Convective Heat Transfer

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Lecture- 11
Integral Solutions- Energy

So, we have looked at the integral form of the momentum equation we have established that how the boundary layer thickness comes out to be almost the same, as what we got through our normal similarity transformation the variations are; obviously, the same.

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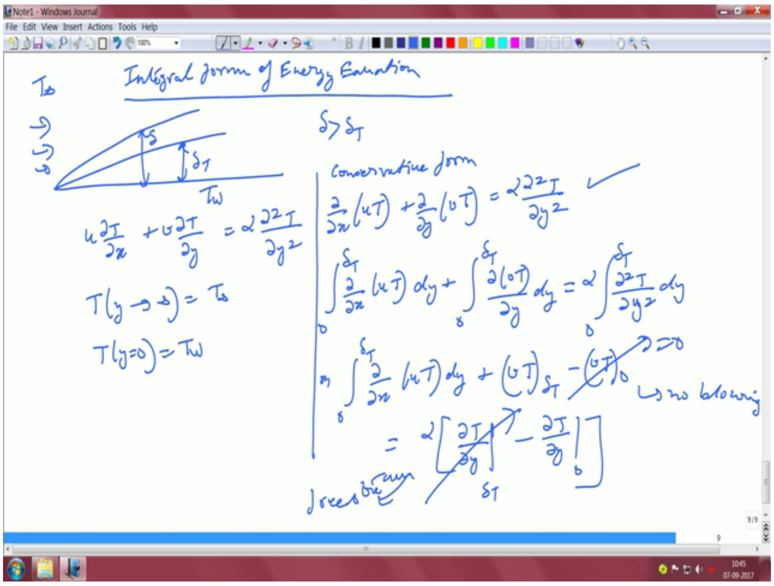

So, in this particular class what we are going to do we are going to look at the integral form of the energy equation, that is what we are going to do here ok.

So, once again as we know that there are 2 forms. So, these are the 2 profiles one can be delta one can be this. So, we are assuming that this is delta this is delta T and the wall is T wall this is T infinity. So, our basic assumption is that delta is greater than delta T which is usually the case, as we said except in liquid metals and other things this is mostly the common type of fluids we will actually follow this kind of a profile right. So, delta is greater than delta T. So, u d T dx plus v d T dy that was the expression that we initially had right. So, T as y tends to infinity is basically T infinity T at y equal to 0 is Tw right that is a wall temperature ok.

Once again let us write it in a conservative form got it. So, now, let us integrate got it. So, this was a conservative form of the energy equation; that means, you include the continuity and this is we are integrating it with respect to y once again, that is what we did earlier. So, once that is done. So, this is equal to 0 right which is basically true because at 0. Now we are not considering suction and blowing we have already written if you want to write it in a proper form that can also be done. So, you can still continue in that particular way, but here v is equal to 0. So, that is what we have taken. So, this is also equal to 0 for the same reason because the thermal gradient disappears as we move to the free stream right.

So, this is basically no blowing and this is basically the free stream. So, these were obvious to you. So, now, what we can do is that, we can now write this particular expression the revised form of the equation.

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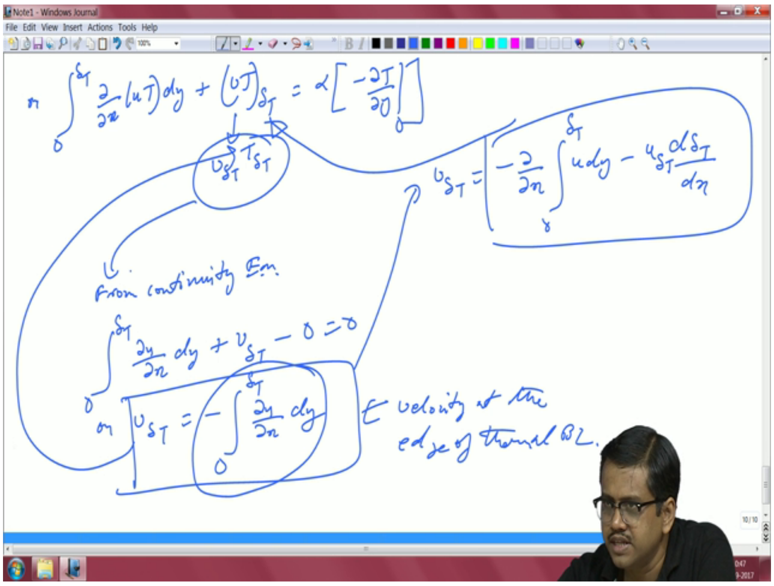

