Convective Heat Transfer

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Lecture – 15
Internal forced convection – Developing flow

So, welcome. This will be the 1st lecture on Internal forced convection. So, so far we have done External convection. So, the External convection does not have any imposed length scale associated with the thing. Now in Internal convection, we will see that there are certain things which we, which makes it a little bit different from internal conviction; internal forced convection.

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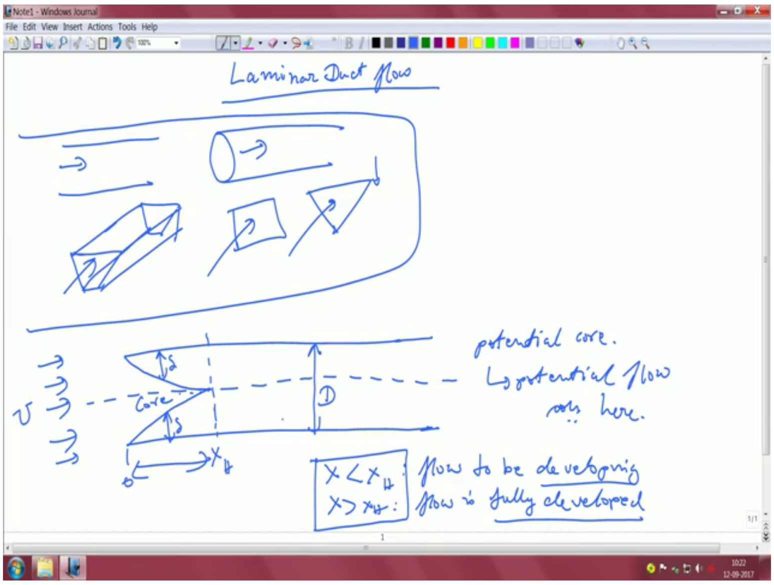

The first thing that we are going to look at is basically Laminar Duct flow; Duct or pipe flow. It can be a channel I mean to like 2 parallel plates. So, if there is a flow going through it, it can also be a pipe with a flow going through it. It can be varieties of other things; it can be like this kind of a pipe also. That will be a little bit difficult that analysis to do this kind of sections. There can be a flow through sections like this, there can be flow through rectangular sections, there can be flow through triangular sections; any type of sections.

So, these are all comes under the under the umbrella of Internal forced convection. So, of course, these things are a little bit difficult to analyze; not impossible, difficult to analyze. So, we will start with a very simple one; Again, a canonical problem to show you how things can be generated. And then, we will take it forward to the next level. So, in this particular case, if we have like 2 parallel plates and then, there is a flow. Once again, this is a uniform flow U, which is approaching the plate right.

This distance between the 2 plates is basically given by D, which is basically equivalent to the diameter. So, it can be a pipe also, but the plate is easier because we deal with Cartesian co-ordinate system, But if the same thing can be done for a pipe, as well. So, there is no hard and fast rule about that. Now, so, if you look at this particular problem now, the interesting feature is that, this is now a bounded flow because there is a boundary on both sides.

So, there is no extension to infinity, also the moment the this flow sees this plate. The boundary layer that we developed earlier will start to develop here as well right. So, the boundary layer is natural it's going to happen; that means, the boundary layer will be the region where the viscous drags will be important; that means, the flow velocity will show a curvature and so, that is going to happen. But now, it will happen symmetrically perhaps from both sides of the plate; that means, from this side of the plate also it will develop, this side of the plate also it will develop. Because there is, we suppose that there is no preferential bias.

So, as it develops, what happens is that these 2 boundary layers then merge with each other. So, this is the boundary layer, delta. This boundary layer now merges with each other. The 2 boundary layers will merge because again, this is not an infinite reservoir. So, the boundary layer cannot go on extending up to infinity. So, it will merge and basically converge with each other. So, beyond that beyond this particular length, you can see that the everywhere in the flow, the viscous drag is important. Because the boundary layers have basically covered the entire pipe, duct whatever you call it.

So, in this particular section, let us mark this important piece of section. Let us call it something like at x H. This is of course, 0. So, when x is less than x H, we call this flow to be developing; when x is greater than x H with a call the flow is fully developed. So, we call the flow to be developing and we say that the flow is fully developed depending on what is the value of x H. So, before x H, let us see what happens. After x H, let us see what happens.

