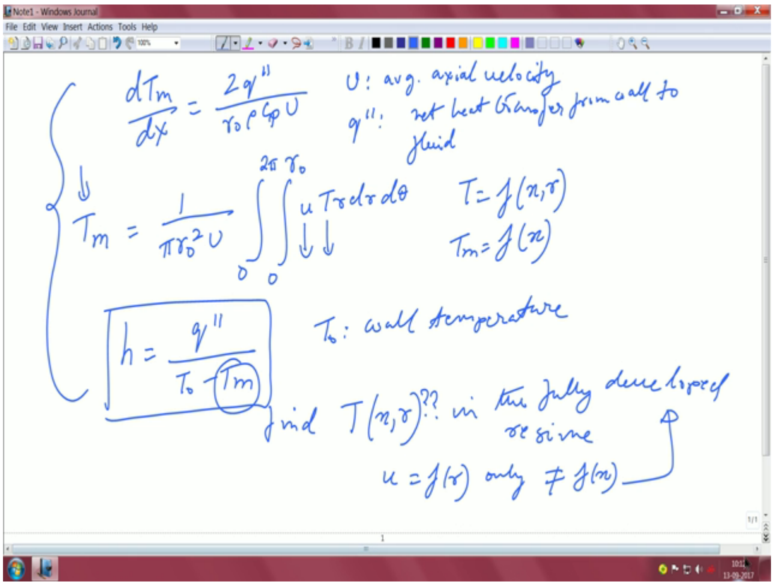
Convective Heat Transfer
Prof. Saptarshi Basu
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture – 17
Mean temperature in fully developed flow

Last class, we actually developed a concept of Mean temperature in a fully developed flow. So, Mean temperature was an useful quantity because it was a radially average to temperature essentially. So, the temperature was averaged in the radial direction and we introduced a concept of mean temperature and we wrote this particular equation.

(Refer Slide Time: 00:41)



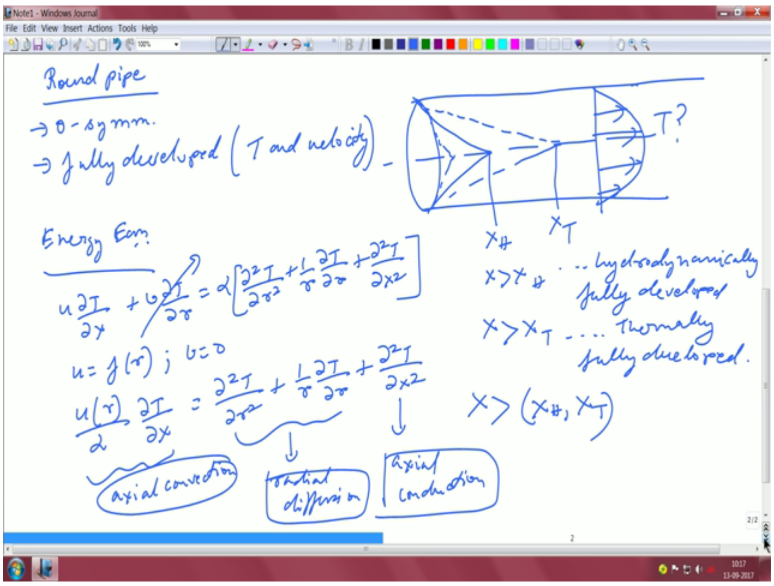
If you look at the presentation r naught rho C p into U where, U was the average axial velocity correct and q double prime was the heat transfer or the net heat transfer; transfer from wall to the fluid. That was a concept that we developed. We also found that while doing, while developing this concept; we found that T m which is the mean temperature is actually given by this we did last class.

So, I am just noting down the key quantities u T r d r d theta. This u is basically the local velocity; T is basically the temperature field. So, T in general is a function of both x comma r whereas, your mean temperature should be a function of x only because we are taking care of the r variation away. Because that is what the integral the purpose of the integral exactly is.

So, also we defined h which is basically the heat transfer; coefficient is basically T naught minus T m. Where, T naught was basically the wall temperature. So, that was the whole exercise that we did in the last class; where, we defined these 3 important quantities. And from here, we thought that the temperature we have to find it in some way. So, our motto is to basically find, what is this temperature field x comma r in the fully developed regime? Like for example, in the case of velocity we found that in the fully developed regime the velocity u was a function of r only.

And it was not a function of x; this we found, in the fully developed regime, in the fully developed regime. What about T? The question remains what about T; whether T is also a function of x is a function of r or it's only a function of r and things like that. That is what we need to answer; because based on that we can find out what your T m is going to be. Once we find T m; all the important quantity once again, is the heat transfer coefficient which is a function of T m in this particular case. So, all these things are kind of coupled with each other. So, T is needed.

