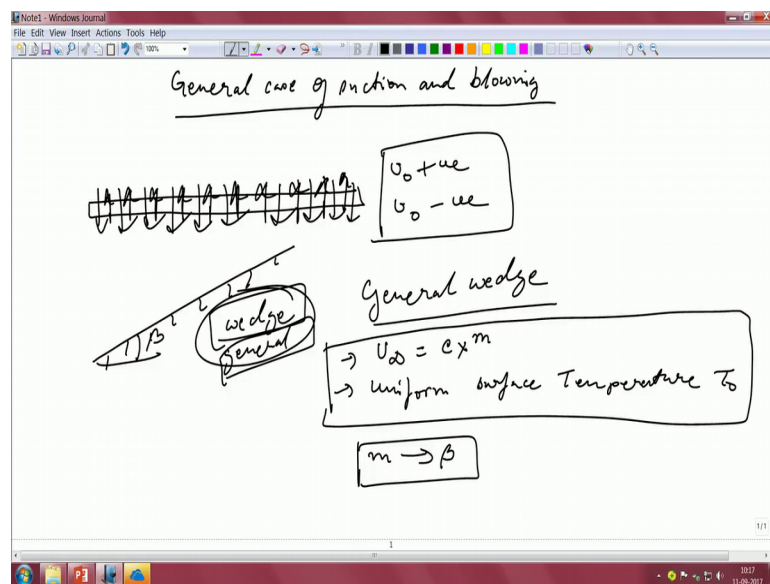


Convective Heat Transfer
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Lecture – 12
Suction and Blowing

So, welcome in this particular class, we are going to do something which is a general case of suction and blowing.

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Now suction and blowing in this particular case, see all the time in the previous classes also we said that, what if there is a there is a is if the plate is not impervious in nature; that means, you have this plate, and it can either built like this. Like a perspiration, or there can be a reverse direction of the flow; that means, it could be porous, and you could have this flow going down like that.

So, for example, if this velocity is upward, we can say it is positive, and v downward we can consider that to be negative right. So, so this is the general case of basically you know what we call suction and blowing, right. The mechanism of suction and mechanism of blowing. Now how to analyze a situation like this, that is the thing that we are going to attempt in this particular class.

So, here we are going to take do a thing which you will we will go into the details a little later, that we are going to take a generalized equation. And that equation will be valid not for just this plate, it can be also valid for a wedge; that means, if the plate is at an inclined angle and if it is still blowing in the same way. But in any case, this part; that means, the wedge part we will devote a full lecture on it. And show that how this wedge can actually you know affect, the what will be the dynamics.

But in this particular case is a general wedge s it is a general wedge that we have taken over here. And the equations that I am going to write, some of the equations we are going to derive a little later. But for the time being for this particular class you are going to take the equation as is, in the later lectures you will understand how those equations are kind of applicable what is the level of applicability.

So, basically this one and the and the general case of a wedge solution will come hand in hand. So, you should ideally look at it in a more integrated kind of a fashion. So, ha for a general wedge, because then we do not have to come back and again do it for a general wedge. So, for a general wedge, the u_{∞} can be written as x to the power of m , now how is it exactly written we will come to that in the next lecture. And it also has a uniform say surface temperature T_{naught} , got it?

So, it is a general wedge in which you have u_{∞} given by $c x$ to the power of m , what m is, we will see m is related to the slope of the wedge how those things are related. We will come a little later where the significance of m will be clear, for the time being you can say that m is some way associated with this wedge angle. If that wedge angle is β , m in some way is related to that β . That is for the given at that. So, it is a function of the wedge angle in some way. What those angles are we will come a little later, but let us concentrate and see what it happens what it does to the suction and blowing.

