Convective Heat Transfer
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Lecture – 14
Arbitrary Wall temperature

So, in the last class, we looked at for the which solution we looked at the momentum part now it is time to look at the energy part of the equation.

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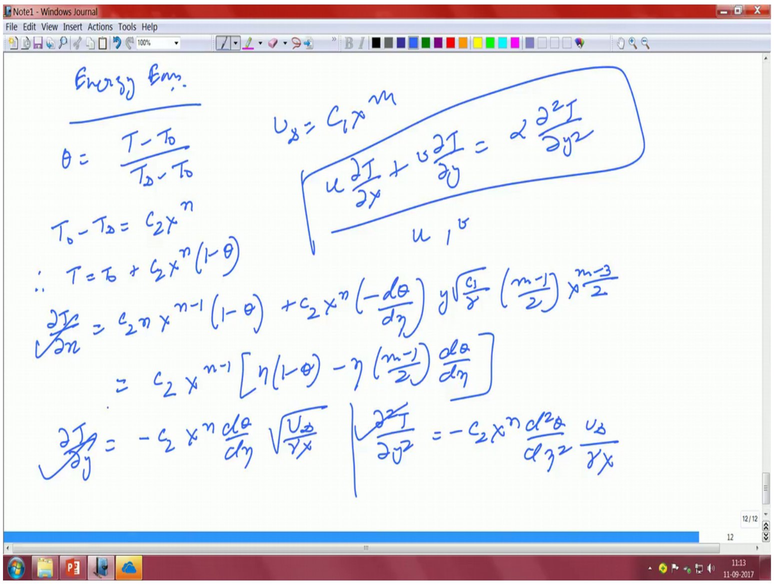

So, the energy equation can be taken as theta is equal to T minus T naught by T infinity minus T naught that we said earlier right. T naught minus T infinity is basically given as C 2 x to the power of n, we do not know the functional form because they may not have the same functional form as your velocity. So, therefore, T naught rather T is given as T naught plus C 2 x to the power of n, 1 minus theta right. So, d T by d x equal to C 2 into n, x to the power of n minus 1, 1 minus theta plus C 2 x to the power of n minus d theta by d eta into y, C 1 by gamma, m minus 1 by 2 x into n minus 3 by 2. So, it will becomes equal to C 2 x to the power of n minus 1, n into 1 minus theta eta n minus 1 by 2.

Similarly your d T by d y, if you do follow the same course of math d theta by d eta u infinity gamma by x. So, you understood what are we what we did. So, it is pretty much just substituting u infinity as and when it is required right. Because remember u infinity is still given by C 1 into x to the power of m, that is how this m comes into the picture right.

So, similarly delta square T d y square is given by C 2 x to the power of n d square theta d eta square u infinity gamma into x. Now once again what do you need to do because your energy equation is d T by d x plus v d T by d y is equal to alpha delta square y by d y square correct? So, we already have evaluated this, we have already evaluated this, we have already evaluated this and we already know what is a functional form of u and what is a functional form of v from our earlier momentum analysis right.

So, it should not be a problem now, what do we do is now put all these terms together because the new terms we have already evaluated in the same way, that we did earlier . So, the final form.

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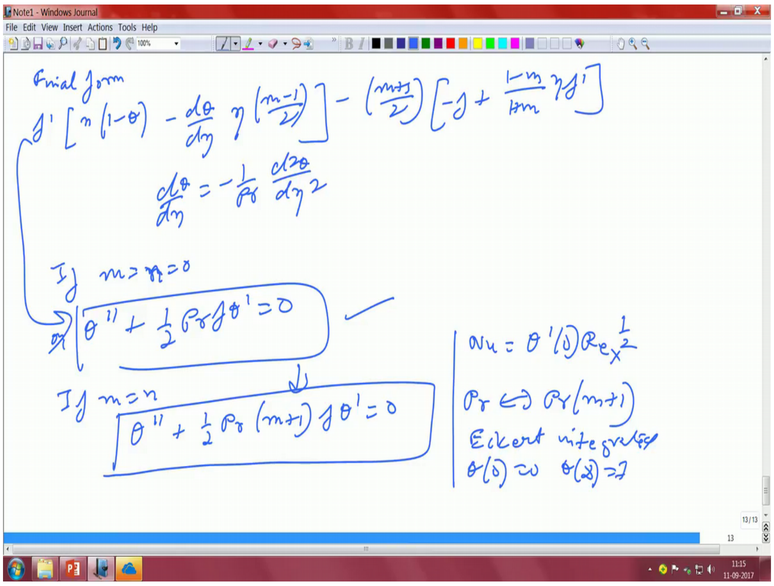

