Convective Heat Transfer
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Lecture – 13
FALKNER-SKAN SOLUTION

In this particular lecture, we are going to look at the Falkner skan class of solutions.

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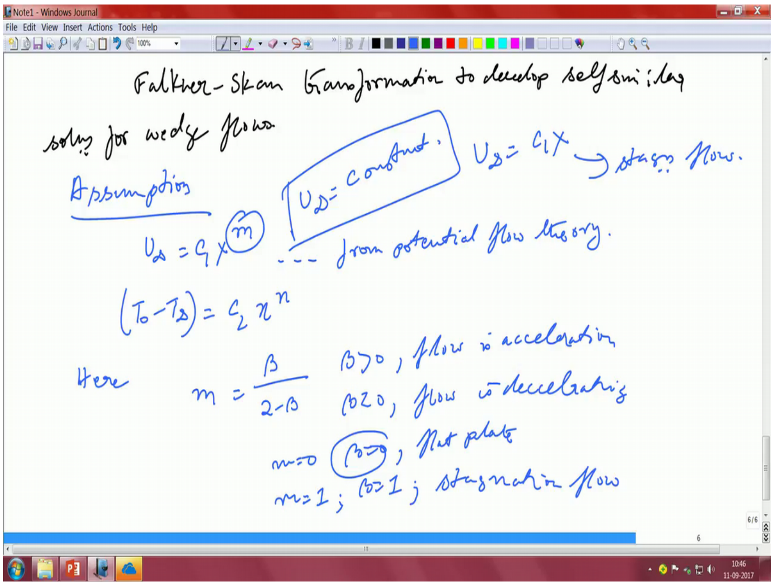

Basically, the Falkner skan transformation to develop, self-similar solutions for wedge flows. The solutions can we have different types you can see.

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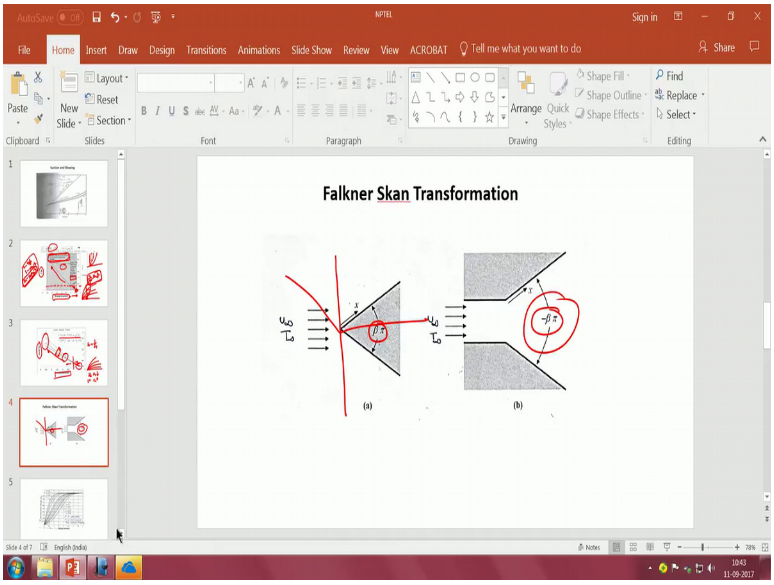

If you look at it here for example, this is 1 type of a wedge and go to the ppt view. This is 1 type of a wedge. The angle subtended beta pi this is 1 type. This is U infinity still T infinity. This is how the direction of X is actually measured.

This is more like a convergent channel kind of a flow. Whereas, the other part that you see, this is also another wedge. Where the flow is a little bit diverging this is minus beta pi. The flow this is the direction of X it is still U infinity T infinity and the angles. These angles can go, it can be anything that you want. The when it becomes flat like this it becomes a hiemenz flow when the angle goes to 0, it becomes basically the flat plate boundary layer. In between and when it goes to the other extreme basically crosses the 90 degree mark you get these kind of solutions.

The Falkner skan class of solutions basically is applicable for any type of wedge where the incoming flow is always uniform, in terms of velocity and temperature. Based on this let us go back. First and foremost, the first assumption 1U infinity is equal to C1X to the power of m. Where does this come from we promised that we will cover the origin of this? It basically comes from the potential flow theory. Outside the boundary layer the potential flow theory dictates that C1 must be equal to CU infinity should be equal to C1X to the power of m.

