Convective Heat Transfer
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Lecture – 16
Hydrodynamic fully developed flow

So, last class we did that developing flow, but only in a very heuristic fashion.

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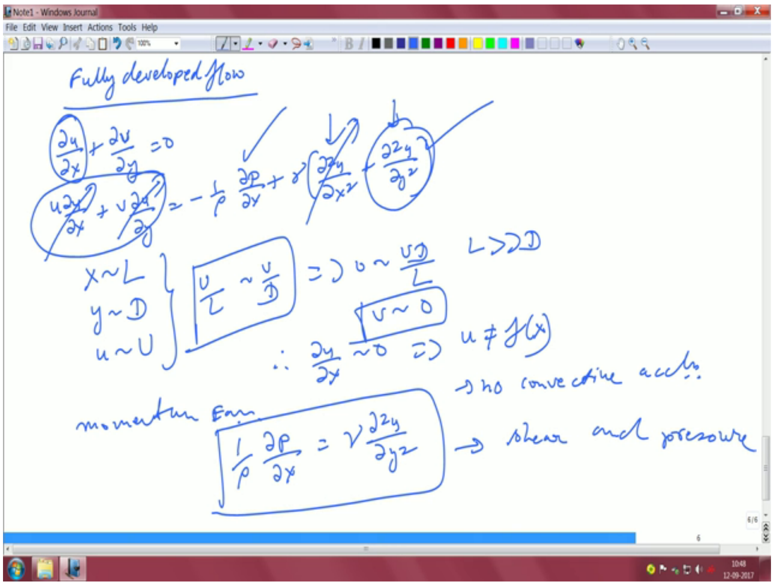

And this time, let us look at the fully developed flow. Where, we said that the flow profile should not change. They look identical; that means, there you can superpose one on the top of the other with distance. So, fully developed flow.

Now in the fully developed flow, I am writing the full fledged navies stokes equation, once again. That's where we will start and we will show that what is the, what are the terms that we can actually neglect. So, you follow the scaling argument here and we will try to show that, how we can get to the revised version. So, here x scales as L as before, y scales as high over D; because the boundary layers have already merged. So, the scale of y is basically D and u the scale of U is basically u that is the, because we saw it is only 1.5 times the maximum velocity.

So, which should be the scale is still the same; we are not going up by one order. So, of course, using this from the continuity equation you have U by L. It should be the same as

V by D. This gives rise to V being equal to U by D by L. Where, L is much much greater than D correct. So, if that is the case that when L is much much greater than D, you can take V to be almost equal to 0; in this particular case. So, therefore, d u by d x is equal to 0; d u by d x equal to 0. Therefore, this leads to that u is not a function of x anymore.

You understood the steps. So, we went from the continuity equation, there you show that V is almost equal to 0 because your [laughter], the length at which you are taking the year entrance is long pipe basically is much much greater than D. As a result of that moment V becomes equal to 0; then, by mandate d u by d x also has to be equal to 0. So, that would mean that u is not a function of x anymore. It can be a function of y; it can be a function of any other things. But it cannot be a function of x at this particular point right. So, the momentum equation then, becomes very simple.

Because, if you look at the momentum equation, the first term vanishes because d u by d x is not a function of x anymore. And the second term vanishes because V is equal to 0. So, and if you look at these 2 terms. Now in the momentum equation that is del square u d x square and y square; of course, we know that x scales as L and y scales as D; L being much much larger than D. So, that naturally only 1 component survives which is this one; this goes to 0 because it's L square.

So, the momentum equation basically if we write it vanishes and become something like this; d p by d x equal to gamma, after all these manipulations. Only 2 terms survives this one and this one that is it. So, that would be an interesting proposition, when you look at the momentum equation in this particular form, you will find that the flow is obviously, moving. But it's a non accelerating flow because the convective derivative which is the convective part has gone equal to 0.

So, there is no convection; no convective derivative. Let's not say convection, no convective derivative or no convective acceleration; that is equal to 0. So, the flow is basically happening due to a balance of pressure and viscosity. So, shear is balanced by the pressure that is what you have; shear and pressure and it's a non accelerating flow. This part should be very clear that it, this is only valid in the fully developed regime.

Now, now that we have this particular equation we don't need boundary layers anymore and we don't need scaling arguments anymore, we can simply solve the equation from the first principles.

