Convective Heat Transfer

Prof. Saptarshi Basu
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture – 18
Uniform heat flux

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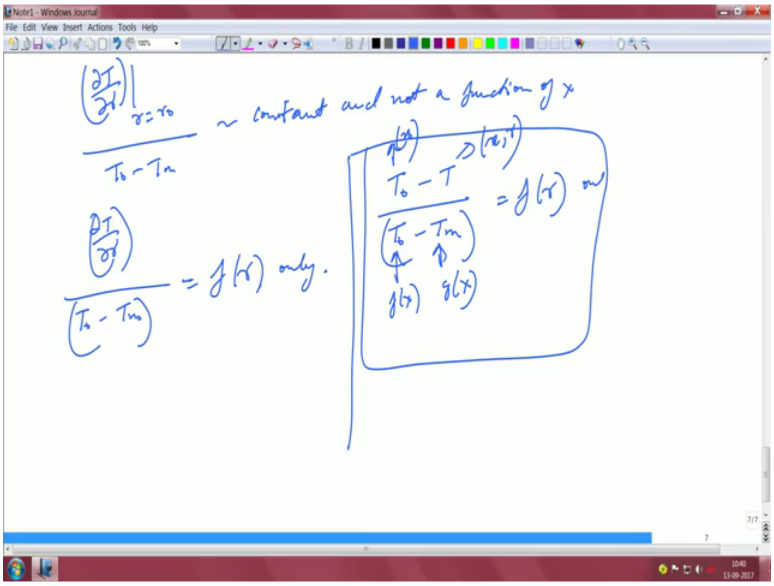

So, last class what we saw? Was that d t by d r evaluated at r equal to r naught, divided by T naught minus T m is basically a constant. Basically, a constant and not a function of r, not a function of r not a function of x, but we know that d t by d r by definition, is a function of both x and r.

So, one can conjecture that perhaps d t by d r by T naught minus T mean will be like a function of r only, because T naught minus T m is a function of x. So, d t by d r is evaluated it one particular point we saw that it was not a function of both where same functions of x. So, therefore, once you divide 1 by the other, there is a possibility, that we can argue that this will be a function of r only. So, based on this definition, let us define what will be the temperature profiles? The temperature profile will be T naught minus T divided by T naught minus T mean this is a function of r only.

Whereas; however, individually these are functions of x, T is a function of both x comma r, T naught can also be a function of x got it, but even though whatever be the nature, together this is only a function of r got it. So, together it is a function of r only. So, this will be the definition of temperature in the fully developed regime. So, in the fully developed regime this will be the definition of temperature that we are going to use.

Now, let us take the very basic definition that before we go into all these things we can take the basic definition now, that let us see what can be what are the different kinds of temperature profiles that we can actually have? So, one will be of course, the uniform heat flux case and one will be the uniform wall temperature case. So, you will look at both the uniform wall temperature and the uniform heat flux both, in the subsequent section. So, let us take the uniform heat flux given over here that is a little easier to start with uniform heat flux.

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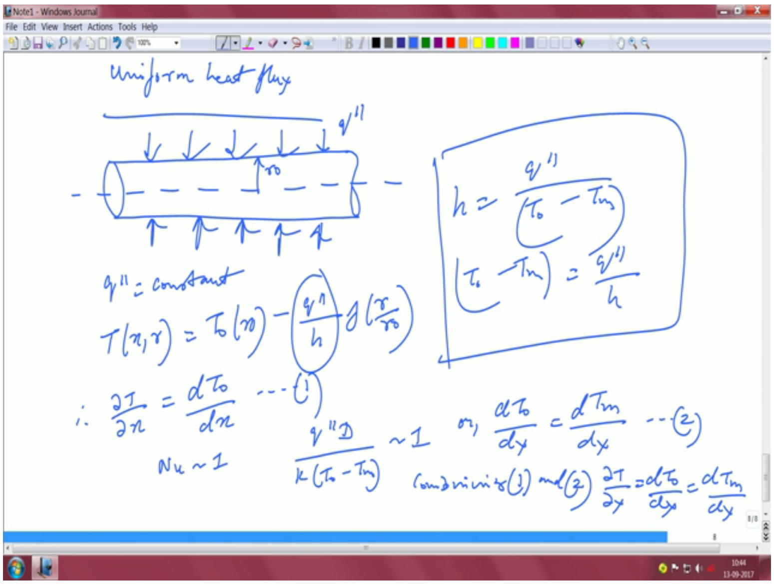

there is the boundary condition has changed basically.