It comes let me just now if m is equal to n equal to 0 typical flat plate boundary layer solution, what we get is theta double prime this equation will boil down to this particular form, which is the correct one as we already know from all other analysis. Now if n equal to m; that means, the temperature and the velocity external temperature and velocity has got the same functional form, this equation will now become theta double prime plus half prandtl number m plus 1 into f theta prime is equal to 0 this is the expression that we also used in our suction and blowing.

So, your nusselt number will be equal to theta prime 0 into Reynolds number to the power of half here what has happened is that prandtl number has been substituted by prandtl number as m plus 1 right. So, it is like a pseudo prandtl number. So, Eckert actually integrated it integrated this once again this kind of thing has to be solved by knowing what is the functional form, integrated it with theta 0 equal to 0 and theta infinity is equal to 1. So, if you look at the presentation format once again.

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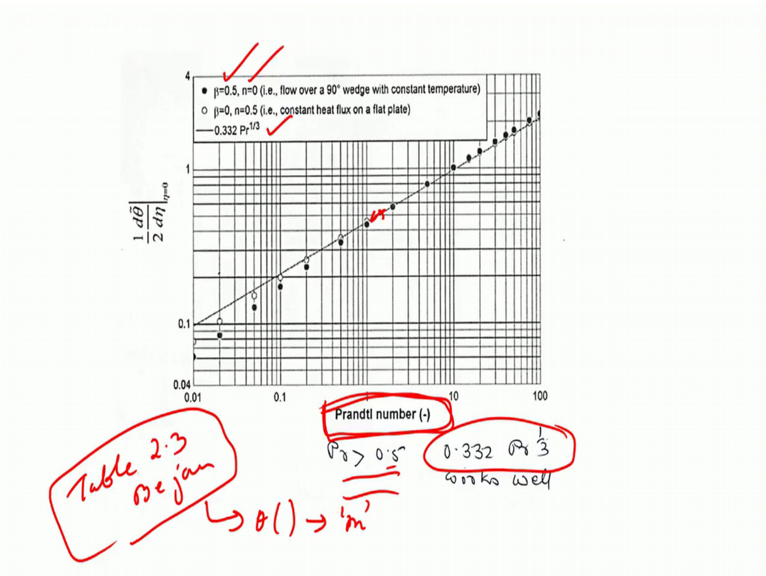

So, this is basically the expression in the line that it is given as a solid line, that is basically the correlation that you are most experienced with, it is plotted with terms of prandtl number. So, as you can see in this particular configuration once again you have beta equal to 0.5 in n equal to 0, beta equal to 0 n equal to 0.5. So, these are do not be bothered about the constant heat flux on the constant boundary profile.

It is not the point over here, but you can see that this is plotted with respect to your prandtl number. So, prandtl number greater than 0.5 the correlation works pretty well basically this 0.332 into prandtl number to the power of one-third correlation works pretty well. Here also can look at table 2.3 of Bejan in order to see that what is the solution for theta for different m. So, that what we are going to do in the next class we are going to give you a chart in which we are going to say that this is the table, which contains all the values of beta. So, based on this we have pretty much finished the case of Falkner Skan with the exception that I will provide a table which may be which will be added with this particular class, where we will say that what is the basic case of basically suction and blowing what is the m parameter all about right ok.

Now, without in order to finish off the external flow, there is one more thing that we need to address which is called the arbitrary wall temperature.