So, the interesting part will be when you look at this particular section, the section, this particular section actually; the developing region of the flow. What do you see that what will happen at the center. That means, in this particular region, this is called the Core or in other words, it is called the Potential Core. So, the name is a potential core. Why is it called the potential core? Because the viscous effects are not important. So, you can apply your potential flow solution, which you learn in your fluid mechanics.

Here, you can apply your potential flow solution here, there is no problem; except here of course, the potential core does not mean that the flow velocity does not change in the potential core. Flow velocity changes in the potential core, but it does not change due to viscosity. So, that is what you are getting in your potential core region. So, in the potential core region, there will be a velocity profile which is different from what the velocity profiles are in the boundary layer.

After it has merged; that means, beyond x by H, we can apply the full fledged navies stokes equation to solve it there. So, in the developing region there are 2 parts, inside the boundary layer; it is basically the boundary layer equation or the navies stokes equation. In the core region, you apply what we call the potential core solution. Beyond x equal to x H, the entire thing is basically navies stokes or the boundary layer solution whatever, we will find it prudent over here to apply.

So, how would the velocity profile actually look? So, let us look at this, once again we can prove that this will be the velocity profile. So, let us once again draw this profile. This is smooth. So, in this particular area as many of you would know by now, the velocity profile is something like this. Forget it, this is not at an angle; this is actually all along the axis of the duct. So, this is a parabolic profile.

We will see how it becomes parabolic and this profile actually; we will find does not change with distance. So, you can take this profile, take this profile and put it here; it would be the same. So, there is no change in the velocity. That is a fully developed profiles feature that the velocity profile should not change. What about here, in this particular section? In this particular section, what will happen, if I draw it properly, you will have that there is a rise from both sides like this and then there will be some kind of a potential core like that. So, up to this, it is basically the boundary layer.

Then after, that it is the potential core, this potential core; that means, the flat region that flat region, that will accelerate as you go to the next level. That means, this is the profile at x 1, if you take another at x 2 you will see that this potential core velocity or the velocity at the potential core, is only 1 single velocity because the flow is uniform there it will increase with distance. And you can readily make out why? That is because as the boundary layers are extending, the flow velocities are changing right. So, in order to maintain a constant mass flow rate through this conduit you have to have an increase in the centerline velocity.

But after the x equal to x H, the velocity profiles will be self similar in nature. So, some of these things we will prove. So, donot worry, but this is kind of intuition that there is an entrance length, where the boundary layers are developing. And then, there is a fully developed length where, the boundary layers do not vary. So, depending on, so, it is not just with hydrodynamic boundary layer. This is a Hydrodynamic; obviously, there will be a thermal boundary layer also which will come in due course, over here. So, let us do our standard analysis and see that if we can extract some scales out of this particular problem.

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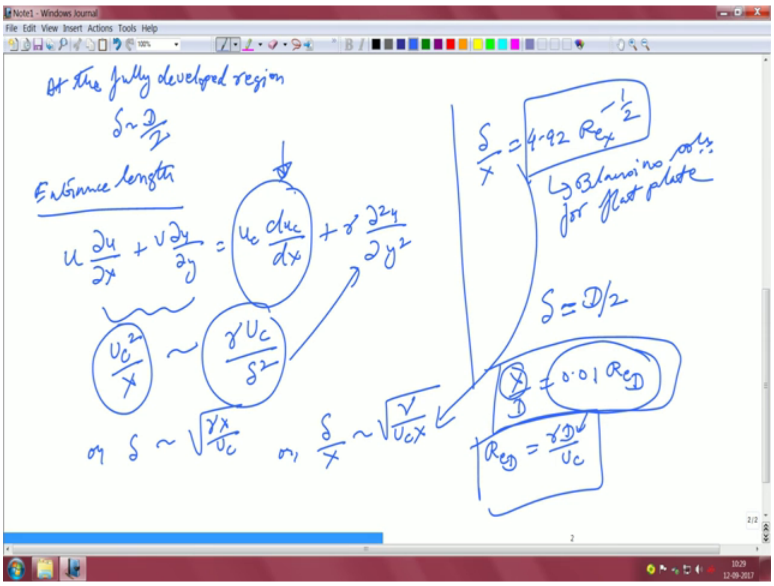

So, At the fully developed region, you know that your delta will scale as D by 2. It has to scale as D by 2. Now in the Entrance length; that means, in the length where the profile is still developing, we can apply the equation. How does this come? It is a same way as your external boundary layer. This particular term comes from the pressure. The pressure in the potential core is basically imposed into the boundary layer and this comes from the Eulers equation therefore.