(Refer Slide Time: 03:42)



So, let us take the formulation based on a round pipe. So, our basic problem is therefore, a Round pipe. So, the equations will be a little different from the Cartesian co-ordinates is basically theta symmetric; no variation in the theta direction and it is fully developed both in terms of temperature and velocity.

Of course, we do not know what the fully developed temperature profile is going to look like; that is what we are going to find out. Velocity profile is already known that is what we did in the last class right.

Now, fully developed means that even the thermal boundary layer, I am not saying that the 2 boundary layers have the same entrance length; that is not the question over here. So, if this is the round pipe that you have. This is the center line. It is possible that this is your x H which is the hydrodynamic boundary layer merged. So, from here onwards x greater than x H is what we call hydro dynamically fully developed; fully developed whereas, the temperature profile may be something like this or it can be also something like that. Whatever depending on whatever it is your pandle number is.

This say call it x T. So, if x is greater than x T, we call it Thermally fully developed. So, if x is greater than x T, we call it Thermally fully developed. If x is greater than x H, we call it hydro dynamically fully developed; the 2 boundary layers over here.

So we are in a regime in which x is greater than both x H and x T; that means, we are somewhere here. So, that will be like your velocity profile which you already know parabolic and we want to know, what is the temperature profile here? So, that is the unanswered question that we have. So, let us write down the governing equation because we already know the momentum equation because the velocities are known variable over here.

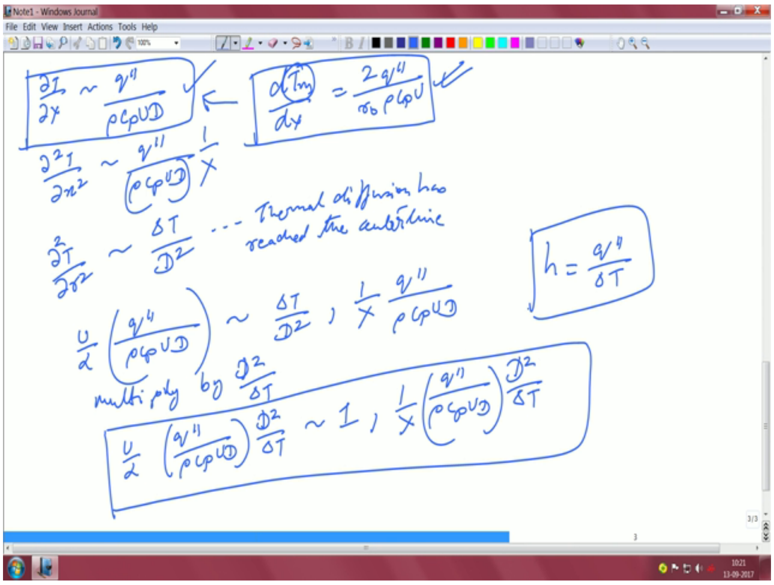
We just go on to the energy equation at this particular point. So, the Energy Equation, u d T d x plus v d T d r alpha T d r square plus 1 by r d T by d r plus del square T by del x square. This is the full energy equation written in polar co-ordinate systems and of course, we know u is only a function of f r and we know v is equal to 0. These 2 things are already pre known.

So, what you can write by simplifying this equation. So, it will be alpha d T by d x is equal to delta square r squared plus 1 by r d T by d r plus d T d x square. So, that will be the combined equation after we take off this particular term. And we write u r, as of u as a function of u r only. Now there are 3 main terms in this particular equation. So, this is what we call the axial convection. There's an axial convection. This is what we can call it to be radial diffusion or conduction.

And this one is called axial conduction. So, there are the different modes. So, you have the axial convection. Then you have the radial diffusion and then, you have the axial conduction. The idea of framing this equation now is that we have to find out which terms are important, which terms we can safely leave out. Obviously, we have to show some legitimate reasons, why we are actually deleting some of the terms.

So, let us do an exercise in scaling once again and let us see whether that makes sense in this particular context or not. So, let us take it term by term once again.

(Refer Slide Time: 08:32)



So, what about d T by d x; d T by d x is nothing but, q double prime divided by rho C p U into D and d square T by d x square is basically q double prime rho C p U into d that X factor comes into the picture; let us put this nicely. So, the first, so, we know d T by d x and we know del square T by d x square.