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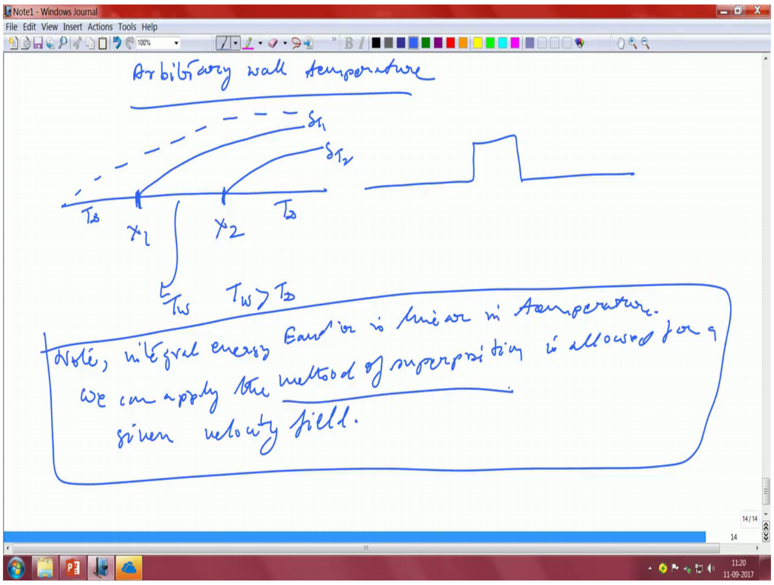

That means, this is a very specialized case and I will show you what that specialized case is. Say for example, this is your generalized flat plate; this is how the boundary layer actually develops. So, say in this particular portion between x 1 and x 2 between 2 limits of this particular profile, the temperature is T w, where T w is greater than T infinity on the 2 sides of the plate this is still T infinity understood.

So, it is a plate in which only one segment of the plate is actually at a higher temperature than the incoming flow field, the other part of the plates are at whatever temperatures in this case it is the same as the ambient. So, if the temperature was the same as the approaching temperature, you know quite well that there will be no velocity profile will be developed right if the if the profile is the same right.

But; however, in this particular case there is one section where the temperature is a little higher right. So, it is almost like this in this particular section the temperature is higher exactly like that. So, what you are going to have if the plate was heated from x 1 right say entire plate was at a temperature T w right, you would get a boundary layer which will start developing from here right. Similarly if the plate is started it is heated from x 2 you will actually have a velocity have a thermal boundary layer which will develop from there right let us call a delta T 1 this is delta T 2 correct. So, you note one thing one important thing when you solve this particular problem, that the integral energy equation that we did right energy equation is linear in temperature is not that so? It is linear in temperature ok.

Hence to solve this particular problem we can apply the method of superposition, superposition is allowed for given velocity field. See the velocity field is unaltered right because it is a one way coupling as we said; that means, the change in the density has got no effect on the temperature. So, the integral energy equation is linear in temperature, and we can apply the method of superposition because it is linear we can apply the method of superposition provided we know what is the velocity profile going to look like right. So, this is the way.

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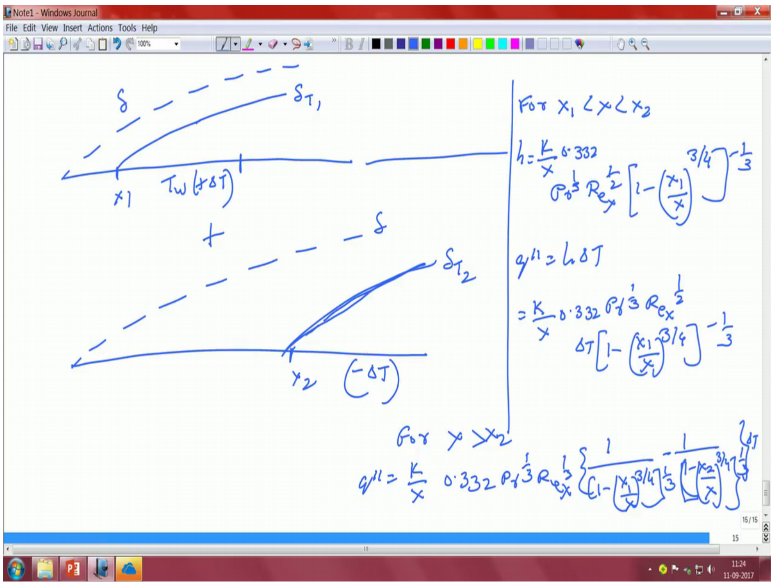

