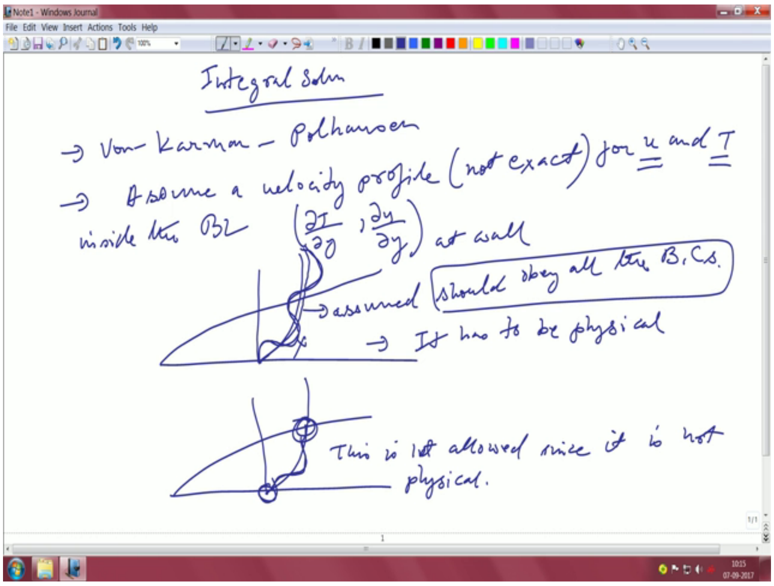
Convective Heat Transfer
Prof. Saptarshi Basu
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture - 10
Integral Solutions- Momentum

So, in the last class, what we did was that, we showed that what the similarity transformation was and how it can predict the right result; at the same time we showed that the scaling was able to predict the correct variation. Now we also suggested in that particular lecture that if it is just the value of the gradient at the wall, be it temperature be it velocity that is all that we need for evaluating what will be the heat transfer coefficient and the skin friction, why resort to all these things let us device something which is a little bit more simpler right, and that is how we suggested that maybe we can have a semi analytical type of an approach ok.

(Refer Slide Time: 00:59)



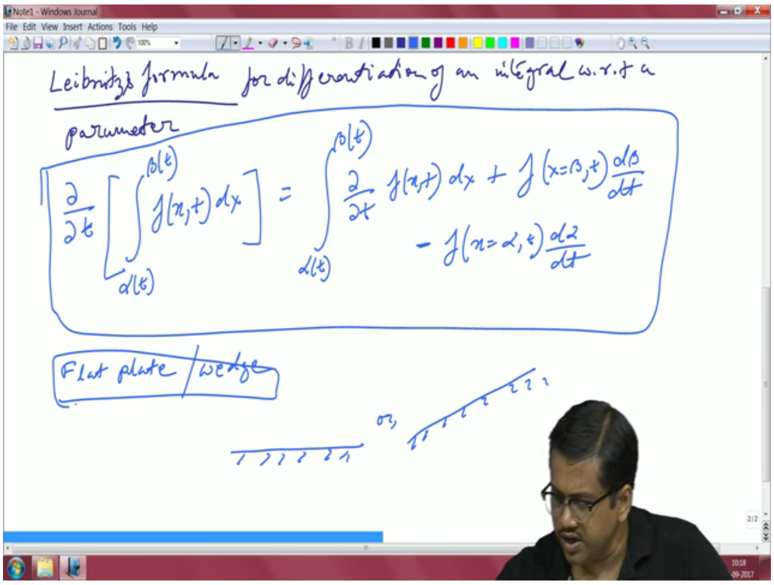
So, based on this we move to something called the integral solution. So, this was like Von karman and Polhausen. So, the main building block of this particular approach is assumed a velocity profile, which is not exact because we do not know without the actual solution what the velocity profile would look like.

