flux. So, Nusselt number T naught equal to constant condition is about 3.66 Nusselt number for q double prime equal to constant; that is about 4.36. both are of the order one, both are constant which agrees with whatever we did say earlier, but this one is slightly greater than this, slightly alright greater than this.

So, you can see that the equations are also kind of very similar that we got the Nusselt number, definitions are similar, the definition of g is very similar, the definition of T m is similar. So, everything else is similar except that we have applied. We have basically applied a new boundary condition and a little bit of a different way to cast the problem to get to this particular figure ok.

Now, in table 3.2 of Bejan you will have a lot of you know for different cross sectional cross section pipes. So, we have done it for circular cross section, say for example, square hexagon, octagon, these kind of cross sections you can get an expression for the

Nusselt number, both for the fully developed regime and for the uniform heat flux condition, as well as for the isothermal wall temperature; that means, the constant wall temperature boundary conditions in all of those cases, you will observe that the Nusselt number value for the uniform heat flux within a particular family, is always higher than the corresponding isothermal wall temperature.

So, that is what you will see always for all the cases, and for fully developed flow for the circle shows the largest value of Nusselt number. Whereas, at the same thing is valid for the isothermal wall also for any other cross section, you actually have a reduction in Nusselt number ok.

So, that is what. So, closer for example, square will show a very low value of Nusselt number, no means in a relative parlance, it will show a low value, but as you increase go to more and more higher order polygons; that means, say hexagon, octagon and all these things; that means, you are closely approaching a circle ,as you increase the number of sides of the polygon you basically approach a circle, you slowly approach the Nusselt number limit of 4.36 for a uniform heat flux and 3.66 for a uniform wall temperature.

But 3.2 and 3.3 of Bejan actually reports and I will provide this as a supplementary material. It is actually all these Nusselt number values are compiled. So, you will not show the details at how they were calculated, but the procedure will be essentially the same as laid down here. Of course, square and other things will be more 3 d in nature ok.

So, it is a little bit more complex than what you can comprehend, but nevertheless heuristically speaking circle shows the largest value of Nusselt number followed by the higher order polygons; that means, in decreasing order of polygons, you have a decreasing order of your Nusselt number.

So, in the next class we will start doing the things on what we call the isothermal delta, if the tube is surrounded by an isothermal fluid then what happens.

Thank you.