Say for x 1 this is one of the velocity profile, one of the temperature profile this is the velocity profile right that develops regardless right that is delta right. So, this is the first temperature profile where this is T w, which is a plus delta T effect, delta T is basically T wall minus T infinity. I am summing it with this particular profile once again this is still delta now it is x 2 right this particular profile has got a temperature like this right which is delta T 2.

Now, this temperature I am taking it as minus delta T right because here we have a plus delta T added throughout the length of the plate right is plus delta T throughout the length, but in reality only the portion x 1 and x 2 is the only portion that is heated. So, therefore, from x 2 if we take a profile which is minus delta T, then basically you are taking into account the effect of that, basically you are subtracting that effect out from the whole picture right.

So, therefore, if this is the situation for x 1, x greater than x 1 less than x 2, h will be given as k by x into 0.332 prandtl number one-third Reynolds this proportion remains the same regardless right. 1 minus x 1 by x to the power of 3 by 4 to the power of minus one-third; 1 minus x 1 by x to the power of 3 by 4 minus one-third q double prime is h into delta T that is given as k by x 0.332 prandtl number one-third, Reynolds number half, delta T 1 minus x 1 by x to the power of three-fourth raised to the power of one-third. This is for this region right for x when it lies between x 1 and x 2 right for x greater than x 2 right your q double prime becomes k by x 0.332, again prandtl number one-third, Reynolds number half.

Now, it becomes a little bit more complicated because now you have the effect of the previous one as well as now you have this minus delta T effect as well right. So, it is 1 minus x 1 by x to the power of three-fourth to the power of one-third minus 1 minus x 2 by x sorry three-fourth raise to the power of one-third this entire thing is multiplied by delta T, let me write it maybe in a more proper form.

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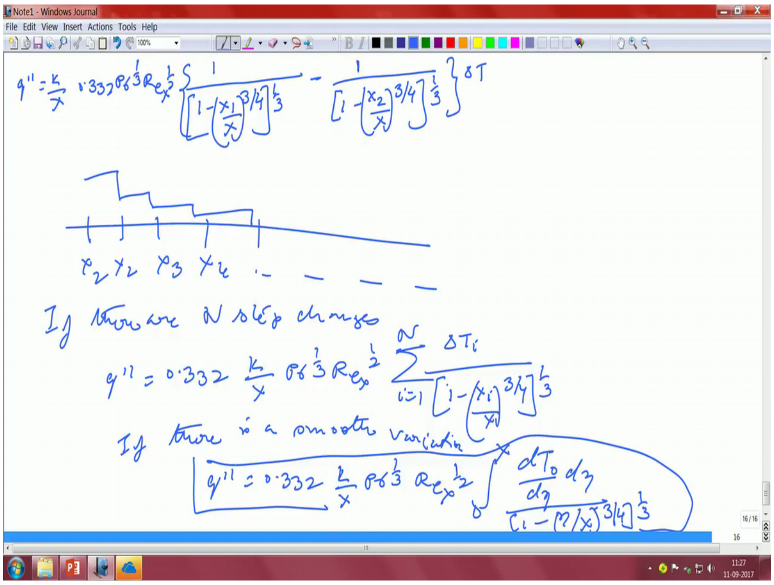

So, q double prime is equal to k by x 0.332 prandtl number one-third. So, that is the expression that you get in together. So, it is basically a reabsorption of heat in the heated section. Now if there are n such step changes it is possible right you can have n number of step changes right. So, you can have situation like this right x 1, x 2, x 3, x 4 like that continued ok.