So, assume a velocity profile for u and t basically right inside the boundary layer thermal or otherwise. And because this this should work because all we need to kind of be concerned are variables like these right at wall all right that is our main concern. So, why not assume something for u something for t such that so; that means, if the profile is like this right the normal boundary layer is like that correct that is what we drew. So, if I can you know draw inexact profile something like that which kind of resembles this, but it does not quite it is not exact because we do not know the form, but if this boundary profile whatever is that as assumed profile. So, to say should obey should obey all the boundary conditions right. So, I cannot have a profile which is like this.

Say for example, that would not be physical, that will not for example, agree with the boundary conditions either. So, one is that it should obey all the boundary conditions and the second thing is that it has to be physical in the sense that it has to obey certain physical characteristics like for example, this profile that I drew over here this curved kind of a wiggly pattern right you know that this profile is not physical because why a profile should be like that right because the high velocity low velocity it cannot exist in a steady flow like, this which is laminar in nature correct there is no mechanism by which why this should happen. So, even though this profile might satisfy the boundary conditions this is not a physical profile ok.

So, let us take this more carefully, because I want you guys to get the car. So, this is the actual profile say for example, I draw a profile like this which kind of agrees you know with the boundary condition here, with the boundary condition there correct, but in between it has got all these wiggles. So, this is not allowed, this is not allowed since it is not physical right that is the most important part, that it has to make common sense right and there is no mechanism through which this flow will make something like this even though the boundaries are still satisfied. So, these are the 2 prime parameters that we need to get a feel of one is that it has to obey the boundary condition second is that it has to obey the physics ok.

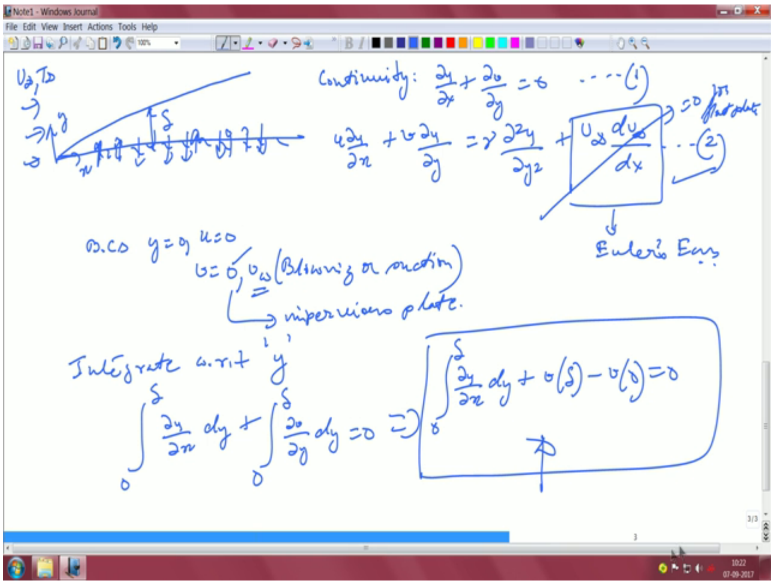
(Refer Slide Time: 04:40)



Let us look at let us before we do this let us put down one particular form which is called the Leibnitz formula. So, Leibnitz formula for differentiation for differentiation of an integral with respect to a parameter parameter. So, we change the pink colour to something like this. So, let us take this particular form, and this will come in very handy this is the generalized form. So, pay attention to the details. So, that is the form that is the first 1 plus minus that is the Leibnitz formula. So, you can see that how this how this will come in handy we will see in a second, but this is the form of the Leibnitz formula ok.

Now we are going to need this when we actually do this an integral formulation. Now let us take the case of let us go back to our problem, let us take the case of a flat plate it can be a wedge also the wedge part we will do the similarity a little later. So, flat plate will be like this, wedge will be something like that; that means, an inclined flat plate whatever it is. So, let us take that particular form and let us try to work out the equations ok.