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$\psi = (u_{\infty} x)^{\frac{1}{2}} f(\eta) \quad \eta = y \sqrt{\frac{u_{\infty}}{x}}$
 \downarrow
 stream function.
 $v = \sqrt{\frac{u_{\infty}^3}{x}} \left[-f + \frac{1-m}{1+m} \eta \frac{df}{d\eta} \right]$
 $v = v_0$ at $y=0$ (i.e. $\eta=0$)
 $v_0 = \sqrt{\frac{u_{\infty}^3}{x}} \left(\frac{m+1}{2} \right) (-f_0)$
 $v_0 = \left(\frac{m+1}{2} \right) f_0 x^{\frac{m-1}{2}} \cdot \sqrt{c}$
 f_0 is not a function of x
 $\therefore f_0 = -u_{\infty}^{\frac{1}{2}} \left(\frac{2}{m+1} \right) \frac{1}{\sqrt{c}} \frac{1}{x^{\frac{m-1}{2}}}$
 $f_0 \neq g(x)$
 $f_0 = f(x)$ if this is not true

Now, following our old similarity transformation. We can write something like this here, where this is nothing but the stream function. The significance of this and eta is of course, the same as the similarity variable, what we had earlier. So, v, that means, the velocity the vertical component of the velocity becomes something like this. So, as you know that v is nothing but the differential of psi with respect to x, right. Psi being the stream function this is once again nothing like the; it is exactly the similarity solution that what we did earlier, but v now is equal to v naught, right. At y equal to 0, which means at eta equal to 0, right.

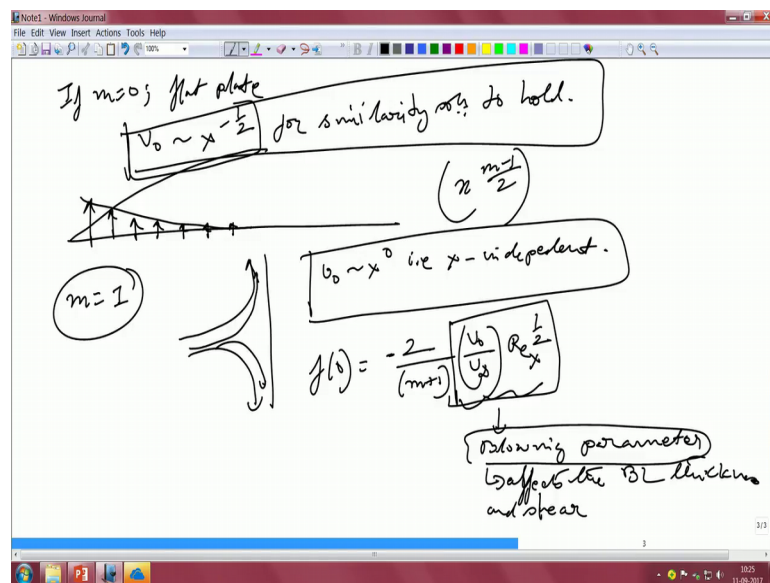
So, v naught will be given by $u_{\infty}^{\frac{1}{2}} x^{\frac{m+1}{2}} f_0$ evaluated at 0 right. So, v naught is also once again, just writing it in a proper way f_0 ; that means, f_0 evaluated at 0, right. Then x raised to the power of $\frac{m-1}{2}$ multiplied by gamma. So, this is the form that we have in this particular case. We also know that f_0 is not a function of x . So, if f_0 is not a function of x . That part we know, therefore, if this is not a function of x , then you can imagine, this is the left-hand side this is the, right. Hand side of the 2 equation right.

So, if f_0 is not a function of x , then let us write f_0 take everything to the other side, it is basically $v_0 = \frac{2}{m+1} \frac{1}{\sqrt{c}} \frac{1}{x^{\frac{m-1}{2}}}$. So, this is not a function of x ; that means, we can say it is not a function of $g(x)$, right. So; obviously, in order for this to be true that f_0 is not a function of x you can imagine

that your v naught has to vary as x to the power of m minus 1 by 2, right. Because then only this and this will cancel each other right. So, if does not if it does not vary by this particular way.