So, this is the pipe once again that you have goes through. So, there is a heat flux this is q double prime this is basically r naught that you have. So, this is a fully developed temperature profile that you have to find out q double prime is basically a constant this is a uniform heat flux case got it. So, we have said nothing about the wall temperature, here the wall temperature is not a constant wall temperature is still given by T naught, but it is not a constant. So, that we will see what that wall temperature is.

So, instead of defining the temperature in that way, let us first say t x comma r T naught x I am writing the functional variation. So, that you can get an idea h f r over r naught that is the definition that we have applied. This is a very simple enough definition once again because T naught minus T m has been basically substituted by q double prime, that is all and using the definition of h. So, if you recall the definition of h what is was q double prime divided by T naught minus T mean.

So, if you have to substitute T naught minus T mean it will be q double prime divided by h that is exactly what we have done over here, this particular term. This is there is no difference between this and this they are essentially the same only T naught minus T m has been substituted by q double prime, because q double prime is a given and it is a constant that is the only reason that we have used.

So, do not lose the track of this particular thing. So, therefore, d t by d x should be equal to d T naught by d x, when you differentiate it with respect to x your q double prime is a constant this function is a function of r only. So, in you are differentiating by x the second term just vanishes. So, this is the first quantity that you get. Now again, we also know that Nusselt number is of the order one correct. So, therefore, q double prime into d divided by k T naught minus T mean is of the order 1 of the order 1.

So, therefore, therefore, from this particular case if this is of the order1 then what can you say or d T naught by d x is the same as d T m by d x. You just take it to the other side and again differentiate it with respect to x you know q double prime is a constant. So, it becomes d T naught is equal to d T m by d x. So, compositely what we get from this is your 2. So, combining 1 and 2, what do we get we get? D t x partial is equal to d T naught by d x that is equal to d T m into d x got it. So, all the slopes are basically kind of the same.

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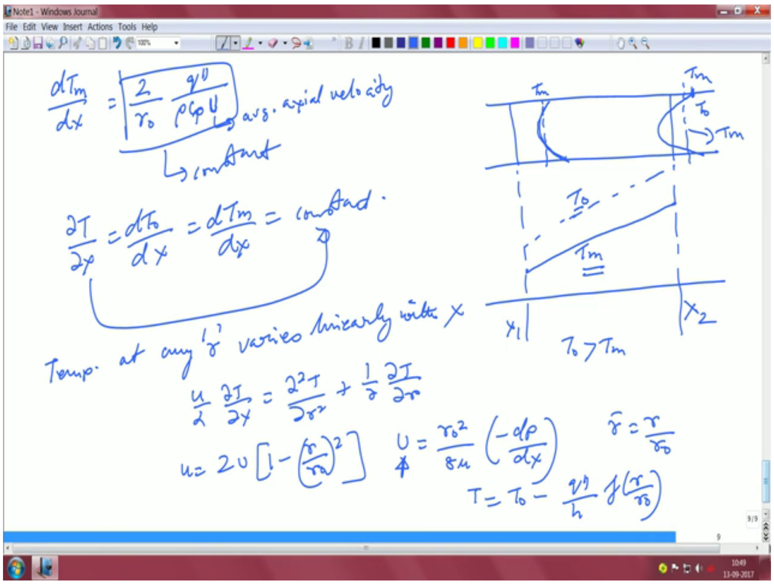

Now, in addition to this, we already know that d T m by d x is basically 2 by r naught q double prime divided by rho c p rho c p into u. This we already know recall this is the average axial velocity got it got it.

Now, this is basically therefore, a constant, because your q double prime is a constant. U is the average axial velocity which is a constant and everything else is a constant over here. So, therefore, as we wrote earlier d t by d x equal to d T naught by d x equal to d T m by d x is equal to a constant. This implies that this is a constant as well d t by d x is a constant.

So, temperature this implies the temperature at any r with varies linearly with x temperature at any r at any r location varies linearly with x. So, therefore, why not take a look at the sample profiles? So, let us take this is as the tube. So, let us take this is one section, this is another section. These are all in the fully developed regimes pull these things down this is you x 1 this is your x 2 al. So, the temperature profile is the mean temperature this is the profile of the meantime this is the full temperature profile, mean temperature will be some out here, that will be the mean temperature.