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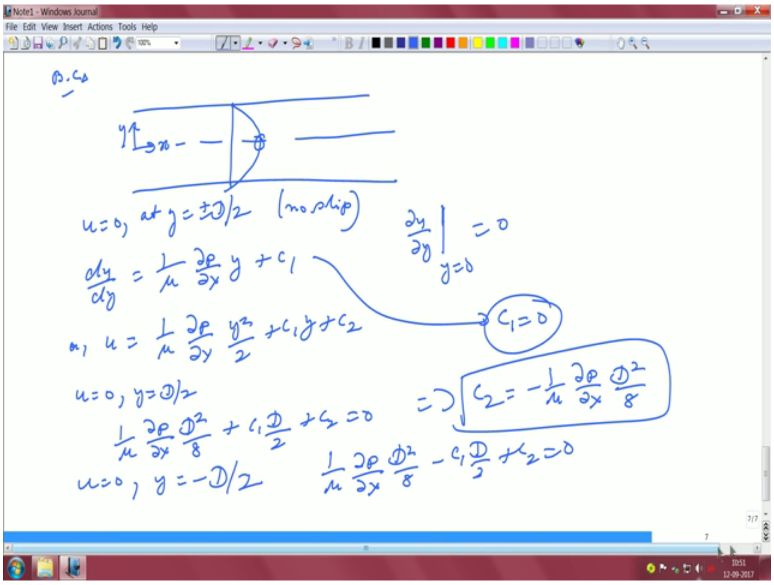

So, we can write the Boundary conditions first. As we write the boundary conditions. So, let's take this as x this is y; this being the access, you can have it in any other way that you want doesn't matter. So, u is equal to 0 at y equal to plus minus D by 2. The simple reason is that these are walls. So, the no slip condition applies; no slip. The equation also can be integrated very easily.

So, d u d y is equal to 1 by mu d p d x into y plus c 1, that's the 1st part of the of the integral and then, you integrate it once again. 1 over mu d p by d x y square by 2 plus c 1 y plus c 2. Using that so, u is equal to 0 and y equal to D by 2 Then you have 1 by mu d p by d x D square by 8 plus c 1 in by D by 2 plus c 2 is equal to 0. So, that is the first one. So, in this particular case as you can see. So, this is the full expression that you get.

Now, u is also equal to 0 and y equal to minus D by 2. So, in that particular way you will have 1 over mu d p by d x D square by 8 minus c 1 by D by 2 plus c 2 is equal to 0. So, this is the second form that you have. So, there is one more, one more thing that we can apply that if you look at the velocity profile, it is symmetric. It is a symmetric velocity profile because on both sides so, there has to be an inflection point.

At the center which would mean that your d u by d y has to be equal to at y equal to 0 has to be equal to 0 in a sense because a slope has to vanish. So, if you look at this from here, you will get your c 1 is equal to 0 because the slope will vanish and from the other part, you can get c 2 is equal to minus 1 over mu d p by d x D square by 8. So, that will be the other part.

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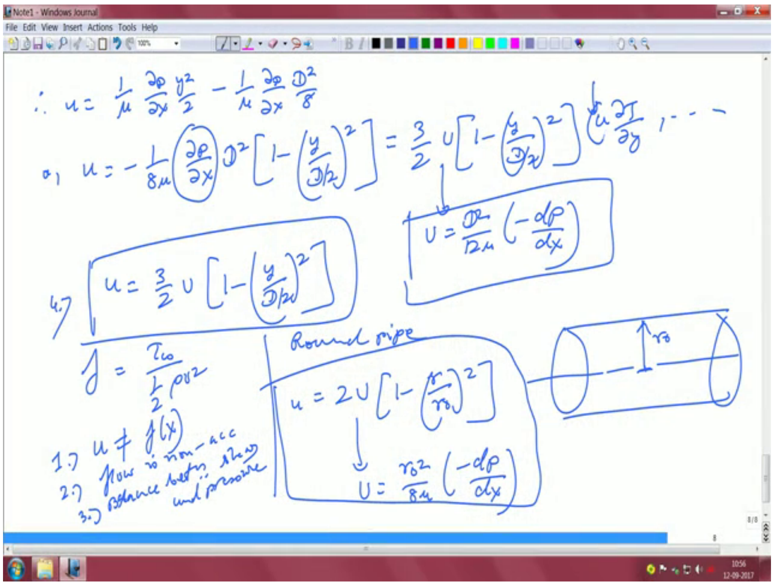

So, c 1 is equal to 0, c 2 is equal to that. Now if you. So, your total u therefore, becomes mu d p by d x y square by 2 minus 1 over mu d p by d x D square by 8 or u is equal to minus 1 over 8 mu d p by d x D square 1 minus y by d by 2 square, got it. This can be further represented as something like 3 by 2 u 1 minus y by D by 2 square. Where this u is written as D square by 12 mu minus d p by d x. There's only 1 pressure gradient. So, you can write it instead of partial you can write it as an ordinary differential as well.