Then you will have an x dependence f naught, will be a function of f in that particular case, if this is if this is not true. Got it? If this is not true that is what is going to happen. So, if that happens the entire similarity solution actually breaks down. So, for the similarity solution to be applicable for the general case of suction and blowing, what we are going to have is that your f naught cannot be a function of x , that is a given. And therefore, your v naught has to vary in the same way as x to the power of m minus 1 by 2; which if m is equal to 0; that means, it is a flat plate basically, right. With no angle right..

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So, v naught in that particular case should vary as x the power of minus half, right. For similarity solution to hold, right. So, that means, as you as your distance x increases, right. It is 1 over inverse right.

So, it is basically comes down in this particular fashion, correct? That is how it is coming down. And as you know that the boundary layer grows up by x to the power of half right. So, the boundary layer as it progresses this actually decays. Whether it is moving up or coming down that depends, right. It is like the whether is a porous matrix or it is like a perspiration.

So, for this to hold, a similarity solution to hold, you have v_{naught} which should vary as x to the power of minus half. Now this m once again we have not said about the nature of m except that it is equal to 0 for β equal to 0; that means, when the wedge becomes a flat plate. Now m equal to 1 corresponds to a stagnation flow; that means, there is a flow somewhat like this. In other words, is also called the Hiemenz flow. So, that is equal to m equal to 1. Once again, we will come across what these quantities are right.

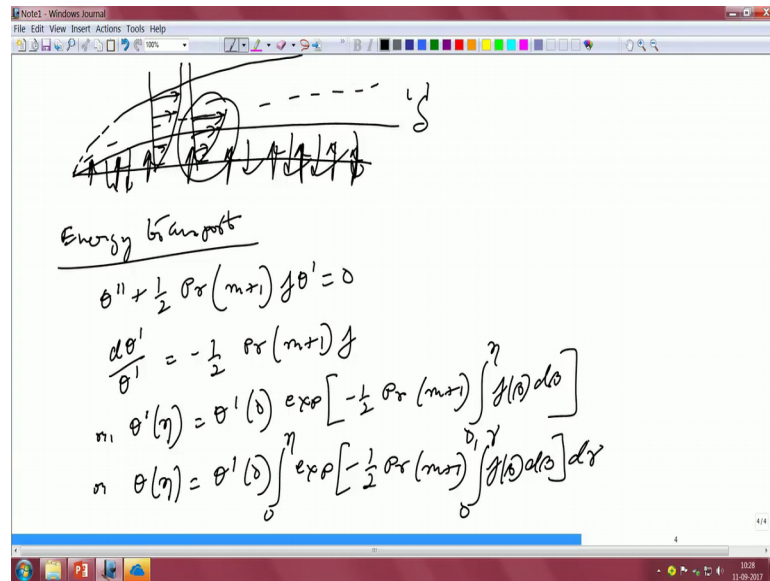
So, in that case v_{naught} should vary as x to the power of 0. That is, it is x independent, got it? It is x independent, got it. So, this is x independent for the Hiemenz flow, it is x to the power of minus half for a flat plate. And for in between any other parameters or any other angles. It should vary as x to the power of minus m minus 1 by 2, right?

In order for the similarity solution to hold, now remember you can have any type of suction and blowing, right. For these kind of parameters if the functional form is like this then only the similarity solution will hold, but in real life you can have other kinds of suction and blowing as well. So, in this particular vein let us define a parameter called f_{naught} which is 2 by m plus 1 v_{naught} by u_{∞} Reynolds number or of half. This particular parameter space is called the blowing parameter..

It effects the boundary layer thickness. Of course, it does and the shear obviously. Because if the boundary layer thickness changes the angles will change. So, naturally the shear will change right. So, in this particular way this f_{naught} parameter this part is called the blowing parameter. Remember, this v_{naught} by you $naught$ into Reynolds number based on x raised to the power of half.

Now, how does suction and blowing actually play a role? So, physically intuitively, so if there is a say a blowing happening like this, what do you expect?.