Similarly, Tminus Tnaught minus Tinfinity where Tnaught is the temperature of the wall is given by C2X to the power of n. N and m are different as of now remember that here we try to establish the relationship between the 2 m is equal to beta by minus 2 by beta 2 minus beta. Where beta greater than 0 implies that the flow is accelerating; that means, it is like a channel flow. Beta less than 0 implies the flow is decelerating; that means, it is basically a diffuser kind of a flow or a divergent nozzle beta equal to 0 implies a flat plate. Beta equal to 1 implies a stagnation flow.

You can see the corresponding M’s you can find out from this particular thing. One thing you can notice is that when beta is equal to 0 M is also equal to 0. The flat plate basically corresponds to M equal to 0 does that obey with this because as you put M equal to 0 in this particular expression, your U infinity becomes a constant. Which is the correct thing for a flat plate with no inclination your U infinity that is your free stream velocity is not a function of X. That is easily you can see that that is the condition over there.

Similarly, M is also equal to 1 when beta is equal to 1 got it. M equal to 1 because beta is equal to 1 that would mean that U infinity should be equal to C1 into X. That would mean this type of thing if you recall your fluid dynamics a viscous fluid flow this actually typically corresponds to a stagnation type of flow. This kind of agrees whatever our strategy was is kind of agrees with the view that we have linked it with beta where beta is basically the wedge angle.

Let us based on this, let us try to see how we can actually best define the whole framework once again the self-similarity solution because that is what we promised.

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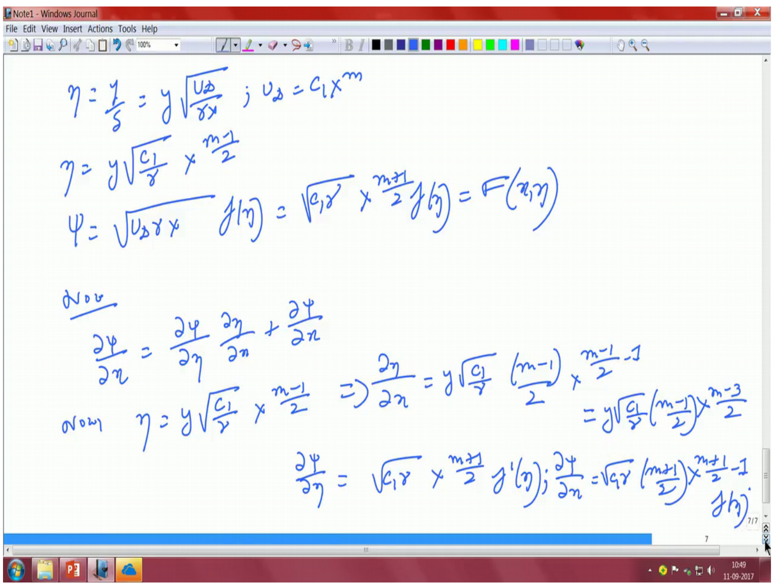

Eta is equal to y over delta y U infinity by gamma X. U infinity is equal to C1X to the power of m. Eta is equal to y. 2C 1 by gamma X m minus 1 by 2. Psi which is the basically the stream function this we already defined in the general case of suction and drawing. Some parameter space like this.

Now, let us do the transformations, d psi by dX the slightly different of how we progressed with the self-similarity solution with a flat plate, but we could have done it also in this way. There are different routes essentially the thing remains the same you are free to use whichever one you are most comfortable with. By d eta d eta by dX d psi by d X now eta is given as yC 1 by gamma X m minus 1 by 2. This is also given. Therefore, this leads to these are all mathematical transformations. It just requires that you just work out the math in more details which gives you got it. D psi by d eta is equal to. Similarly, d psi by dX equal to, this is after this after this. These are the different types of transformations that we are actually doing over here.

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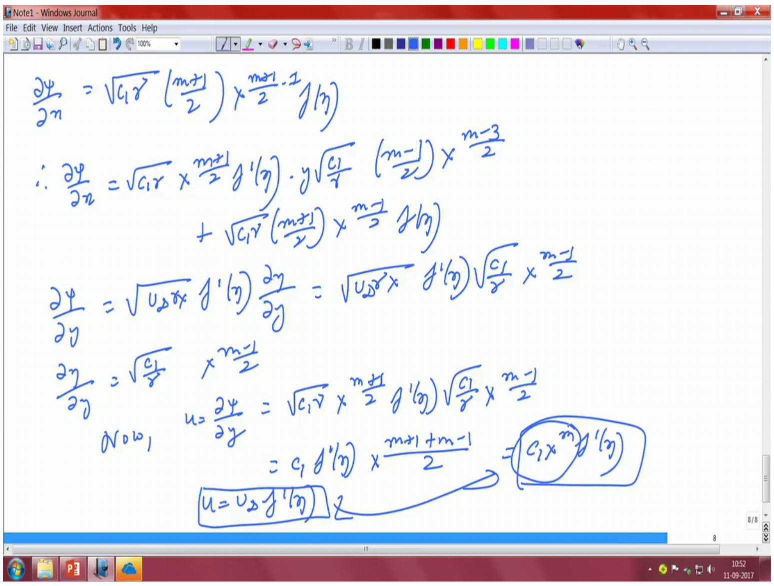