Now, as got it that is a revised expression right of course, looking at the nature of this particular term vT delta T and T delta T, we need to find out what this term is all about right that is what that term is. So, for that we would need to use the continuity equation right we can use the continuity equation for this right.

So, v delta T it is basically given us 0 to delta T then du dx into dy right. So, that is the v delta because we needed that expression, which you can substitute it there right of course, T delta T you already know what T delta T is going to be correct. So, if you do them. So, now, that you have got this particular expression ironed out now how to proceed to the problem. So, this particular expression you remember that this is given that the velocity at the edge of thermal boundary layer right that is what it is correct ok.

So, we can once again apply the Leibnitz rule over here to this particular guy, if we do that then what we get is v delta T will be given by d by dx 0 to delta T u dy minus u delta T d delta T by dx right. So, now, we take this value of v delta T and substitute it here right we have to substitute it there. So, let us look at once after substitution what happens.

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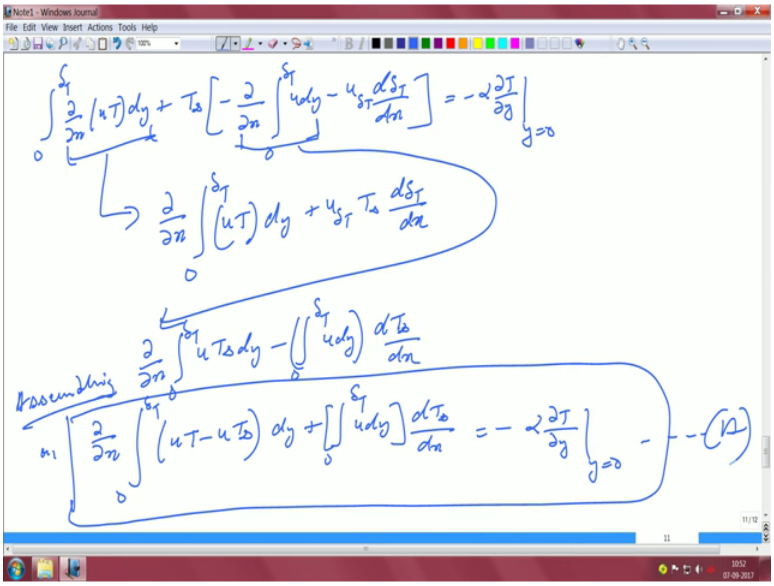

So, the first term remains the same plus T infinity minus d by dx y equal to 0 right. So, let us take term by term. So, that we can it makes a lot of sense to do the math properly the first term let us do that ok.

So, the first term can be written as again applying Leibnitz. So, that you can see how the Leibnitz is coming in handy right. So, applying Leibnitz we can get this and not writing the 0 boundary condition anymore which was a part of Leibnitz because you already know that that is going to be 0. So, that is the first term that we get this particular term. So, that was this term only this particular term now can be written as that is the second term of the series right can be written as d by dx 0 to delta T u T infinity dy minus 0 to delta T u dy d T infinity by dx right. So, that is also that the second term that we are actually writing over there.

So, now that we have finished writing these 2 terms, now let us kind of assemble things a little together because we have all this term spread all around assembling you should practice this algebra well in your leisure time just. So, that you know this basically algebra is not much of a complicated math either right. So, that is the total temperature profile that you get.