(Refer Slide Time: 07:05)



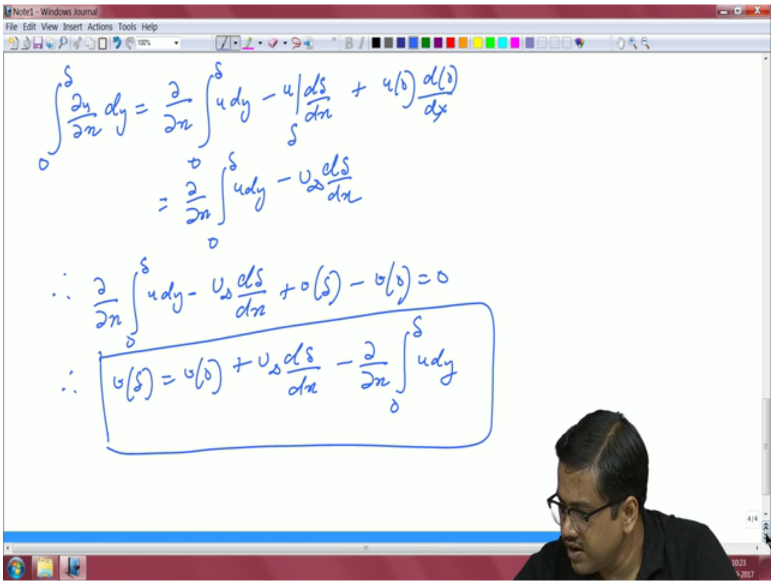
So, it is a standard this is the flow u infinity t infinity, this is x and y, this is the corresponding delta the thermal analysis we will go a little later. So, the continuity equation is du dx plus dv dy is equal to 0 this is 1 the x momentum is du dx plus du dy. I will tell you the origin of the last term this is 2 this origin of the last term is comes basically from the pressure gradient right this is basically nothing comes from the Euler's equation correct outside the boundary layer.

So, for a flat plate this is equal to 0 right or a flat plate it is equal to 0 for flat plate, but when you are dealing with a plate which is inclined this would not be equal to 0 right. So, this is basically the variation of the free stream velocity with x that is all that is right and that comes from the Euler's form. So, the boundary conditions we already know y equal to 0, u equal to 0, v can be equal to 0 it can be also equal to some other quantity which is called a blowing or suction, we will cover this a little later. But it is like if this plate is kind of porous; that means, there is a flow coming up like this that can be accounted for by v w or if the flow is porous and the fluid is actually seeping through. So, that can be given as v w. So, it is normally will be 0 for impervious plate, we do all these special cases later I am just writing it in a common way. So, that later on we can use that we do not have to come back to the integral formulations once again, but most of these terms are 0 as I said this is 0 this is also equal to 0, but there are special conditions as I say, that in some cases there can be a flow that is suction or blowing right or there can be this flow outside the boundary layer may actually vary.

So now, what we do is that we take the continuity equation and we integrate it with respect to x oh sorry with respect to y, because that is the y variation we are trying to take out. So, du dx dy plus 0 to delta dv dy into dy is equal to 0 right. So, that is the form that we have this leads to 0 to delta, du dx dy plus v delta minus v 0 is equal to 0 very simple right that is the form.

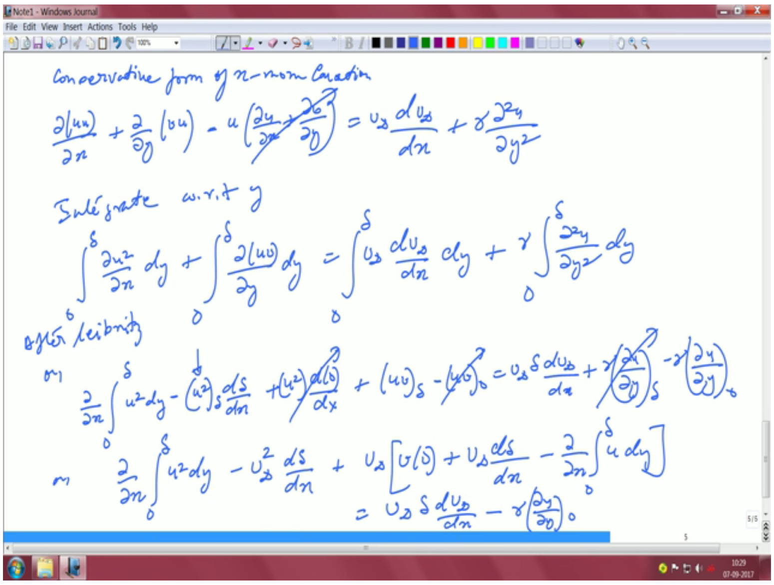
Now, we are going to apply Leibnitz rule on this, we already established the Leibnitz rule now we are going to apply it first to our continuity equation ok.