So, the pressure term is substituted by Uc duc by dx and that comes from the potential core solution. So, now we can do our standard scaling analysis on this. So, the 1st 2 terms, in this series u x square by X. So, is actually proportional to got it. So, this will be u c square by X and the 2nd term, this particular term will be that. So, this gives you delta scales as gamma X by u c or gamma by X gives you gamma U c by X, that is what you get out of this.

Now you already know from your Blasius solution that delta by x is equal to 4.92 into Reynolds number to the power of minus half, am I right. So, that is what you know; that is going to happen. You substitute that over here, you substitute the expression for that over here. Because this is a scaling argument that is an exact equation that you got from your, from your Blasius solution. So, this is of course, not strictly applicable, but we are going to take a leap of faith and use it over here. So, this is Blasius solution for flat plate.

Now in addition to that, we apply 1 more equation that delta is equal to D by 2 at the point, where the entrance length ends. So, combining these 2 equations therefore, you will get X by D will be equal to 0.01 Reynolds number to the Reynolds number D. Where, Reynolds number with respect to D is actually gamma D by U c. It is not the same as Reynolds number to the power based on x. So, the length scale is just D instead of that.

So, this gives you that your X that is at the point where the 2 boundary layers actually join is given as a function like this. It is given as a form as I have mentioned in this particular equation. It is roughly, what we call 1-10th of the Reynolds number. So, X by D ratio. X by D ratio at the point where you actually have the 2 boundary layers merged, is given by 0.01 of Reynolds number based on D.

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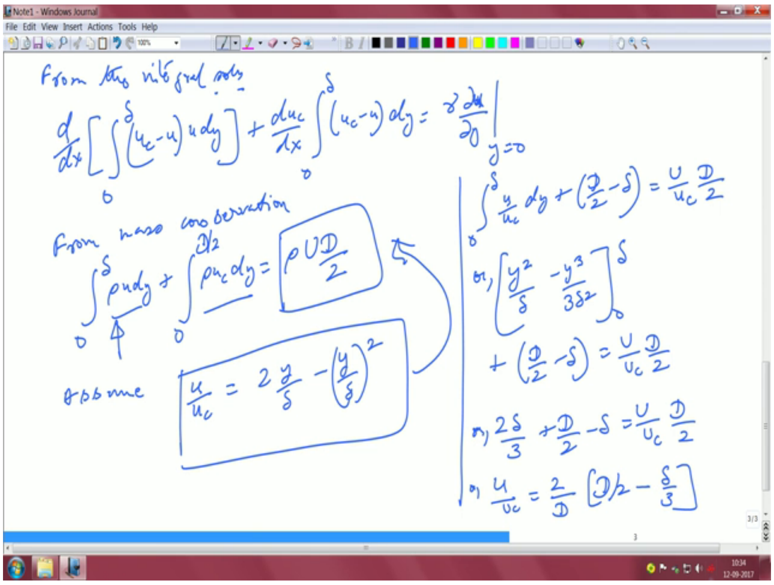

Now, from the integral formulation we can try to extract the same thing because this was done rather heuristically. Because we just took the scaling from the Blasius solution and kind of applied it here, which is strictly not the correct thing to do, but we kind of got away because these can be constituted as 2 parallel plates; 1 plate is at the top, 1 plate is at the bottom. So, it you can consider it like to be 2 parallel plates kind of independent with respect to each other.

Now, from the integral solution, integral formulation, we can apply it directly. Look at this form u c to u into d y plus d u c d x, that is the integral formulation that we had earlier. From mass conservation, what we can write? Try to understand this particular expression, at any point in the entrance length. The total mass is given by this, this is what the mass that is entering inside the duct.