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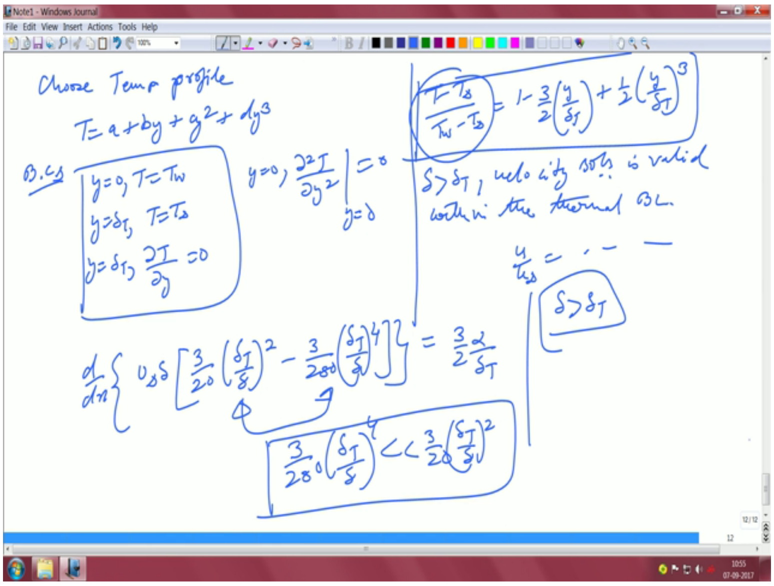


Choose these are the boundary conditions, and similar to the velocity boundary condition right. So, similar to all these boundary conditions we get this also. So, choosing the temperature profile then putting in the boundary conditions we get this.

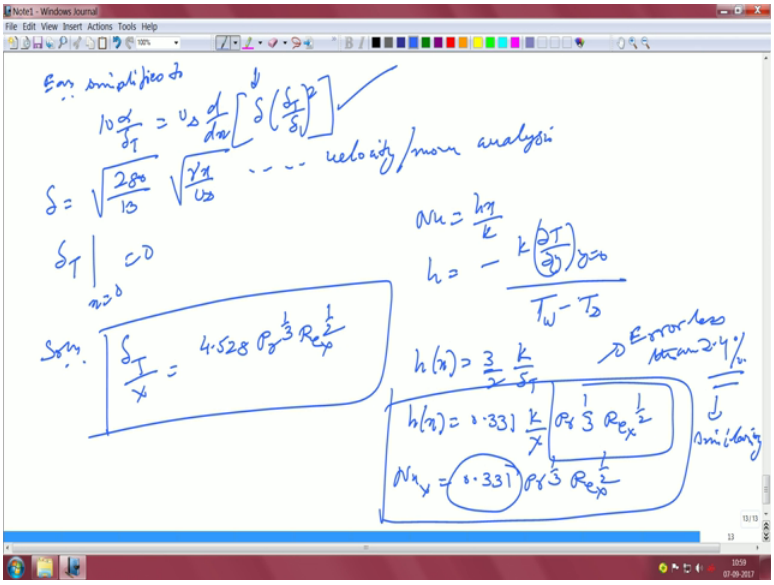
So, if you now go through the math by putting in all these boundary conditions, and I already shown it for the velocity boundary layer there is no reason to show it once again reiterate it once again for the temperature boundary layer, that how you actually find them right, but you can readily see depending on our boundary conditions, this is the T wall temperature, 1 minus 3 by 2, y by delta T very similar to the velocity boundary layer except that that it is a one minus because of the opposite nature of the slope right. So, that is what you get right if you plug in all the numbers right ok.