(Refer Slide Time: 10:41)



So, by Leibnitz rule this will be du dx dy will be u dy minus u at delta d delta by dx plus u 0 d 0 dx right that is a lower boundary 0 is basically the lower boundary right. So, this gives you d by dx 0 to delta u dy minus u infinity d delta by dx of course, this goes down to 0 the second the yeah the third term. So, therefore, now combining the full continuity equation therefore, there is a full form that you get. Now of course, v equal to v at 0 you can put whatever values that you want we and delta you can put, whatever values you want and all these things are incorporate. This is the most generic form of the equation that we can write got it ok.

(Refer Slide Time: 12:32)

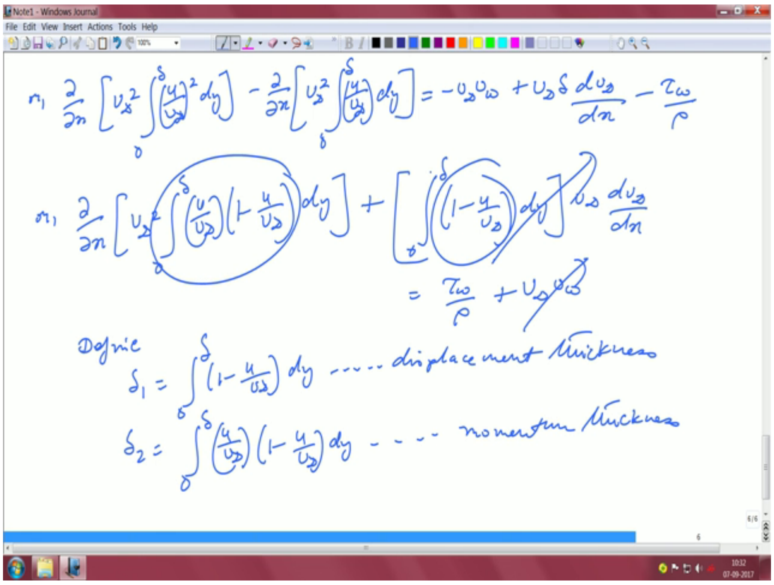


So, now that we have taken care of continuity, let us move on to the momentum. Let us write the momentum equation in a conservative form this basically goes to 0, I am putting the whole thing. So, that one it is easier for one to understand that how we are proceeding throughout the problem right. So, that is the whole form of the equation again integrate with respect to y. So, this will come as this got it that is a full form now again you can apply your Leibnitz I am not going to go through the steps, but you can apply your Leibnitz accordingly. So, the form that you are going to get after Leibnitz, full version of this full equation ok.

So, you can see a few things over here, which we can eliminate and some of those things should be pretty common sense as well that what are the parameters that we can eliminate. So, let us see that which are the parameters that we can safely eliminate. If you take your pick say for example, few things to note what is u square at delta that must be u infinity square right u and delta is actually equal to u infinity square right. Similarly this particular parameter; obviously, will go to 0 we know that for sure right uv, whatever is the nature of v even if there is suction or blowing u is equal to 0 always. So, that should go to 0 at the same time of course, this parameter will stay depending on what you want your uv to be and or will it stay we will see to that in a little bit ok.