So, as you can see so, therefore, your final expression for u becomes 3 by 2 u minus y by D by 2 square. That is the final expression that you have where u is given in that particular functional form. Similarly your f which is your friction factor which is tau y wall by rho u square can we also evaluated. So, tau wall you can easily evaluate it as mu into d u by d y at the wall and you can divide it by rho u square. So, all these expressions you can find out, find out quite easily.

Now, let us talk about for a round pipe. So, this is for a duct, 2 parallel plates. For a round pipe the essential expression is still the same given by 2 U into 1 minus r by r naught square in that case. Where this U is basically this is small u by the way this r naught square by 8 mu d minus d p by d x. So, it is a round pipe like this, this is r naught which is the radius of the pipe. So, it is very similar expression. This time, you have to take care of it by using the using the polar co-ordinates in this particular case.

So, so, what did we do? We found that indeed the u the fundamental take away things, u is not a function of x; that was the first point that we took. Second point was that flow is non-accelerating. It is a balance between shear and pressure and there is an exact solution that exists which is given by this; that would be your fourth point. There is an exact solution; that means, you know in the fully developed regime. Why did we get away? Because a non-linear term of the momentum equation got cancelled; Because there was no convective derivative to begin with.

So, taking advantage of that we were able to now get to this particular situation, where the flow velocity actually, there was no acceleration term. So, therefore, it offered an exact solution of this particular problem and that is exactly what we did in here. But; however, a few things to note in the developing region; obviously, this is not true the flow is accelerating in that particular reference frame u is not a function of f x only, u is not a function of f x that statement does not hold water. And so, therefore, you there you have to solve the whole momentum equation which is what we did in our integral approach.

So, you can see over here that once the flow becomes hydro dynamically fully developed it has got an exact solution; that means, in the energy equation where do you have basically u t d t by d y and terms like that, it can be easily applied. This u can now be taken and easily applied over there. You don’t have to you can solve it a priori and you can take that solution and you can apply it in the momentum equation provided, in the energy equation provided, provided that you are operating in the fully developed regime of both the cases.

That means, you are in the hydro dynamically fully developed regime. And of course, the temperature or the energy equation is also in the thermally fully developed regime. So, or it can be the at least the flow has to be hydro dynamically fully developed, in order to apply that u and directly place it over here. So, that is what we have done and for the for the round pipe we have seen that we have given an expression for the round pipe as well.

Now, let us see that for temperature, what are the key concepts that one needs to learn before one takes up this particular problem?

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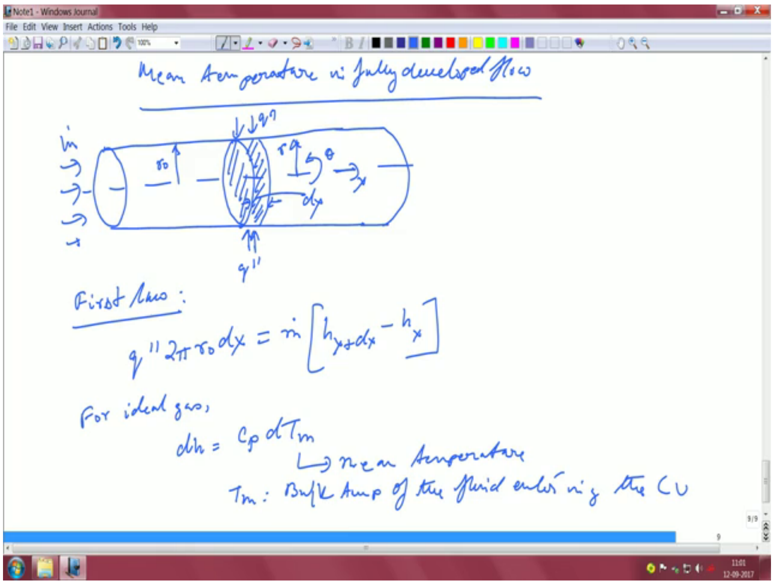

So, let us introduce the concept of Mean temperature, in fully developed flow. What do we mean by that? Let us take a pipe, round pipe. As I say round pipe and ducts they are not very different. This is of course, the centre line. This is of course, r naught theta is obviously, in this direction. So, it is theta symmetric. So, this way is x that way is r.