Similarly, d psi by dX. This is the string function approach to the similarity solutions that flat plate can also be done in a very similar way, but we chose to show 2 approaches in 2 problems. That you can adopt any one of them. Therefore, d psi d y. Now, we already know this is your d u. Therefore, this will be and u is basically equal to U infinity f prime eta. You can see that it kind of agrees we started with this particular expression first. In our previous approach you can see that these 2 things are basically equivalent to each other. Except U infinity instead of a constant now it is C1X to the power of m.

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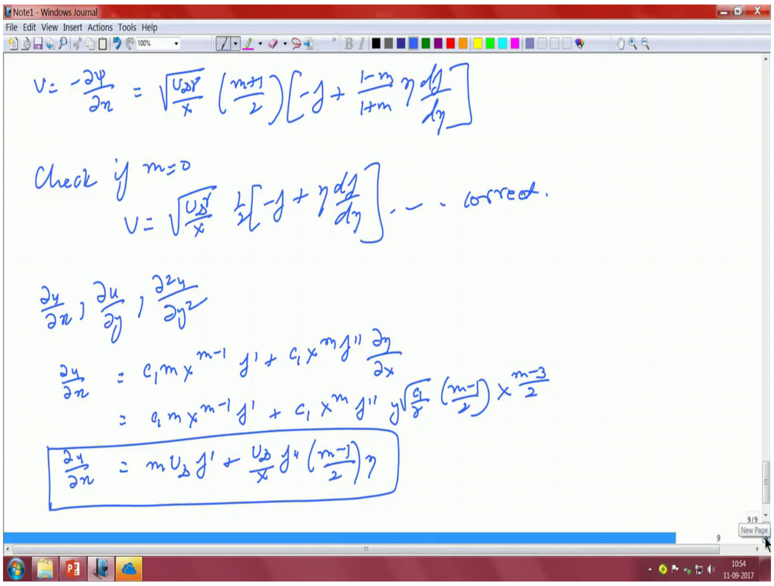

Now, given by that if I write the full expression because there are a few steps which I am going to skip over here that you can work out. This is what we wrote in our general case of suction and blowing also, this comes basically like this. Now if we can do a check if m equal to 0. Which is basically the flat plate boundary layer your V becomes equal to U infinity into gamma by X into half minus eta d f by d eta. This is correct. This is quite correct.

Now, our to go to the momentum equation we need to find out terms like du dx plus du dy d square u dy dy square etcetera, etcetera. All those things needs to be found out. Let us write du dx is going to be long because we are carrying a few terms with us now. F prime plus C1 is equal to got it. That is du by dx we are doing it term by term. That we can collect it at the end. Du dy will be equal to.

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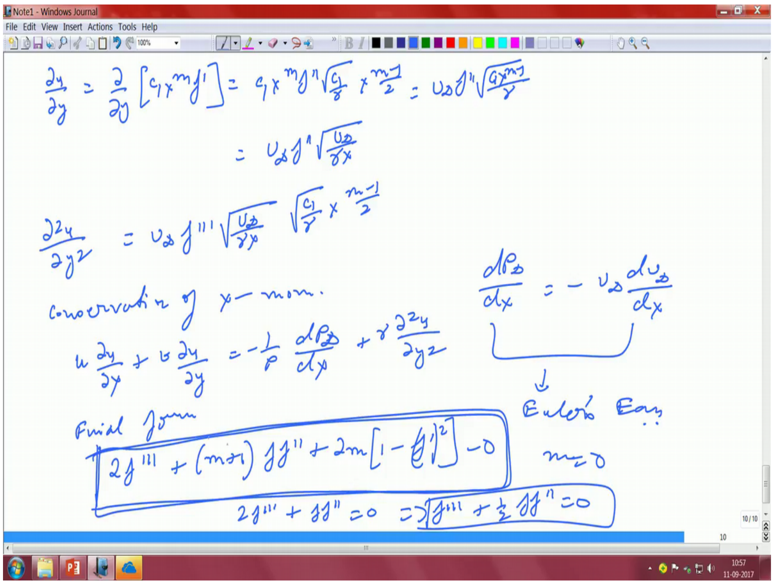

That is the first derivative similarly, that is the other term now we feed all this information to the conservation of X momentum conservation of X momentum which is basically u du dy plus v. Here of course, your pressure term dP by dX is minus U infinity d U infinity by dx this comes from the rulers all.