So, that is rho UD by u and u is constant. So, this is the total mass that you are flushing in. Now inside the boundary layer, it will be rho U into d y. Just outside the boundary layer, it is rho u c into d y right. So, these 2 terms when they are added, it should give you the final expression. So, this is the conservation of mass. There is no ambiguity in this. This is the conservation of momentum. The previous equation where you can also see this is of course, u. Donot. So, where we have just done the momentum expression in a very similar way, that we saw earlier.

So, these 2 things are already kind of known and it's very intuitive. So, you can assume now, like last time I am not going through the whole steps. Because you can work it out yourself, you can assume a polynomial profile. And then, you can evolve through the boundary conditions because that is not the intention the intention is to just give you guys an idea that how this to be done. So, you can assume very similar to the velocity boundary layer over a flat plate.

You can constitute a similar polynomial profile for u by u c in that entrance length. That means, in the developing part of the whole thing; so that you can easily do. So, now, applying this particular form into the continuity equation, that you have here. What you will get is, that is what we have done over here. And now, if you just substitute, now this into this particular form, you will get this is y square y delta y cube by 3 delta square, 0 to delta plus D by 2 minus delta

Now, it is easy to integrate 2 delta by 3 plus d by 2 minus delta U by U c D by 2 or in other words, u by u c 2 by D D by 2 minus delta by 3.

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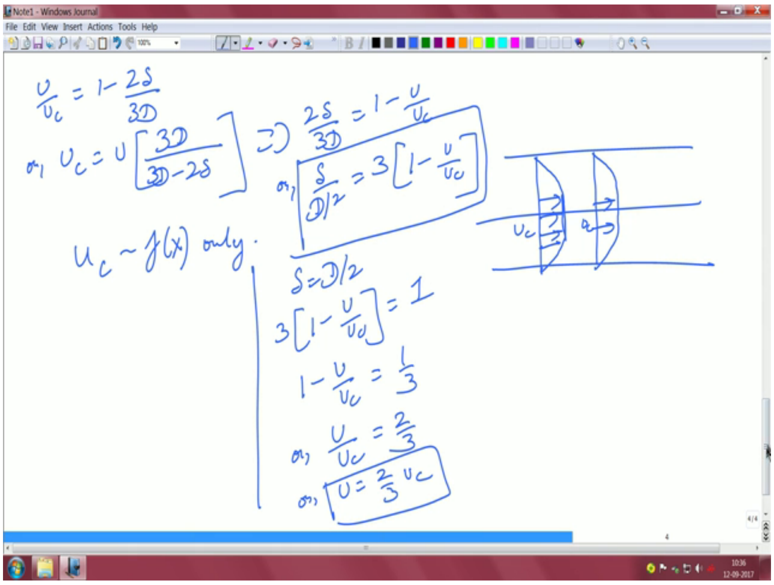

Then, U by U c or U c is equal U to 3 D divided by 3 d minus 2 delta or in other words, from here you get 2 delta by 3 D into 1 minus U by U c or delta by D by 2 3 into 1 minus U by U c. So, here of course, is you know u c is a function of x only. That is the reason we could do such a lot of things, as a function of x only. This has got no dependence on y because at any particular y section you saw that u c was constant.

If you donot recall, this is the way. So, if you recall the profile at any particular section. So, this is the straight part. This is what your u c is, the core velocity. That is not a function of y, but if you go to a different section because the boundary layer will be like this. This will actually; will not be a sharp kink like that. So, there will give you this U c and that u c is different. So, U c is a function of x, but it is not a function of y.

So, this is what you get as your u c and. So, this is fairly easy. So, you know that delta by d by 2 is basically 1 minus u by u c. Now when delta becomes equal to D by 2 equal to D by 2 then, 3 into 1 minus U by U c becomes equal to 1 or 1 by U by U c is equal to 1/3 or other words, U by U c becomes equal to 2/3. So, this u then, becomes equal to 2/3 of your u c, at that particular location. Obviously, so that is what you get. Similarly, if you take this particular form.