So, there is of course, an m dot amount of fluid that is flowing through this particular pipe. There is a net heat transfer that is happening from the wall into the fluid, as it passes through a, through this pipe and particularly we are interested in this small segment that you see over here. And that is what we are most interested in. So, we apply first law, the first law of thermodynamics over here. So, we say that q double prime just look at the expression 2 pi r naught into d x that is the d x is this elemental section.

So, this is the total amount of heat that is dumped into that elemental section d x leading to m dot. So, the m dot is a mass flux that was coming in. So, the enthalpy of the fluid should have increased in this particular fashion. So, whatever is coming out on the other side should have a little bit more enthalpy than whatever is entering that is because of the amount of heat that you have actually supplied.

Now, for ideal gas d h is given by C p and d T m where, we are introducing T m as the mean temperature. We will see what this is in a second, mean temperature. So, T m is basically the bulk temperature; temperature of the fluid entering the control volume. This is a control volume. So, the bulk temperature of the fluid that is entering inside into the control volume is given by T m. So, all that whatever may be the temperature distribution of the fluid doesn't matter.

It is the average temperature at that particular section. In this particular section whatever was the average temperature that is what we have taken, that is your T m. So, this T m changes by a little bit because you are dumping more amount of heat. There's a fluid which is coming with a mean temperature, you have dumped x calories or joules of heat into the fluid that naturally the mean temperature will rise a little bit. As a result of that the enthalpy of the fluid also changes by that little bit. So, that enthalpy is given by d h is equal to C p into d T m, that is a change in enthalpy is linked to the change in the mean temperature.

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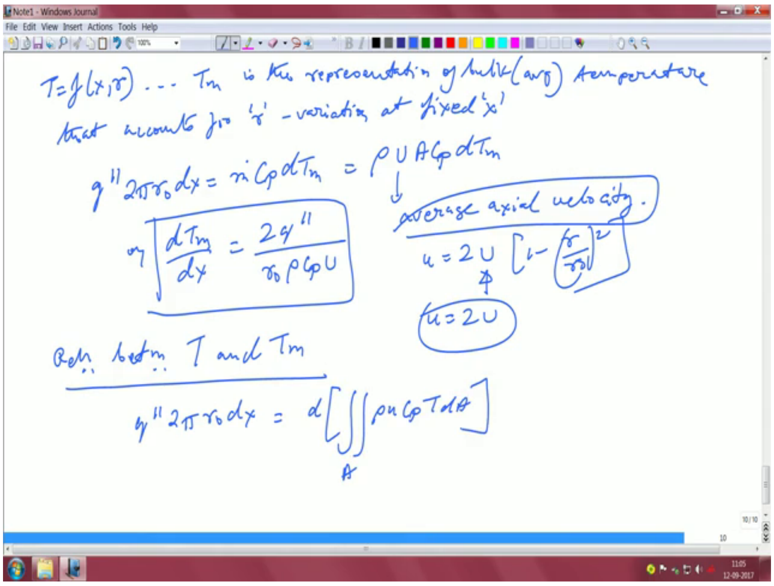

So, normally your temperature T would be a function of both x and r because it will be a function and it is theta symmetric. So, therefore, there is no variation with respect to theta, but with respect to x and r there would be a variation of the temperature.

So, basically your T m is the representation, a representation of bulk or in other words, average temperature; bulk at the average temperature that accounts for 'r' variation understand what 'r' variation; that means, is the variation across the radial direction at fixed 'x'. So, at any fixed x if you averaged out over r the temperature that you are going to get is basically your T m. So, you have q double prime 2 pi r naught into d x equal to m dot C p d T m; m dot C p into d t m.