And of course, on the other hand if you go to this side of the equation, you will find that what about the nature of for this should this go to 0 as well. Because you are looking at the shear stress at the edge of the boundary layer right where the flow the velocity should have actually become equal to u infinity all that gradient is very small, that is what we are arguing. So, or therefore, we can write this equation in a more nicer form this part remains the same plus u infinity, if you take the u infinity u v, v is equal to actually u infinity. So, this is will be given as v 0 plus u infinity dx and a d infinity dx minus du dy is equal to infinity delta into du infinity by dx, minus gamma du dy evaluated at 0 right. So, that is the total expression that we get ok.

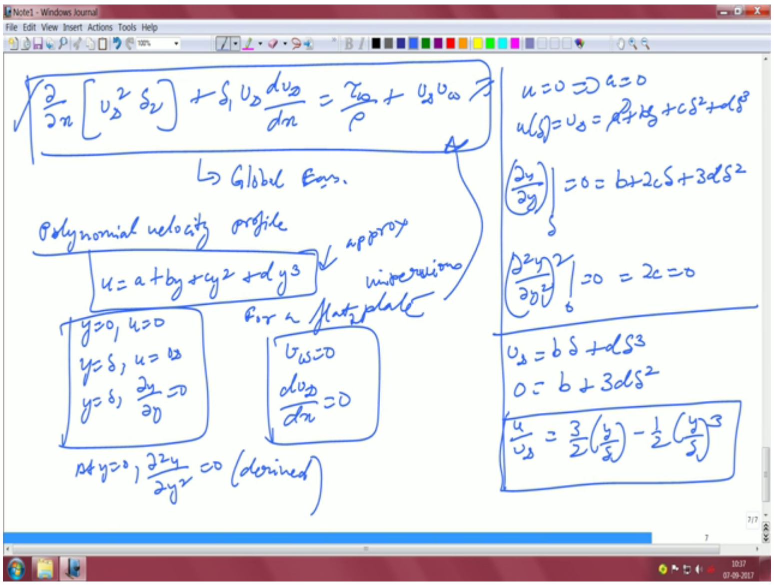
(Refer Slide Time: 17:38)



So, now doing some more algebra taking u infinity square out 0 to delta u by u infinity square in dy minus d by dx squared 0 to delta u by u infinity dy is equal to minus u infinity, this is whatever that v w was suction blowing it can be 0 also, suction blowing or 0. So, that is what we are arguing over here, plus u infinity delta du infinity by dx minus tau by rho. So, that would be the other part of that term. So, the tau wall minus row is basically nothing, but du by dy. I evaluated at the wall right. So, this is the total expression that you get and simplify this by a little bit u by u infinity dy plus got it now. So, this is the total form of this equation, now in many cases you will find that this will go to 0 this will go to 0. So, that will be the form that we will be left with in the absence of any suction and blowing or in the absence of any free stream variation in the free stream velocity right.

Now, based on this we can define basically 2 parameters one is called delta one which is basically this dy, this is got a name it is called the displacement thickness, may have heard about it in your fluid mechanics course right. So, this is displacement thickness the other one is basically what we call the momentum thickness. So, the displacement thickness and the momentum thickness. So, these are 2 parameters one basically represents this one basically represents that right ok.

(Refer Slide Time: 20:45)



Now, putting these 2 terms in this is the global equation got it. So, this is the global equation right. So, we have reduced the differential equation to an integral form. So, far this particular equation is exact there is no problem we have not assumed anything. So, if you take a differential equation you can arrive at this form without much of an assumption. Whatever is the boundary layer assumption that is the still valid except that we have not assumed any velocity profile as of now we are going to do that ok.

So, let us assume a polynomial velocity profile why a polynomial velocity profile? Because it should be able to satisfy the boundary conditions that are at hand and a polynomial kind of can represent without any kink the boundary layer velocity profile right. So, it can be other profiles also by the way. So, long as it makes physical sense. So, this is a common polynomial profile abcd typical third order polynomial. So, y is equal to 0 at u equal to 0 right and y at y equal to delta u is equal to u infinity, at y equal to delta du by dy is equal to 0 right these things we know right we already know these parameter space ok.