So, at a different cross section you will have that there will be some variation and the mean temperature will be somewhat like that T m at that particular location. So, if you now, plot the variation of the mean temperature you will find that your mean temperature goes up.

So, this is your mean temperature profile, the wall temperature as you can see from that is also going to go up, again linearly the slope is the same. So, it is also the wall temperature it also goes up. So, it is a linear both are linear so, the wall temperature goes up linearly, the mean temperature also goes up linearly. Now, as you can see that T naught is definitely more than T m, that is why we have 1 over the other because that is a wall. Wall is the maximum temperature over here maximum temperature over here.

So, it will be naturally more than the mean temperature because of the curvature you can see the curvature over here and you can easily make out that the wall temperature will be the highest. This is basically T naught, this is basically T m, if you there is any ambiguity. So, that is the wall temperature and the mean temperature so, between any 2 locations x 1 and x 2 you will see all of them going up like that.

So, therefore, we have established that in the case of a uniform heat flux these 2 like follows like 2 parallel lines kind of thing. So, the wall temperature also increases T m also increases. Now, let us look at the situation that how this can be now solved in a proper fashion. So, again the equation is still the same that d t by d x is equal to d square t by d r square plus 1 over r d t by d r. U we already know is given by 2 u into 1 minus r by r naught square. Where, u is basically given by 8 mu d p by d x got it.

So, that is the original thing that we said this was the average velocity that, we said and this was the whole profile after of the velocity. Also, the temperature profile we also defined that was equal to T naught minus q double prime by h f into f by r naught. So, that was the 2 definitions that we had and r bar is basically can be taken as r over r naught in the normalized variable. So, based on these quantities we can now reduce substitute them back in the energy equation.

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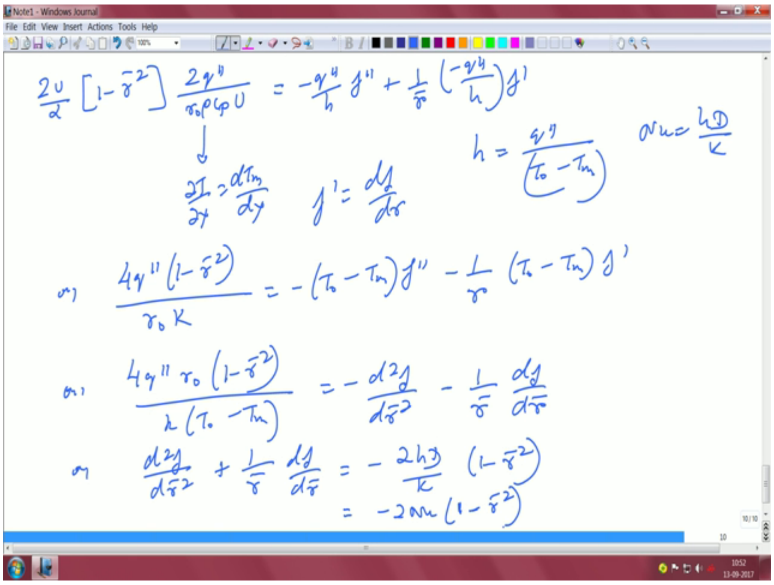

So, it becomes u by 1 minus r bar square into 2 into q double prime divided by r naught rho c p into u. Why can I substitute that? Because d t by d x this is because d t by d x is the same as d T m into d x. That is a substitution that we have made minus q double prime h into f double prime, plus 1 my r bar r bar q double prime by h f prime. Where h, is given as q double prime divided by T naught by T mean.

So, this is the very standard equation or in other words now, we can write it as 4 q double prime 1 minus r bar square, divided by r naught into k, is equal to minus T naught minus T m into f double prime, minus 1 by r T naught minus T m into f bar. Now, all this f prime is basically given as d f by d r. So, you can convert them to the corresponding d r bar coordinate system. So, that is that is fairly easy to do and you can also introduce Nusselt number in the mix, because Nusselt number is given by as you know h d by k. So, based on that what we can do is we can write the full equation now.

So, I will do one more step so, that you get a feel or d square f d r bar square plus 1 over r bar d f by d r bar, 2 h d by k k into 1 minus r bar square. So, this can be further written as minus 2 into Nusselt number into 1 minus. So, I have skipped a few steps, but you can easily follow that. What all we have done is converted it to the r bar coordinate system. Now, now let us solve this particular equation then.