Now, what will be your m dot in this particular case? m dot can be represented by that average axial velocity that you have that we just derived a few movements earlier. So, this can be written as a rho U into A into C p into d T m. Rho U m C p u into d T m or in other words, you get d T m by d x is given by 2 q double prime divided by r naught rho c p U. So, this is the average axial velocity that we already derived earlier, that U, that remember that in the velocity profile if you have forgotten.

We already had an expression for u; u was 2 U to 1 minus r by r naught square. Do you recall that? So, this is nothing, but that average velocity that we are concerned with here now. So, so you can see that for example, even in this particular expression you know that the u is maximum at the center. So, when you actually put r equal to 0, u becomes equal to 2 U; that means, twice the average velocity.

Now, this is the average axial velocity therefore, that is what we have used over there. Remember we said that we are going to use the momentum equation as much as we can. So, this establishes one fundamental relationship between d T m; that means, the temp mean temperature and the corresponding velocity and the heat flux. This establishes that and how the mean temperature varies.

Now, what is the relationship still between T and T m? That is what we are going to do before we end this lecture. T and T m, we said that T varies with x and r; T m is basically takes into account that r variation at fixed x. So, there must be a relationship between T m and x which we need to establish. So, that is what we are going to do. So, the relationship between T m and T over here, if you write this expression; once again in the proper form; what you will get? This area integral C p T d A, that is a total change if you take into account, the temperature, this is the actual temperature which is a function of both x and r.

So, you had integrating it across the area; that means, across the cross section cross sectional area. So, there you have both u this u is the fundamental variable is not the average and this t is also the native the temperature or the temperature at each point.

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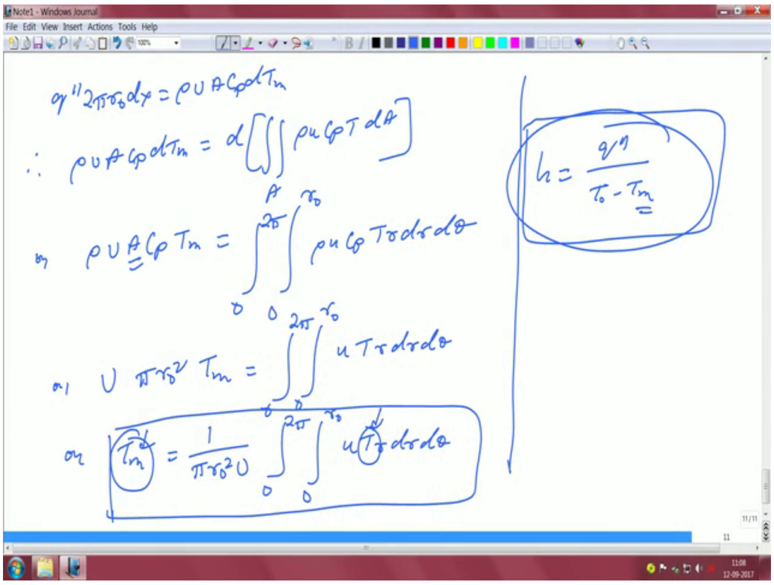

So, based on this expression, also we know that 2 pi r naught d x is given by rho u d C p d T m, that is also we have just derived. So, therefore, rho u A C p d T m is equal to d the area integral rho u C p T d A.

Just we have substitute we are just replacing the temperature with that. So, in other words or rho u A C p T m basically have to integrated over the 2 variables, rho u C p T r d r d theta. This is the area or in other words, this will be U pi r naught square for this, this is the average area I mean this is the area not average area into T m is equal to 0 to 2 pi 0 to r naught u T r d r d theta or T m is equal to pi r naught square by U 2 pi 0 to r naught u T r d r d theta.

So, this is the expression that you get links T m with the corresponding T and h there is the heat transfer coefficient in all these flows will be given by q double prime into T naught minus T m. That will be the case for all the, all the all the cases that we are going to do here.

So, basic point is that we require T in order to calculate T m; otherwise, you cannot calculate h. So, the problem is not gone you have just transferred the problem to a new variable which is T m which is nothing but it's more like than integral approach. Where, we have basically integrated out the whole thing; in a way that it is understandable.

And so, we have shown that how the mean bulk temperature is related to the actual temperature and we have also cast that the heat transfer coefficient perhaps can be written in terms of the mean temperature only in this particular case.

So, in the next class we will see what is the nature of this T m; How does it vary with distance and with different conditions, uniform heat flux or uniform surface temperatures that we will do in the next class.

Thank you.