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Initially the boundary layer say is something like this, right. With suction and blowing when you actually pump in momentum in that upward direction. The boundary layer is supposed to go like that right. So, there is a displacement of the boundary layer in the upward direction right.

So, in other words, the slope if this is this was the slope for the for the boundary layer after it was kind of blown outward. This slope over here will be much more you know much more higher, in that way because the boundary layer thickness is now lower. In this particularly at the same location, if you talk about location to location at the same location before blowing the boundary layer thickness was delta after blowing it say it is delta prime..

So, obviously, delta prime is more than delta similarly when you actually have suction; that means, when you have it in the opposite direction the boundary layer becomes thinner. So, the gradients actually gets enhanced. So, the gradients will get enhanced. So, this actually proves that why we are going to get a change in shear, and obviously, a change in boundary layer.

So, let us look at the energy transport what does it do to the energy transport. So, here we have easily explained that if delta changes; obviously, you know that the CPCF or basically the drag is dependent on delta. So, naturally as delta changes that also changes. So, the energy transport if you recall the energy transport that we did earlier. This is I am

just adding a term called $m + 1$, in this particular case, we will come to the significance of $m + 1$, once again when we do the suction when we do the Falkner skan class of solution.

Once again assume that this is the energy transport. Let us let us assume it in this particular way. So, this is that let us do a little bit of mathematical jugglery on this, or theta eta straightforward integration, now theta infinity is equal to 1, we all know that, I'm not going through the steps once again..

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The image shows a handwritten derivation in a software window titled 'Note1 - Windows Journal'. The equations are:

$$\theta(x) = 1$$

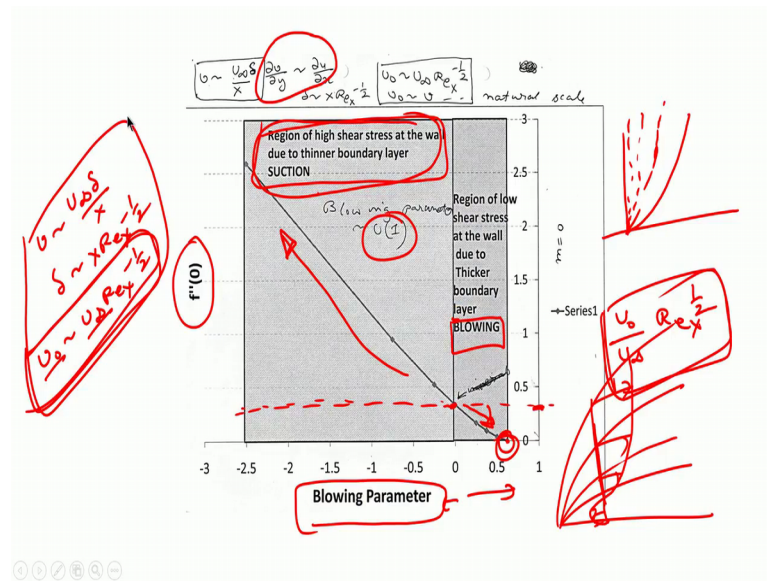
$$\theta'(0) = \frac{1}{\int_0^{\infty} \exp\left[-\frac{1}{2}\rho_0(m+1)\int_0^{\eta} f(\eta) d\eta\right] d\eta}$$

$$\therefore \theta(\eta) = \frac{\int_0^{\eta} \exp\left[-\frac{1}{2}\rho_0(m+1)\int_0^{\eta} f(\eta) d\eta\right] d\eta}{\int_0^{\infty} \exp\left[-\frac{1}{2}\rho_0(m+1)\int_0^{\eta} f(\eta) d\eta\right] d\eta}$$

Theta 0 theta prime 0 0 to infinity. So, therefore, theta eta; that means, the total function. So, this is the total expression that we actually get for theta and eta.

Now, for a wedge we will see how this x actually varies, but it is already incorporated in that $m + 1$, right. Now if we look at therefore, the solution that how this is going to really you know look like when you actually have suction and blowing. Let us look at the ppt or we will show that how that thing actually happens..