So, you can have step changes like this, you can have any combination that you want right. So, if there are n step changes, it becomes a summation now, y equal to 1 to n delta T i, 1 minus x I by x to the power of three-fourth one-third this is a step changes. So, you can add all the steps depending on whatever it is like delta T 1 is negative positive, you can do it in whatever way that you want right. Now if there is a smooth variation q double prime will be given as 0.332, k by x and the number one-third Reynolds number x to the power of half 0 to x d T 0 by d eta by eta this is the dummy variable, 1 minus eta by x this is only raised to the power of three-fourth, 3 fourth and one-third to the top of that right. So, this actually shows you that this is how the for any type of temperature profile; that means, if there is a variation in the temperature profile that we have across a plate, you can actually represent it in this particular fashion right. Now what to do if there is a similar situation in which we have a uniform heat flux kind of a consideration, how to add those things also ok.

So, that will take a little bit of time. So, what we are going to do is that we are you remember last class we actually posed that what happens when there is a uniform heat flux situation right. If you recall that is what we did in the last class where we said that if there is a uniform heat flux, the integral formulation how it can be done, do you recall that that was what we actually did. Now if that is the case we what we are going to do in this particular class or in the next class, that we are going to look at that particular solution hopefully you guys have already done it by now. So, you can cross check that what will be the answer.

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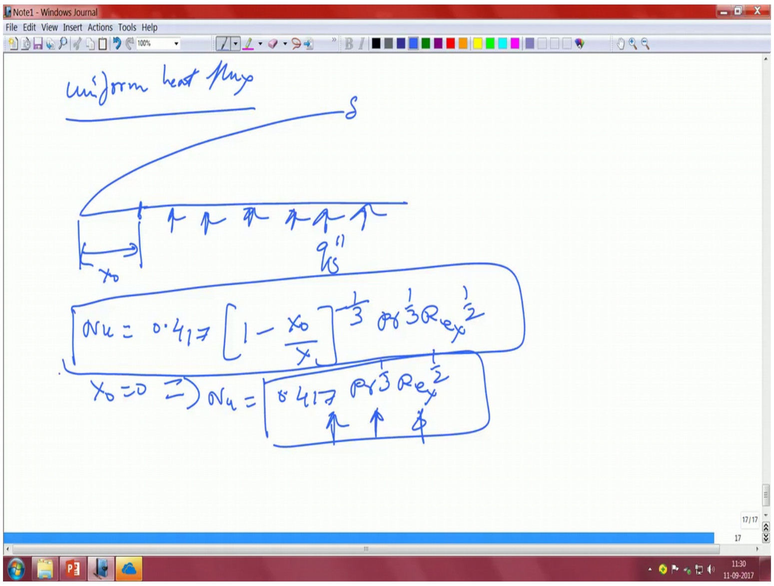

But in any case if there is a let me just give you the general profile for a uniform heat flux consideration, when there is an unheated again there may be an unheated section right the this is delta still goes up as it is from x 1 you are pushing in a uniform heat flux right, let us call that q s double prime. So, you are pumping in a lot of heat flux after say not x 1 say it is x this distance is say from here to here, from here to here is x naught right.

Now, this is no this similar than the problem that I gave you, where the uniform heat flux was everywhere right. So, in this case the answer will be the nusselt number will be 1 minus x naught by x raised to the power of one-third, prandtl number one-third, Reynolds number x to the power of half right. When x naught is equal to 0 which is the case which is the exercise that I gave you the nusselt number is 0.147, prandtl number one-third Reynolds number x to the power of half right. Now in the next class we are going to look at how we got this before we wrap up this particular section right that how we got this particular profile because we did it only for a uniform wall temperature I gave this as an assignment. So, we are going to do it in the next class and show that how this thing actually works while on the other hand for any unheated starting length this is the form that you are going to use right; that means, if there is an unheated starting length a portion of it there is no heat. So, in that case this is the profile that you have to follow right.