Now what we do? Is now you put all these values of u v dU dX and all these values and the final form of the equation. Therefore, now it is term by term substitution. That is the total expression that you got, in this particular. As you can see once again if m is equal to 0. What will happen? This equation will become 2 f triple prime plus f into f double prime this entire term goes to 0. You will get a 0 over here. This will lead to f triple prime plus half f by f double prime is equal to 0, which is like the flat plate boundary layer equation. We once again get the same thing the flat plate boundary there.

But this is the total expression when you actually have a wedge solution. This is the general class of solution more generic than your Falkner, that then your Blasius boundary layer.

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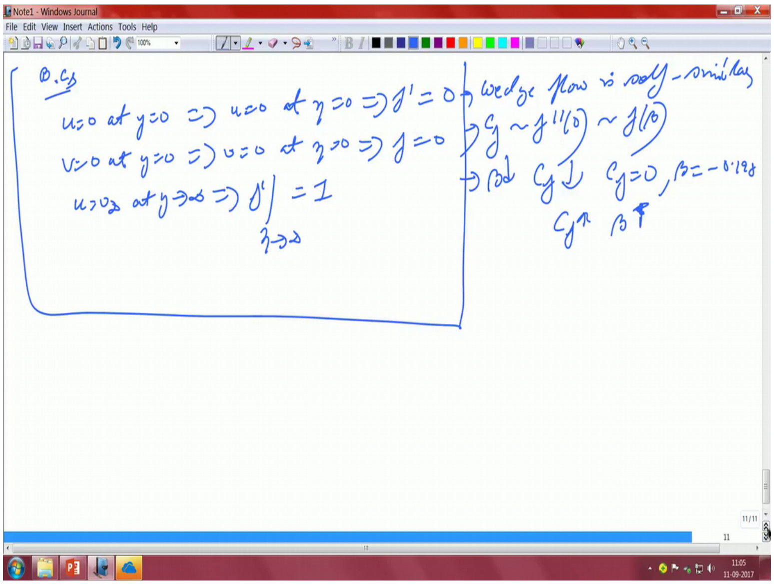

The boundary conditions will be u equal to 0 at y equal to 0 and leads to u equal to 0 at eta equal to 0, leading to f prime is equal to 0. V equal to 0 at y equal to 0 leads to v equal to 0 at eta equal to 0, leads to f equal to 0. U equal to infinity at y goes to infinity leads to f prime as eta progresses to infinity is equal to 1. These are the sets of boundary conditions that you have.

Now as soon as these things are done. Let us now look at what the boundary layer profiles are going to look like before we go to the energy equation, and see what the energy equation should look like.

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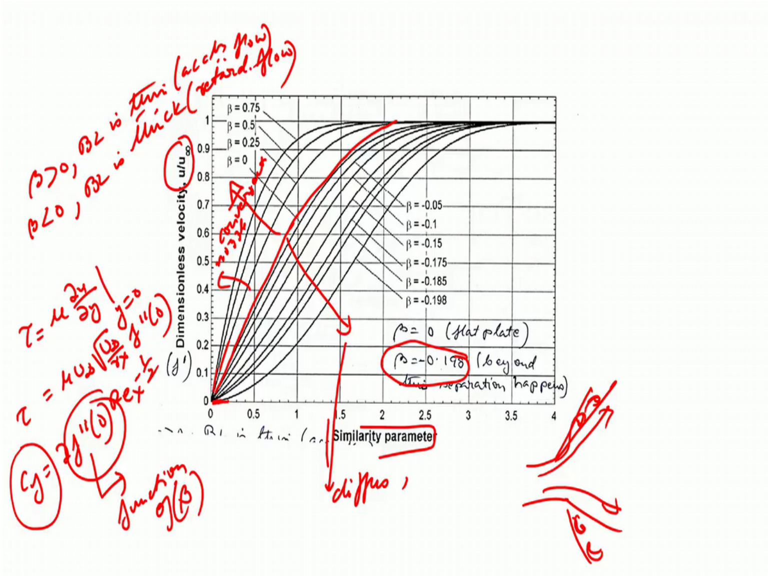

Let us look at this particular expression over here. These are for the different betas this is basically nothing but the velocity profile u by U infinity. As you can see beta equal to 0. Once again is a flat plate boundary layer let us mark that line. This is basically your flat plate boundary layer. Basically, a flat plate boundary layer in this particular direction you can see the boundary layer is basically sagging a little bit, that is where the point is; that your beta starts to actually become negative and in this particular direction on the other side the beta is actually increasing. This corresponds basically to a diffuser that corresponds to a convergent nozzle all.