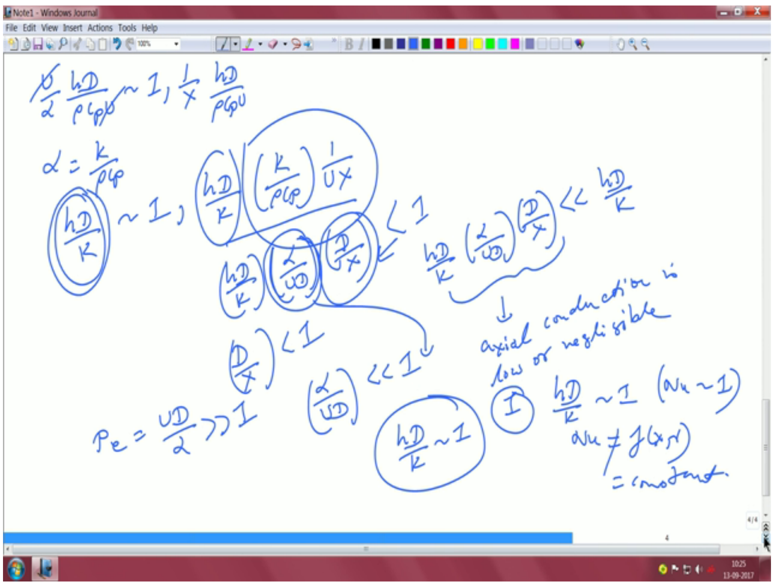
These terms will be known right now. And of course, there will be the last term which is basically d T d square T by d r square. Now that will be given as delta T divided by D square because we already know that r has reached to d; that means, the center of the pipe. So, basically this comes because of the fact that Thermal diffusion has reached the center line.

So, let us now take term by term. So, the 1st term will be U by alpha q double prime rho C p U D; that's a first term. This should be delta T by d square. So, that is basically the radial term; the radial diffusion terms. And you have 1 by x q double prime rho C p U into D; that is axial conduction term. Now pay attention this how we got this because we used the corresponding formulation, if you remember that how we found out the d T by d m kind of a term; it comes readily from there.

You may recall the term that d T m by d x was actually equal to 2 q double prime r naught rho C p into U. This was already pre given. So, using that we are actually utilizing that to find out, what are the scales of this particular factor is going to be. Because the mean temperature gradient is also similar to the actual temperature gradient; because we are more concerned about, because d T m by d x is basically take, has taken care of only the radial part, axial part variation is still there. So, that is exactly what we have used in this part that scaling; utilizing this which we already did earlier.

So, now that we have all these terms let us multiply all the terms by. So, multiply by D square by delta T, all the terms. So, the first term will become u by this q double prime rho C p U D into D square by delta T; this is proportional to 1 comma 1 over X q double prime rho C p U D into D square by delta T. That is the terms that we get. And we also know h is given as q double prime divided by delta T. So, let us look at term by term once again, using this definition; this is basically your heat transfer coefficient.

(Refer Slide Time: 12:30)



Based on this, based on the definition you have h D by rho C p into U proportional to 1, 1 over X h D rho C p U.

Now, if you look at this first term, this U and U gets cancelled with each other and you have alpha into rho C p. What is alpha? Alpha is basically k by rho C p. So, the first term becomes h D by K that is proportional to 1 or of the order 1. This becomes h D by K into k by rho C p into 1x over U X. This is the 3rd term.

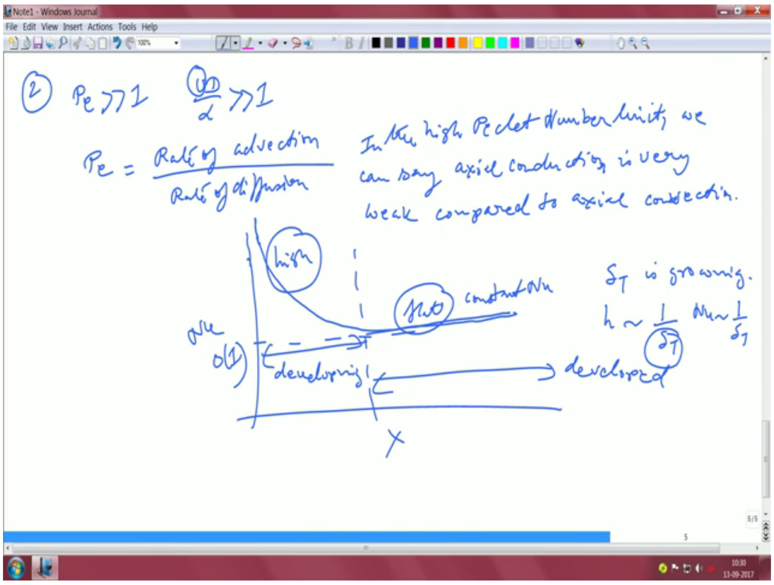
So, this we already know that this is readily the definition of your nusselt number; definition of your nusselt number. This is h D plus some more terms that are situated over here. So, let us look at the nature of this particular term in general. So, this h D by K is the first term, alpha by U D is the second term and then, D by X is the third term. I have just written it in a more compact particular notation.