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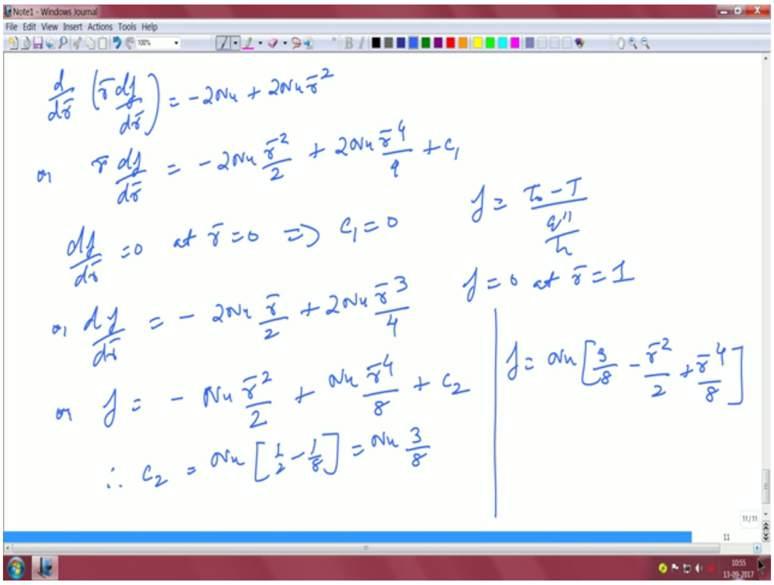

So, d by r bar or d f by d r bar is equal to 0 at r bar equal to 0. So, this leads to C1 is equal to 0 or d f by d r bar minus 2 Nusselt number r by 2, plus 2 Nusselt number r cube by 4. Integrate it one more time, f is equal to Nusselt number into r square by actually this will become 2 now, plus Nusselt number r to the power of 4 by 8 plus C2.

Now, f as we know is T naught by T divided by q double prime by h. So, and h of course, we know so, f will be equal to 0 at r bar is equal to 1, because it will be the wall at that particular point. So, therefore, C2 will be equal to half minus 1 by 8 to be equal to Nusselt number 3 by 8. So, therefore, the total form of f will be Nusselt number 3 by 8 minus r square by 2 plus r to the power of 4 by 8.

So, that will be the total expression that we will get. For f, because f is the thing that we have to solve utilizing the boundary conditions that we just now mentioned, that f prime is equal to f equal to 0 at r bar equal to square r bar equal to. Now, that we have it now let us write the full eq therefore, your therefore, your t is given as T naught.

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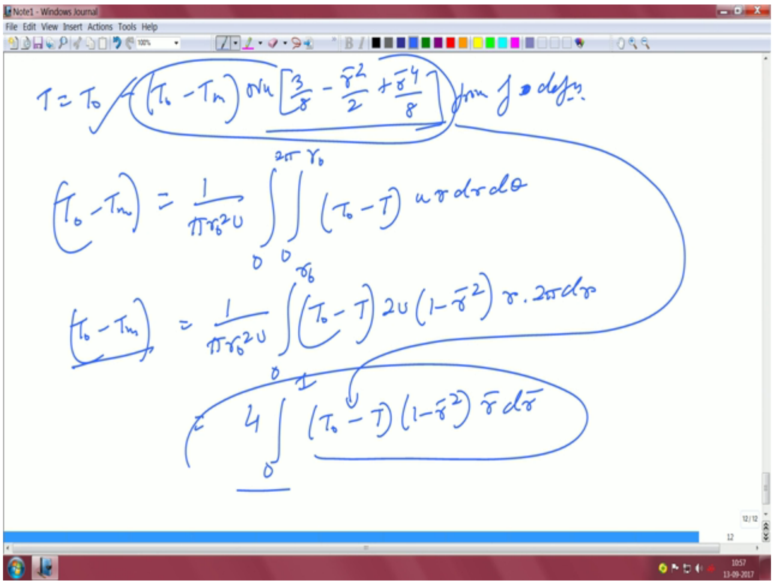

minus T naught minus T mean into Nusselt number 3 by 8 r square by 2 plus r to the power of 4 by 8.

So, this is the if you consider f if you open up f. So, that is what you get so, from f basically from f definition. So, that is what you are going to get now. So, of course, you can take things. So, this T is basically a function of both x comma r alright, now my original definition of T m. If you recall was 1 over r naught into u divided by this is this was 2 pi r naught T naught minus T u r d r d theta do you recall.