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So, let us look at first, that how the boundary layer is going to change, using quantitative information, what I did was draw qualitatively, what is going to happen.

Now, if you look at the annotations that I have actually made, if you recall that your v is a function of u infinity into δ by x , isn't that? So right. And δ is; obviously, x into Reynolds number minus x to the power of half right. So, v naught was actually equal to u infinity into Reynolds number to the power of minus half right. So, these are the 4 scales that I have written on the top of this particular thing alright because the first scale comes from the fact that $d v$ by $d y$ is the same as $d u$ by $d x$, right. Because that is the continuity scale that we have.

Now, what we have done is therefore, we have plotted this blowing parameter which we already defined in our previous class. And $f''(0)$ is nothing but the slope, or the corresponding shear, you can talk about it as a shear. So, you have basically 2 series. One is blowing parameter, and one is the suction parameter. So, this part which is the positive side is basically blowing and this part; which is the negative side is basically suction right. So, this part is positive this part is negative right.

So, as you can see that when there is when you continuously go on blowing more and more; that means, you are increasing the blowing parameter, right. If you recall what the blowing parameter was we just did it let me write it down over here. So, that there is no confusion x to the power of half right. So, that was the blowing parameter, right.

And as you can see the order is also we have just proven the order over here, this is like v naught scales as u infinity into δ into Reynolds number to the power of minus half; that means, they are of the same order. So, that is why you see the numbers are also of the same order, right. They are all about 1 of the order 1, that is what exactly what we have d_1 over here.

So, what do you see as you increase the blowing as I say that the boundary layer is blown outward right. So, therefore, there is a sharp decay in the shear. Because here if you look at it. So, this is the shear that you found normally for a flat plate. So, the middle one is an impervious flat plate right. So, as you can see over here, that there is a lot of blowing, and because of that there is a sharp decay in the shear stress right.

Whereas when you actually have suction; that means, the boundary layer should become thinner. You see over here that there is a sharp increase, almost from about 0.3, right. All the way to about close to 2.7 right. So, it is roughly a one order increase in the shear stress. So, there is a very high shear level, when you actually have suction at the wall, right. And the blowing parameter is always remember of the order one comes from this particular scaling and from our definition.

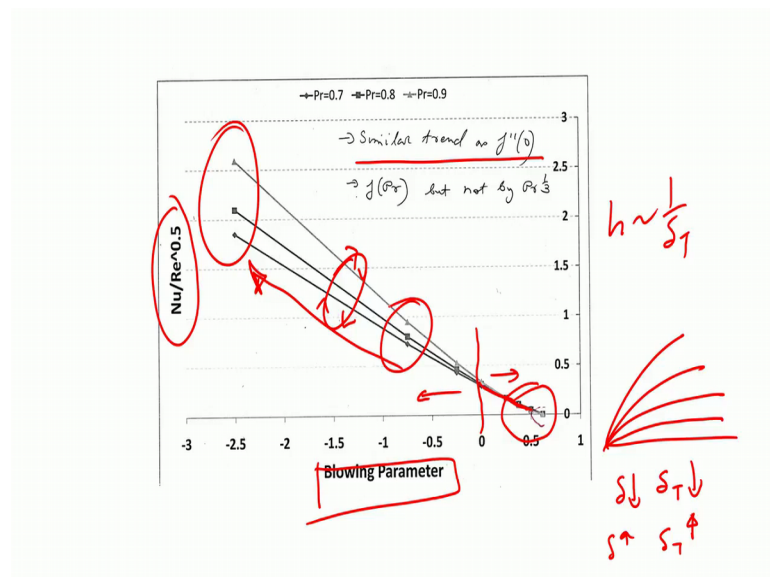
Now, let us look at if the suction side we went up to this. Whereas, on the blowing side we found that it becomes equal to 0 very fast, right. Do you see that it becomes 0, right? After we cross about 0.5. So, around point 6.7 it becomes equal to 0, why that is the case of that is because, as you blow the boundary layer more and more, this is how the boundary layer is increasing as you increasing the blowing, right. As the after you increased it beyond a certain amount the flow starts to separate. The flow starts to separate not exactly like that..