Now, we already know that D by X is a number which is less than 1 because X is the length scale. So, X would be normally more than D. We are talking about pipes like that. So, this term will be less than 1; we know that. This particular term only; only the one which I have circled. Now, what about this term? We do not know anything about the nature of this term like alpha by U D. The nature of alpha by U D is unknown.

But; however, however if we define our number called P e which is called peclet number and we say that that is U D by alpha and if we say that that peclet number is much much greater than 1; that means, alpha by U D should be much much less than 1. In other words, this second term within the bracket will be much much less than 1; in that particular case what will happen? This entire term h D by K into alpha by U D into D by X, all of these term will be much much less than 1 or rather instead of saying much much less than 1 much much than h D by K, which is the term on the left hand side or in other words, we can say that axial conduction, conduction is low or negligible.

So, in other words, we are left with only the term involving the axial convection and the radial diffusion or in other words we are left with only h D by K proportional to 1. So, 2 important things that we have got out of this exercise. The first one, the most important thing is that h D by K is of the order 1; this means that nusselt number is of the order 1; that means, that nusselt number in the fully developed regime is not a function of x or r right not a function and it is basically a constant. That’s the first thing that we got.

(Refer Slide Time: 16:07)



Let us look at what is the second thing. The second thing that we said it the most important thing that we said is that peclet number is much much greater than 1 or in other words U D by alpha is much much greater than 1. This is a new quantity that we have defined. So, what is peclet number therefore? Peclet number is basically given by the Rate of advection divided by the rate of diffusion.

So, this is what is peclet now. So, that means, high peclet number means that the rate of advection is much much more than the rate of diffusion. So, in the high peclet number limit, in the high peclet number limit number limit, we can say that we can say that the axial conduction is very weak compared to axial convection; that is exactly what peclet number actually says; that means, your rate of advection is so high compared to the rate of diffusion, that you can actually neglect the axial conduction effect; because it's a very small number in the high peclet number limit.

That means, if your flow is very sluggish say for example, that is barely creeping and this peclet number much much greater than 1 limit is not applicable. Then, what you cannot apply you have to carry the radia, the axial conduction term with you. So, you cannot simplify this equation in this particular way.

So, this is like an eye opener, we saw 2 very important things through this scaling argument; one is that axial conduction in most cases are negligible because we are dealing with a convection problem with reasonably high peclet number, that is always the case whatever flow that you are going to deal with. Because you have this factor you alpha is normally a very small quantity.

So, if U is substantially large this number will be probably substantially large than, I mean more than 1, but the most important finding was that based on these 2 limits, you have that your nusselt number becomes a constant. So that means, whatever; that means, if you have to draw heuristically; that means, if you look at this particular plot and this is nusselt number this is X, X being the distance from the inlet of the pipe right.

Now, intuitively as you know that in. So, let us say that this is basically the developing region and this is basically the developed region. So, we are basically looking at the nusselt number in the fully developed region. Now in the developing region we all know that the boundary layer thickness is growing; it has not touched the middle of the pipe as of now. So, naturally as you know still that h is proportional to 1 over delta T; that means, nusselt number is also proportional to 1 over delta T.

So, that means, the nusselt number value is going to be very high in the developing region of the pipe. So, if you talk about it like this, the nusselt number value will probably come out as very high, but as this delta T goes on increasing, the nusselt number value is coming down. So, in the limit it will become something like this.

So, it is very high here and it is absolutely flat here and this value that you get is of the order 1. So, anywhere between 1 to 10, something like that it will be your nusselt number value; regardless of whatever you do. So, it is very high initially asymptotes and comes and becomes this flat line which is not a function of x anymore in the fully developed regime. So, that is a very important conclusion that this is the region of high nusselt number, this is the region of constant nusselt number or a heat transfer coefficient.