So, this actually completes. So, what we are going to do as a part of this lecture we are also going to show you that table, which shows that how the nusselt number and other thing varies with m. So, that is still remaining that we will add, but in essence the entire external forced convection is kind of complete with the exception that we are going to just do the uniform heat flux case, which were given as a homework problem I am going to just do it here. So, that you can cross check that whether you are on the correct path or not. So, using this, this is where we wrap off we are going to look at internal forced convection from next class onwards.

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Table 2.3 Local Nusselt number $Nu/Re_x^{1/2}$ for laminar boundary layer flow over a wedge

		Pr				
β	m	0.7	0.8	1	5	10
−0.512	−0.0753	0.242	0.253	0.272	0.457	0.570
0	0	0.292	0.307	0.332	0.585	0.730
$\pi/5$	$\frac{1}{9}$	0.331	0.348	0.378	0.669	0.851
$\pi/2$	$\frac{1}{3}$	0.384	0.403	0.440	0.792	1.013
π	1	0.496	0.523	0.570	1.043	1.344
$8\pi/5$	4	0.813	0.858	0.938	1.736	2.236

So, just to add as I was stating that, if you look at that with when there is a flow over a wedge we wanted to know for example, what will be the local nusselt number values and that is exactly what we are putting up over here.

So, if you look at this list. So, this is your beta the 1 that we said is the wedge angle, this is the corresponding transform variable which is m over here and these are the different prandtl number. So, this is prandtl number less than 1 regime, this is prandtl number greater than 1 regime. So, as you can see beta equal to 0 corresponds to the flat plate situation right. So, prandtl number equal to 1 is 0.332, as we knew earlier from our similarity transformation.

So, as we. So, this is for different sets of prandtl number, what are the different values that you get. So, similarly m equal to 1 basically corresponds to the hymens flow condition or the stagnation flow. So, those values are also given over here. So, as you can see the nusselt number seems to be a function of the prandtl number for all the wedge angles, but as you can see that as we increase the wedge angle; that means, you are going into a more and more convergent nozzle kind of a situation; that means, the flow is accelerating a little bit more, you can see that the value of your nusselt number correspondingly increases alright.

So, this is kind of obvious the same reason that we explained earlier, that when the flow is accelerating you have a very thinner boundary layer so; that means, your nusselt number or your heat transfer coefficient should actually go up right because it is inverse of delta T right. So, this is what you have you have an increase in the nusselt number as you go on increasing the beta. So, this is what I wanted to show you guys and this is once again the same table when you actually have suction all these are taken from Bejan.

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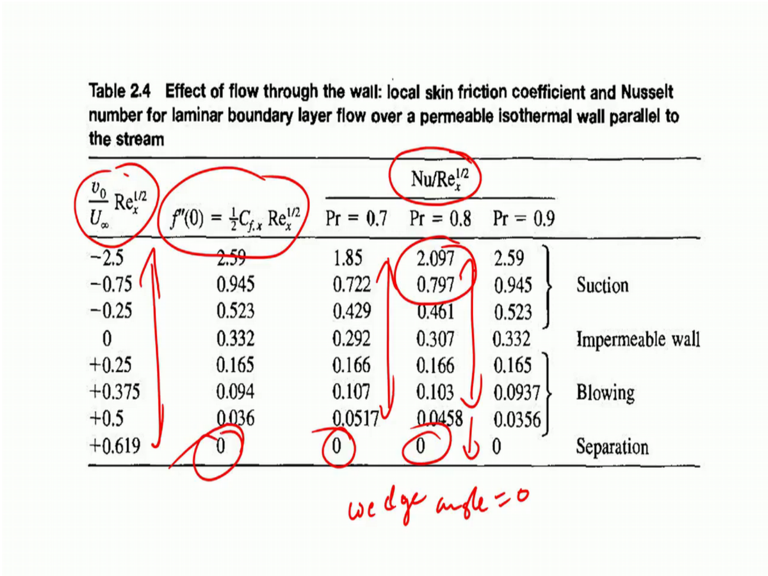