So, that was the original definition of T m when we actually did it. So, therefore, T naught minus T m well now if you can put the terms in within the bracket so; that means, you can integrate out this thing in a little bit of a better way 2 u 1 minus r bar square r into 2 pi d r. That is the equation. So, this will give you 4 into 0 to 1, T naught minus t 1 minus r bar square r bar d r bar. That is the expression that you will get.

So, therefore, we have basically 2 terms one is this, which basically gives you T minus T naught is equal to this. The other one is T naught minus T m is given as a function of this. If you look at it so, therefore, what we are going to do now? That we are going to take this definition and substitute it here. So, T naught minus T we are going to substitute it in terms of T naught minus T m in this expression.

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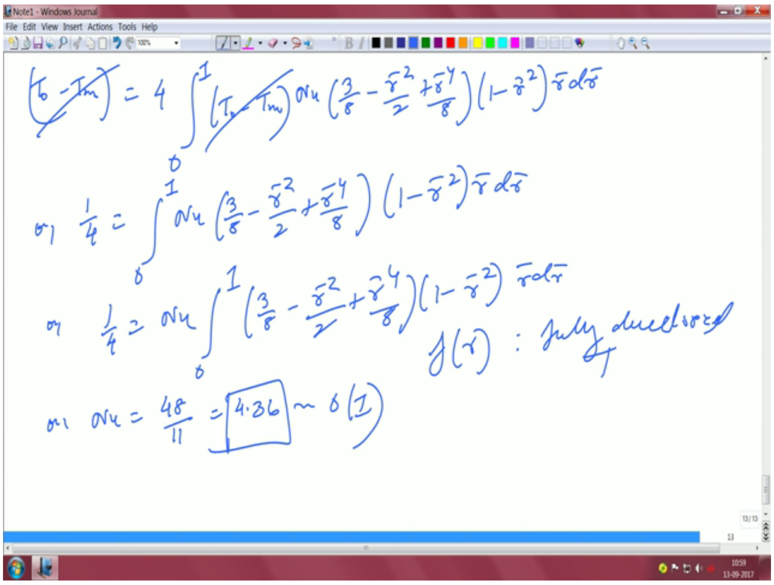

So, T naught minus T m is equal to 4 into 0 comma 0 to 1. Again, T naught minus T m we are substituting it now. With the Nusselt number 3 by 8 r square by 2 r to the power of 4 by 8 into 1 minus r r bar square r d r bar. So, that is the substitution so now, this gets cancelled that is because T naught or T m is not a function of r anymore. So, that you can take it out of the integral.

So, what you are going to get is one forth is equal to 0 to one Nusselt number 3 by 8 minus r square by 2 plus r to the power of 4 by 8 into 1 minus r bar square r bar d r bar or. You take the Nusselt number out because it is a constant by definition. So, the rest of the integration becomes easy. It is basically an integration with respect to r.

So, if you do this integration, do it you will get 48 by 11 which is 4.36 that will be the answer this is of the order one. So, our scaling analysis was correct the exact value is 4.36 which has come out through this exercise that we followed. So, 2 things we did it f r represents what we call the temperature, the fully developed temperature. Temperature that we have found out what the fully developed temperature was, if I have to write it once again you just note this thing down because.

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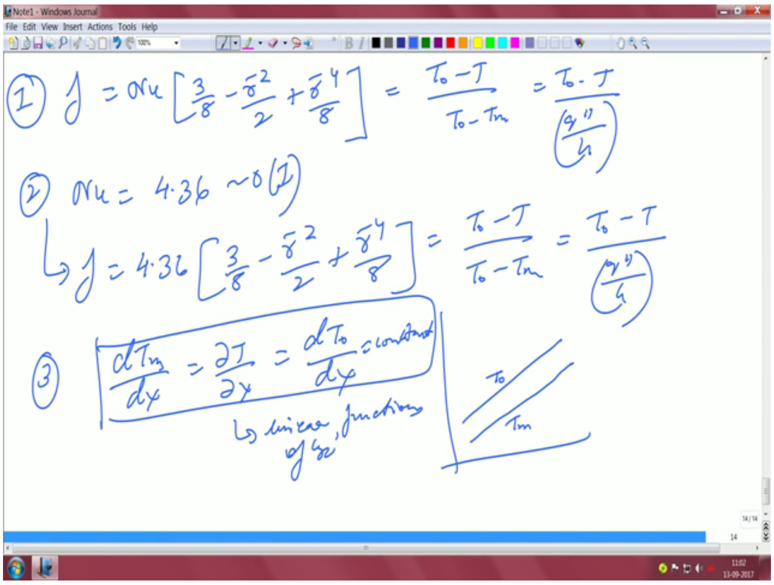

These are the findings like fact findings is given as Nusselt number 3 by 8 minus r square by 2 plus r to the power of 4 by 8. And what was f if you recall? What was f? F was basically T naught minus t by T naught minus T m or it can be also written as T naught minus t divided by q double prime by h whatever it is.