Now, if delta is equal to delta T the velocity solution is valid within the thermal boundary layer right is valid within the thermal boundary layer that was the assumption that we made initially also because it is a thick velocity boundary layer. So, whatever the velocity solution was is kind of imposed on this as well. So, what we need to do now is that we need to pass on this temperature profile, inside the full temperature that we got here. So, call this equation a right that is where we need to substitute and we already have the velocity expression ironed out right because your u was already kind of known right. So, it is not very difficult to do this ok.

So, we are not going to go through the whole math steps for that, we can just write down some of the expressions that we have. So, if you evaluate all these integrals and other things what you will get is as follows d by dx u infinity delta T delta let us put a bracket here or let us make this bracket like it. So, that this can become now this is delta T by delta square minus 3 by 220 delta T by delta to power of 4 close this bracket is equal to 3 by 2 alpha by delta T that is all that you get ok. So, that is all that you get after you substitute these expressions and if you evaluate.

So, basically you have to put in u by u infinity that we already found out and this particular guy over here to the previous expression and you just need to go through the motion now of course, here there is a problem because we have now 2 variables delta T and delta, to make things a little bit more dicey right. So, in order to avoid that, we already know that delta is greater than delta t. So, it is very natural if you compare these 2 expressions for example, right which is delta T by delta we know that the fourth order term will be much much lower right than this a this is 3 by 20 that is 3 by 280. So, there is one order difference right there right and on the top of that this is square, this is to the power of 4. So, naturally there will be more such variations over there. So, you can safely assume that 3 by 220 delta T by delta by delta is much much lower than delta T by delta to the power of 4 is much much lower than 3 by twenty delta T to the power of delta square ok.

So, that is already an assumption based on the fact that delta is greater than delta T. (Refer Slide Time: 14:49)



So, based on that So, this simplifies equation simplifies 10 alpha over delta T is equal to u infinity d by dx delta T by delta square close bracket, we already know that delta is given by 280 by 13 gamma x by u infinity that comes from your velocity or slash momentum analysis that we did and of course, the other boundary condition is that delta T at x equal to 0 is equal to 0. So, that is the starting point. So, the solution of this particular expression becomes delta T by x is 4.528 into prandtl number to the power of one third Reynolds number to the power of x to the power of half. So, that is the expression that we get delta T by delta.

Now, we already know Nusselt number is defined as hx by k and h is defined as k just to recap vty equal to 0 divided by ts minus or T wall minus T infinity Ts right that we already knew right. So, now, it becomes a very simple thing that hx then becomes 3 by 2 k by delta T all right. So, our h x becomes 0.331 into k into x prandtl number to the power of one third Reynolds, number to the power of x to the power of half right and nusselt number becomes 0.331 it is the same remains the same prandtl number to the power of one by third Reynolds number to the power of half right.

So, this is actually very close to the error is actually less than 2.4 percent. So, you can imagine that it is a very robust way. So, all we have done is that we have got this expression and then we just substituted for delta and then we just integrated the whole thing which gave us this particular form, which preserves once again that prandtl number to the power of one by third Reynolds number to the power of half dependence and factor sits in front the error is less than 2.4 percent compared to the similarity right. So, I think it is a very powerful tool as you can see, without doing much of a thing we are able to get this perfectly fine result right using just this kind of analysis.

Now, we will this is we did for the constant wall temperature, we can also do the same for the constant wall heat flux. I am not going to do the full thing, but what I am going to do is then I am going to post the whole thing and it will be left up to the students to follow it up in the form of an exercise ok.