Table 2.4 Effect of flow through the wall: local skin friction coefficient and Nusselt number for laminar boundary layer flow over a permeable isothermal wall parallel to the stream

$\frac{v_0}{U_\infty} Re_x^{1/2}$	$f''(0) = \frac{1}{2} C_{f,x} Re_x^{1/2}$	$Nu/Re_x^{1/2}$			
		Pr = 0.7	Pr = 0.8	Pr = 0.9	
−2.5	2.59	1.85	2.097	2.59	Suction
−0.75	0.945	0.722	0.797	0.945	Suction
−0.25	0.523	0.429	0.461	0.523	Suction
0	0.332	0.292	0.307	0.332	Impermeable wall
+0.25	0.165	0.166	0.166	0.165	Blowing
+0.375	0.094	0.107	0.103	0.0937	Blowing
+0.5	0.036	0.0517	0.0458	0.0356	Blowing
+0.619	0	0	0	0	Separation

So, you can take a look at Bejan and you can see that you can study this different types of tables. For example, here of course, what we have done in one particular table you have the nusselt number, you have the skin friction coefficient, and this is have the blowing this is of course, over a flow over a permeable isothermal plate there is no wedge angle over here. So, wedge angle is 0, as you can see that suction and blowing how does the nusselt number values varies you can easily see that ok.

So, as you know when it is suction the boundary layer is pulled inwards. So, naturally you have an increase and it is pulled out it will go to 0 at some point because of separation. So, this is basically separation all around.

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Table 2.5 Local Nusselt number $Nu/Re_x^{1/2}$ for laminar boundary layer flow over an isothermal wedge with blowing ($Pr = 0.7$)

	m							
$\frac{v_0}{U_\infty} Re_x^{1/2}$	−0.0418 (β/π = −0.08)	−0.0036 (−0.0072)	0 (0)	0.0257 (0.05)	0.0811 (0.15)	0.333 (1/2)	0.500 (2/3)	1 (1)
0			0.292			0.384		0.496
0.0239	0.103							
0.25			0.166					
0.333						0.242		
0.375			0.107				0.259	
0.5		0.0251	0.0517					0.293
0.518				0.087				
0.558					0.109			
0.667						0.131		
1								0.146

Similarly for an isothermal edge if you have suction and blowing that part we did not cover in details, but this table you can still study. So, why some of the data are disjoined? Because not all the experiments were done not all the analysis were done for all the cases. So, that is precisely what it is. So, it is a wedge where you can actually have suction or you can have blowing, so that kind of a situation. So, all the datas are basically compiled over here, once again if you look at the class of the data you will see that; obviously, they also make the same sense because here you can see this actually decreases, for the same reason because you are blowing your blowing is actually increasing correct. For any given condition you take any set any m in this particular series you will find that it is actually going down ok.

So, the wedge we already know what the wedge angle does right on the top of that if you have suction or you have blowing, you are basically additive or subtractive kind of an effect. So, that is what is happening. Moment you have blowing you reduce the nusselt number value, because you are increasing the boundary layer thickness in a relative basis. For example, this is the boundary layer this is the nusselt number for no blowing if you look at this part of the thing right as you go on increasing blowing this actually reduces how much it will reduce that it depends on the calculations, that depends on the on the boundary layer profile these done for only 1 prandtl number you can do the same for other prandtl number also prandtl number of 0.7 is roughly equal to that of air.

So, that is how this thing has been constructed. So, we end this particular particular lecture where we actually have shown that with wedge what are the different values of nusselt number, because you cannot solve the equation exactly. So, you have to show you the end results, and also we have shown that for example, when you actually have suctional blowing what will be the values of the nusselt number, and when you combine these 2 effects together suction and blowing and wedge effect then what you get. So, all these things are kind of narrated in the table form, and all of them make common sense whatever we argued about the boundary layer all of them make sense when we actually solve it and the value seems to agree with our intuitions.

So, we end this before we now in the next lecture, we will go to internal force convection.

Thank you.