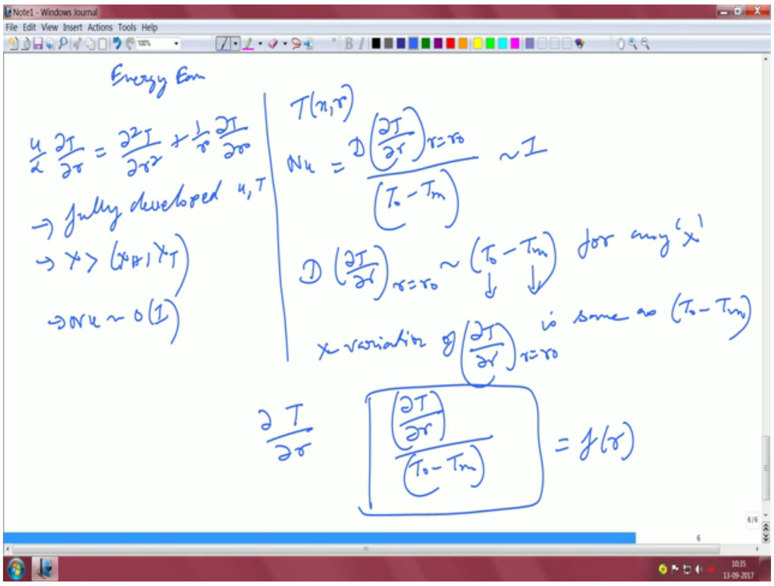
So, this particular finding is one of the most important thing which you did not get in a flat like boundary layer. There the nusselt number was still a function of Reynolds number raised to the power of half if you recall. So, here of course, the nusselt number no longer remains a function of your x anymore which is one of the most crucial findings that you have.

So, now what we are going to do? Now, that we have established the full fledged equation. Now the next half of the business will be to determine what will be the temperature profile; we still have not done anything to the temperature profile, all we have done is that we have made a couple of very interesting observations. We have reduced the energy equation to a very nominal form by neglecting most of the terms.

So, there are three purpose things that we achieved; we reduced the energy equation. We saw that in that limits nusselt number becomes equal to 1 and we saw that of course, axial the peclet this is a limit, this analysis is only valid for high enough peclet number. So, if peclet number is low then, you cannot apply this particular limit anymore.

So, let us look at the first energy equation and let us look at the at what we can do with it.

(Refer Slide Time: 22:02)



So, energy equation now, in its revised form is u by alpha d T by d r sorry writing too close to that; u by alpha and d T by d r is equal to. So, that is the first thing and once again, this equation now you see is devoid of the actual conduction term. So, once again, fully developed U comma T the interest. So, X is greater than x H comma x T; that means, we are in a region which is far away and of course, we got that nusselt number is of the order 1.

Now, let us look at how can we define a temperature profile in the fully developed regime, which will be a function of r like in your velocity profile we had velocity was a function of r only. Temperature is not that simple because T is a function of x comma r, but can we do some variations on this T. So, that it becomes a function of both r only only with respect to r.

But normal T is a function of x and r. So, last lets take the definition of nusselt number, the primitive this definition; it's r equal to r naught; that means, at the wall T by T m this is of the order 1. So, that would mean there is a D over here that would mean that d T by d r r equal to r naught is the same as T naught minus T mean; for any 'x'.

Remember that T naught and T m both can be functions of x only because of wall temperature and the mean temperature both are functions of x because mean temperature by definition you have removed the variation with respect to r.

So, on the left hand side whatever it is on the right hand side whatever it is, for any particular x, this relationship is very valid right. So, therefore, we can argue that the x variation, variation of d T by d r at r equal to r naught is same as T naught minus T m naught minus T m is not that so.

Because otherwise, for any x it won't be acceptable. So, for any x this relationship is valid; that means, the variations on both sides must be the same function of x. So, that takes kind of cancel each other out.

Yeah. So, now, we can have an important. So, based on this; based on this particular definition we, but T is normally, but when you define d T by d r say for example, so, d T by d r is basically nothing but the a variation of the radial temperature profile. Now when you look at d T by d r, what do you think that d T by d r's function is going to be? So, d T by d r is a quantity, d T by d r at r equal to r naught; that means, you are specifying it at a particular thing that has an x variation, but that x variation is basically taken care of by whatever is the T naught minus T mean variation is.

So, if I kind of conjured this up that d T by d r by T naught minus T mean, what will be the definition; what will be the variation of this quantity? Because this quantity therefore, the upper quantity is a function of both x and r, the lower quantity is a function of x only. We know that when you evaluate it at any particular location, for any r it kind of shows the same kind of variation. So, is there is there a possibility of writing this particular equation using this particular equation and stating that this is going to be a function of r only; that is what our main objective is.

So, we will stop here. In the next class, what we will see is that we will find out that how we can conjure up a definition for the temperature field, a definition for the temperature field which is a function of r only.

So, see you next class.