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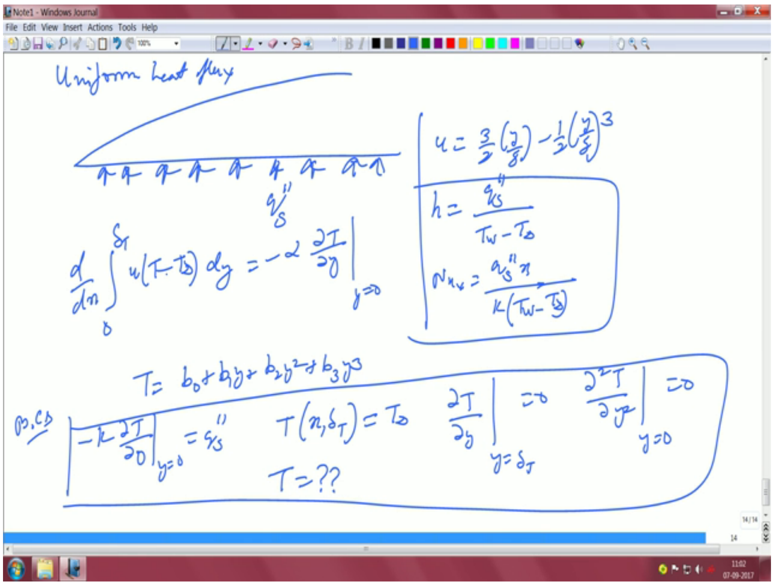

So,. So, uniform heat flux that would mean that, it is the same plate the same type of boundary layer is growing except that instead of a temperature, fixed temperature you constantly give a constant heat flux q so; that means, you have put it say for example, on an isope on a heater and you and the heater is connected to a power supply which supplies a constant heat, the previous case you can consider it more to be you are placing it in an isothermal path right. So, that you have keeping the plate no matter what the temperature is always kept at a constant level right, that it is always in equilibrium with each other. So, this is how you are transferring the heat it can be anything, it can be qs double prime say for example, is the heat ok.

So, once again the integral formulation once you write it, that does not have any such problem right. So, you can easily write the same thing in the same way that you did earlier 0 to delta T u T minus T infinity dy right that is what you wrote earlier, d T by dy evaluated at y equal to 0 right this expression still remains the same there is no problem with this your velocity profile also remains the same. So, I am giving you the hints the velocity profile still remains y delta minus half y over delta cube right all this thing remains the same got it. Only the expression for h now has to be recast in a certain way. So, instead of that we are writing it in this particular form wall minus T infinity, and Nusselt number is q s double prime into x divided by k, T wall minus T infinity right. So, these are the 2 expressions slight change because of the constant heat flux condition ok.

You can once again assume the temperature to be the same b naught plus b 1, if you do not want to write it in abcd you can write it in this way also, y cubed etcetera the boundary conditions now are k d T by dy, at y equal to 0 is equal to q s double prime the first boundary condition. Second boundary condition is at any x and delta T it is equal to T infinity which is also not difficult to once again d T by dy at y equal to delta T is equal to 0 and d square T by d y square at y equal to 0 is equal to 0 right. So, these are the boundary conditions the 4 boundary conditions that we have. So, applying. So, what you need to do is follow the motions after this that you need to apply the boundary conditions ,to find out what the temperature profile is going to look like. Once you get the temperature profile it can be easily substituted now into this T minus T infinity expression and you can find it.

So, I will just give you the temperature profile finally, before we wrap up. So, what we can see over there is that.

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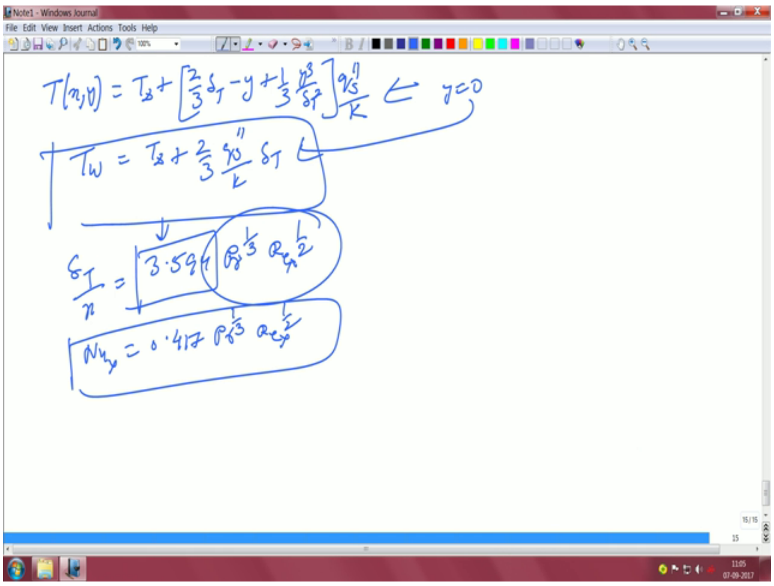