Now, as you can see from a diffuser what happens very standard things as you I have diffuser what happens is that the flow is basically facing what we call an adverse pressure gradient. As the pressure as this diffuser angle becomes larger and larger the flow will at some point of time starts to separate and that separation happens at around beta equal to 0.198 approximately close to minus 0.2 as you can see you the boundary layer profile is almost equal to 0 because, the shear stress is nothing but the slope at this particular point. As you can see the boundary layer is sagging and it is actually equal to 0 at the walls; that means, the wall shear stress should be equal to 0, at this particular point.

As you go on increasing the diffuser part, chances of flow separation increases and as the flow separation increases, you actually have separation of the shear layer as you have separation of the shear layer the flow basically therefore, separates on the other side;

however, the boundary layer becomes sharper and sharper as you can see. It becomes sharper and sharper this is based on the similarity parameter which is basically your eta and therefore, the shear stress will also become higher and higher.

We can write it here when beta is equal to 0 boundary layer is thin. This is corresponds to the accelerating flow, which is basically the nozzle flow. Beta less than 0 means the boundary layer is thick, this is like a retarding flow. Tau is basically given as u dy at y equal to 0. Which translates to tau is equal to mu U infinity U infinity by x double prime is equal to 0. Therefore, c f should become 0 into Reynolds number to the power of minus half this is a function of beta. Which is quite obvious from this particular graph that as you increase beta. Your U by U infinity profile changes as a result of that your f double prime equal to 0 profile also should change because, of that you are going to have this kind of a thing this is quite apparent.

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In the next plot if you look at this, this is flow over a wedge. This part is once again the beta the same thing. This is like as I say it flow through a diverging channel and this is the flow over a wedge. As you can see that this is basically nothing but your shear stresses the wall shear stress. This is your f double prime equal to 0. As we saw c f was proportional to this correct.

We can see in this particular case at 0 this is what is whatever the value that is you get from yourself similarity solution. As you go to about point minus 0.2 everything separates and you basically get 0 shear happening over there beyond this there is separation. Here of course, you can see that the shear stress will continuously increase. It is very much like there suction and blowing kind of a stuff except that here the boundary layer thickness is varied just by changing the channel wedge. That whatever is a wedge angle we are varying that and we are able to get this extraordinarily nice profiles coming out of this.

Based on this, we can certainly say that c f will be reduced is reduced as beta is reduced, become c f is equal to 0. At the point of separation at the point of separation c f becomes equal to 0. From this particular profiles that what we have done over here. We saw 2 important things is that when the flow is accelerating the boundary layer is always thin, when the flow is decelerating the boundary layer is always thick. That is proven through this analysis and the shear stress which is dependent on the profile the boundary layer profile essentially, they are basically the angle at the wall that also changes with the wedge angle.

Now, coming back to our genre. Therefore, we can readily see estimates that wedge flow is self-similar. It is self-similar and cf is actually given by f double prime 0. Which is a function of beta that we saw as beta goes up or other beta comes down cf also comes down. Cf is equal to 0 when beta is equal to minus 0.98, cf goes up when beta goes up.

These particular things and if you look at what we did, all these mathematical calculations yielded that this is the basic governing equation. Which we basically solve in a very similar way that we did earlier; that means, using the shooting scheme and things like that, but even without the shooting scheme and without all this thing the basic problem remains that the solution pattern remains the same, except that the boundary layer is accordingly modified depending on whatever is a value of this m going to be; that means, whether it is this m is nothing but your beta. Remember n is your beta actually and the special forms can all be derived from this particular expression.

We are left with in the next class what we are going to do is that we are going to look at the energy equation now because the energy equation comes after this. Now we have solved the momentum we know how it looks like we have explained what is the nature of this profile is going to look like, but we are going to see that through the energy equation what we can get that whether the energy equation is any different or whether it predicts something very similar that also we will see by looking at the at the results.

In the next class we are going to go into the energy equation mode, but the wedge now it is very clear that how we got those equations in suction and blowing? Suction and blowing physically kind of represents the same thing as your wedge solution. It is all about changing the boundary layer thickness of the boundary layer. If the boundary layer profile changes everything changes right from your shear stress to your wall heat transfer coefficient.

Thank you.