So, this is the definition second finding is or that is what we found that Nusselt number was 4.36. So, essentially you can say f is equal to 4.36 into 3 by 8 minus r to the power of 2, r to the power of 4 by 8 is what is your definition of the temperature profile. This is all for a uniform heat flux case. So, this is like the corollary of this particular thing, we also found out that the Nusselt number. So, we have found out the exact variation. So, this Nusselt number is of the order one as we decided earlier, we have found out what is the definition of x going to be. Similarly, we also found out another important thing that d T m by d x is the same as d t by d x, is the same as d T naught by d x.

So, we said that all the these 2 temperatures well have not drawn exactly parallel. This will be T naught this will be T m. So, both of this temperatures varies linearly with x, linear functions of x is this is equal to a constant, linear functions of x which was a surprisingly interesting tool, which gave us this particular thing. You can also find out what is going to be your mean temperature now, because you have already got the definition of mean temperature. So, you can find out what is the mean temperature profile is also going to be, but utilizing all these things we have got these 3 important correlations or if this 3 important findings out of this entire exercise.

So, this is vitally important because, next we are going to look at the uniform wall temperature case. So, this was a uniform heat flux case and we saw that this is how the temperature profiles vary, this is how the Nusselt number value will be and this is how the temperatures the different temperature gradients are related. So, these are the main 3 so, if this is like a summary that you these are the main 3 events that, you should be really bothered about when you move on to the next. So, uniform wall temperature we will start pose the problem and finish it afterwards.

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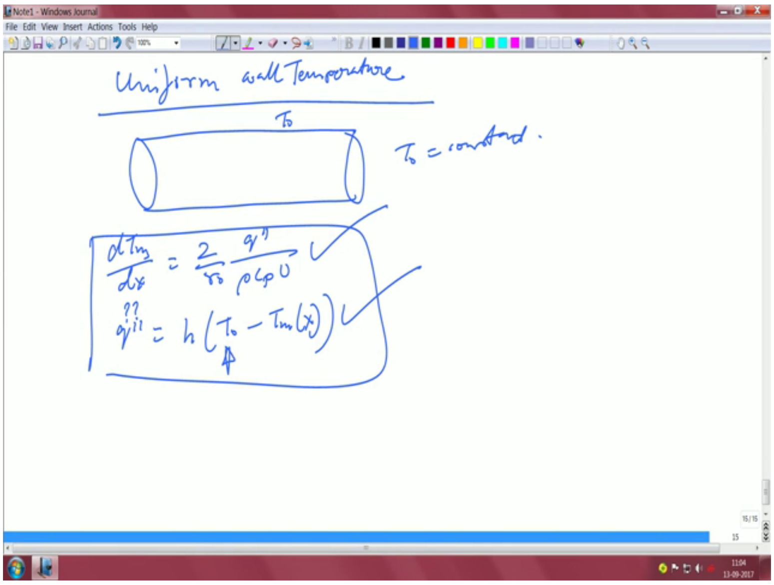

So, uniform wall temperature this is a very similar thing. Now, except now that the wall are fixed at a temperature of T naught. So, T naught is basically a constant, T naught is basically a constant and your d T m by d x is still given by the same thing, q double prime rho c p into you and q double prime is basically h into T naught minus T m into x.

So now, your so using these 2 parameters and using our definition that what it can be we will see in the next class. That, how this heat transfer for this particular system can be analyzed? Everything else is the same except that the heat flux is unknown over here. Only the wall temperature is known this is applicable regardless that d T m by d x and this is also applicable regardless.

All of this can be written in that same fashion, but except we do not know what is the value of this q double prime going to be, but we know what is T naught it is a given to us. So, you will see that how this particular problem pans out in the next class. So, you will finish the uniform wall temperature case and we will see where we get to that.

Thanks.