But say you have increased it to the limit that it has reached the so, here the flow will actually separate. Because the gradient is slowly becoming shallower and shallower, if you look at my hand it becomes slowly shallower and shallower and shallower, till at a point when it becomes equal to 0. And after that there should be actually what we call a separation.

So, the flow actually separates beyond a certain point, because the gradient the slopes will become like this. So, this is the original one, this will be the one after some time, this will be after some time, this will be after some time. So, at one point of time this

particular point will actually separate. So, that is the reason why blowing you cannot go to high levels of blowing because it will separate the boundary layer, right. Whereas, on the suction side you can go very high because it is a favorable kind of a phenomena. So, that you are actually you are compressing the boundary layer in such a way that your shear goes on and on and on it increases more and more there is no separation effect as such. So, that is an interesting thing.

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Now, the blowing parameter let us look at it in terms of the you know the heat transfer. Because the heat transfer is the other half of the story, that we have for which we said that what will be the what will be the theta we derived the theta. So, if you look at it here it is Nusselt number, divided by Reynolds number to the power of 0.5, right. So, because that is the Reynolds number scaling is always hal. And we have different Prandtl number families after that. So, this is once again the blowing parameter space. So, once again you have this point at 0, right. Which is basically the flat plate impervious flat plate this part is basically is blowing this part is once again suction.

Once again, the same thing what will happen? When as you know that when you make the boundary layer smaller; that means, you make delta go down delta T; obviously, also goes down, right. Delta T also goes down the thermal boundary layer also goes down, correct? Similarly, when you make this go up, delta goes up delta T also goes up right. So, it is actually has a very similar kind of connotation right. So, in this case what you

see is that you are basically your Nusselt number by Reynolds number scaling it goes to 0, right. Right about the same time when your separation actually happens, right. Because separation means you lose the boundary layer altogether..

Whereas, it goes upward; that means, you have an enhanced heat transfer because of the lowering of ΔT , because if you recall h is proportional to $1/\Delta T$. So, as ΔT goes down h actually goes up. So, therefore, you have a sharp increase. So, this is a very similar trend as $f''(0)$, right. Very similar trend as your shear. Except for the fact now with Prandtl number dependency, you have different types of variations. The curves do not actually fall on the top of each other they are actually a little bit spread out.

So, that is what you see over here the curves are a little bit spread out. And you see that for Prandtl number as you go on changing the Prandtl number, the slope of these lines actually change, but; however, they seem to merge beyond the for all the blowing cases, but whereas, for the suction cases there is a little bit of difference; that means, there is a little bit of branching out of this particular curve, but otherwise they are very similar to your $f''(0)$.

So, what suction and blowing does is that in a nutshell when you do blowing you actually increase the boundary layer thickness both for thermal as well as for thermal and as well as for momentum boundary layer as a result you reduce the heat transfer coefficient, and you reduce the shear stress. Whereas, on the other side if you go to the suction side of things, what happens is that you are reducing the you are as you go on increasing the or decreasing the blowing parameter..

Or, that means, you are increasing the level of suction, your boundary layer thickness becomes very short, as the boundary layer thickness becomes very short both your wall shear stress; that means, you are basically a drag as well as your heat transfer coefficient both shoots up, these are the things that you must take out from this particular piece of argument. And this is the happens because suction makes the boundary layer smaller whereas, blowing blows the boundary layer off. Now this is valid for all types not just for flat plate, it is also valid for wedges it of any particular angle. But for the similarity solution to hold, we already know that it has to vary in a certain way this suction parameter has to vary, or the blowing parameter has to vary in a certain way; that means,

v naught has to vary in a certain way, which is basically given by x to the power of m minus 1 by 2. So, when once those things are obeyed, this is the nature of the graphs that you have. So, in the next class we are going to look at the wedge.