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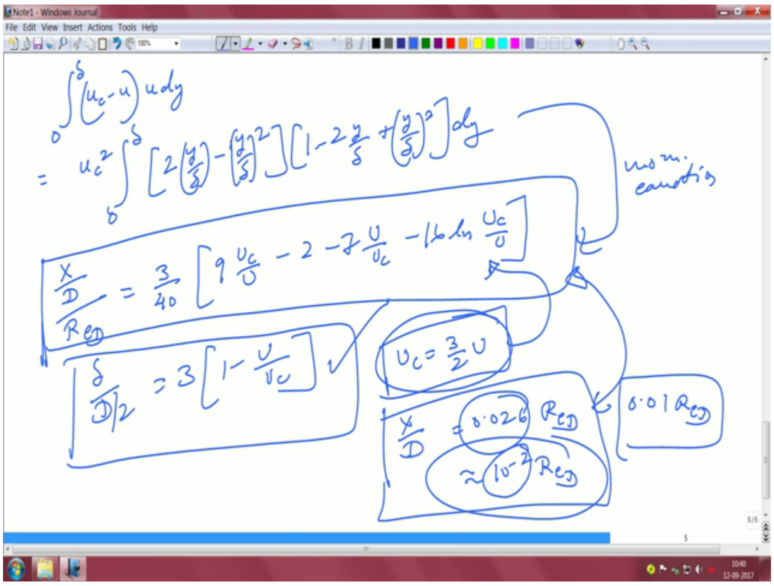

And now do the momentum equation. So, 1 of the key terms that you need to evaluate in the momentum equation is basically you c minus u into u into d y. So, if you do this properly, it will be u c square. Once again, u c is not a function of y anymore; it will be 2 y by delta minus y by delta square 1 minus 2 into y by delta plus y by delta square into d y.

So, if you follow through these steps, if you integrate the whole thing out and you put that back in your in your momentum equation the final form that you are going to get is

X by D divided by R e D is equal to 3 by 40 9 U c by U 2 minus 7 U by U c 16 ln U c by U and this is already we showed the steps.

So, this is left as homework, you come from here to here and this one we have already shown, how to do this. So, you just use this in the momentum equation; you will get this there is a little bit of cumbersome math that is involved. So, in other words, as we said earlier. So, in this case as you saw this also U c is actually then, 3/2 of U. So, it is other way of writing the same thing.

Now this particular expression which is basically the X momentum equation. Now if we add the edge of the boundary layer where we already saw that U c becomes 3 by 2 of U; that means, the centerline velocity becomes 1.5 times the mean velocity, if we substitute that over here, in this particular expression because they all are U c and U combinations of that. So then, this expression yields X by D becomes equal to 0.026 into Reynolds number D.

This is almost of the same order as 10 to the power of minus 2 Reynolds number. So, what we got from Blasius was 0.01 Reynolds number D. So, this is not way of at all this is of the same order this is still ten to the power of minus two. So, here X by D at the point of boundary layer merger is basically scales as 10 to the power of minus 2 Reynolds number.

So, this is the thing that you should remember that this is how we got. So, recapping the whole thing, what did we do? We took the integral equation and the mass conservation equation; mass conservation equation yielded this. From here, we showed at the edge of the boundary layer your U c becomes exactly 1 and a half times of the free stream entry velocity.

We substituted that in the momentum formulation and we got that these were the expression which is basically 10 to the power of minus 2 into Reynolds number which is exactly the same as what we got in terms of our crude Blasius analysis. This is also crude because this is still integral; but we were able to cast it and show that it is approximately 10 to the power of minus 2.

So, as we said. So, there are 2 things that got established your U c actually varies with x, it does not vary with y and U c is actually a lot higher than the free stream velocity u.

This is obvious big; this is because your velocities you are losing you are there is a velocity deficit due to the boundary layer. So, naturally the mean flow or the U c has to accelerate to make up for that. So, that is pretty obvious that, but this value of 1.5 is something that you should keep in mind.

This is of course, at the edge of the boundary layer, at the edge of the fully developed regime. That means, when the after this the fully developed regime completely kicks in. So, through all this exercise we proved that the enter in the entrance length the boundary layer is still growing. U c is still changing; that means, the shear stress will still vary quite a bit, after that in the develop section we will see, we proposed that in the develop section the velocity profile does not change at all.

With at different sections we will prove that, obviously, in the next lecture we will prove that why that is the case. And we will also show that the next, we will also say that what happens now to the heat transfer because of all these flow situations. So, this should be now very clear to you that how we got the developing section and how this developing section can be analyzed and how the flow velocity will change in the developing section.

Of course, we will come to the temperature part when the, when we deal with the developing thermal boundary layer.

Thank you.