If you substitute all these things the temperature profile will look T infinity plus 2 by third delta T minus y plus 1 by 3 y cube by delta T square qs double prime by k that is the expression that we get ok.

Now, surfaces set if you put y equal to 0, the T wall temperature becomes T infinity plus 2 third qs double prime y k into delta T this is the wall temperature if you put y equal to 0 here you will get this right. So, based on these 2 expressions now you can you are in a position right because your wall is now taken care of, you know what is your wall temperature going to be. Now you have to all that you have to do is there to substitute this expression in the parent equation, and try to see that what will be the final number right. So, the final expression if you manage to do this properly will be 3.594prandtl number one-third Reynolds number half. So, the coefficients do change a little bit the coefficients do change a little bit, but this variation still remains the same this of course, remains is a different number now right. So, that is your delta so; that means, when you invert it you are going to get a different expression a lower expression for you Nusselt number right right. So, based on this your Nusselt number should be if you do it properly it is going to be 0.417 and. So, the higher value, Reynolds number to the power of half. So, the Nusselt number will be a little higher because this coefficient is a little lower. So, that is the reason why you should have your nusselt number a little higher. So, what we have done? We have shown that for the 2 sets of conditions constant wall temperature and constant wall heat flux it is possible to cast the expression in a very similar way right.

So, let us just recap what we just did, let us take the momentum equation what we have done is that we saw the momentum equation right what we did is that we integrated out the y variation completely using Leibnitz rule, and using the pre existing boundary conditions we integrated out the whole thing right. As soon as we you are able to integrate out the whole thing we got an expression which has only dependent on x right. So, there we substituted it and solved we assumed a velocity profile, a velocity profile which depends on y and then once we did the integration we found that what will be the delta; that means, the boundary layer thickness. Once we knew delta c f and all the other parameters we can determine it very easily.

Now, coming back to the temperature profile a basic assumption is that weather is delta is more than delta T which is the most common type of fluid that we see, that delta is more than delta T using that expression what we did we started with the energy equation substituted the velocity assumed a profile for temperature again integrated the whole thing out and finally, found out how delta T varies with respect to x. And once delta T is known as we know the h the heat transfer coefficient is inversely proportional to delta T.

So, we were able to knock out the heat transfer coefficient and Nusselt number is nothing, but the heat transfer coefficient minus the it is a non-dimensional number. So, we got the correct expression for that too. So, it is a very simplistic approach by which we have assumed central pieces that we have assumed polynomial, for both velocity as well as for temperature right.

And this polynomial made sense because we were able to satisfy all the boundary conditions. In fact, that is how we found out the coefficients of the polynomial and we found that the results that we got varied by less than 5 percent right with the actual similarity transformation, which was very difficult which involved a lot of math and other things is a very simple integration just a straightforward integration, substitute the values of the y; that means, a variation with y integrate out with respect to y and you get an expression which is a function of x only ok.

So, in the next class what we are going to do we are going to, look at some of the special cases which we alluded here that they are going be suction and blowing that can be something called a wedge which seems to be a little different from what the flat plate boundary layer is. So, we kind of just touched upon that next class what we are going to do we are going, to look at the general case of suction and blowing and after that we are going to look at a new class of solutions which is called Falkner Skan which is basically valid for this wedges those wedge inclined plates and things like that where the free stream velocity is a function of x so; that means, there will be a pressure gradient within the boundary there. So, see you next class.