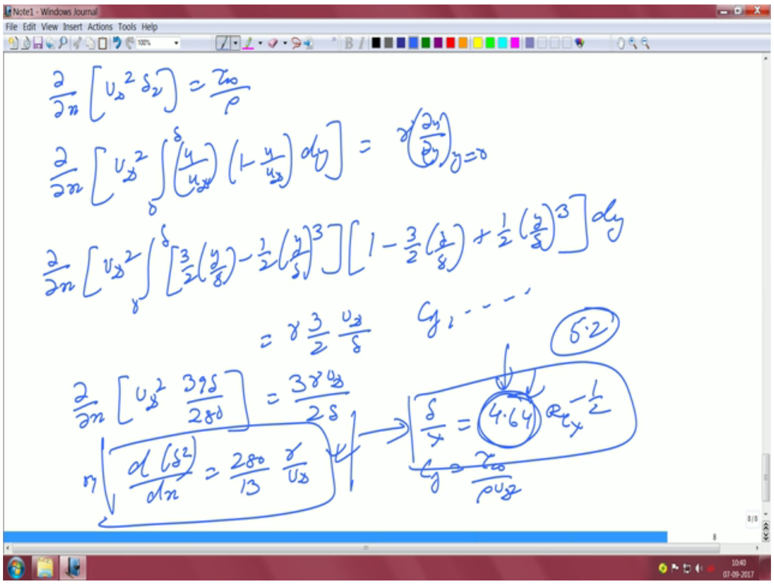
So, if we now try to put at y equal to 0 what happens and for a flat plate before we go to that for a flat plate, this up this equation gets revised a little bit because your vw is equal to 0, and du by dx is equal to 0 also right flat impervious plate right. So, for these 2 plates this is added assumption can be done ok.

So, now the point is very simple, let us now derive that what will be the values of these coefficients because that is what we need to plug in right we need to plug in over there. So, the we need a fourth boundary condition because there is a b c d right. So, we need one more boundary condition over here for v. So, for that particular case at y equal to 0 you are assuming that d square u dy y square is equal to 0, this is called a derived boundary condition. So, based on this let us work out the profile.

So, from u equal to 0 leads to a equal to 0 right u delta equal to u infinity leads to a plus b delta of course, a is being equal to 0 plus c delta square plus d delta cube d delta cube that is u infinity and write du by dy at delta is equal to 0. So, that would imply b plus 2 c delta plus 3 d delta squared is equal to 0 say thirdly dy square and 0 is equal to 0 implies 2 c is equal to 0. So, based on all these things accumulating all these parameters, we have u infinity is equal to b delta plus d delta cube got it, and the other one is 0 equal to b plus 3 d delta square right. So, you can solve for b and delta from these expressions.

So, therefore, u by u infinity becomes 3 by 2 y by delta minus half y by delta cube the velocity profile that we get assuming that this is the parent equation we have taken care of 2 of the terms and this is the velocity profile that we have assumed right. So, this is approximate that is exact. So, this is approximate and this is the velocity profile that we get not it.

(Refer Slide Time: 25:48)



So, for no suction and blowing particular case and for a flat plate this is the equation. So, therefore, if we write taw wall by a row that will be the form right of the expression. So, now, if we put this thing together, this is basically what we wrote earlier remember u by u infinity dy plus du dy at y equal to 0. So, now, you plug in all the parameters as u infinity square you know to delta 3 by 2 y over delta minus half y by delta cube, that is the first term the second term is 1 minus 3 by 2 y over delta plus half y by delta cube right into dy, that is equal to 3 by 2 u infinity by delta all right. So, now, it becomes du by dx u infinity square 39 delta by 280 is equal to 3 comma u infinity by 2 delta or d delta square by dx equal to 280 by 13 gamma by u infinity right ok.

So, now it is very easy because delta only depends on x that we already know. So, in this particular case if you do this you will get delta by x equal to 4.64 into Reynolds number x to the power of minus half; C f will be given by tau wall by p u infinity square right. So, you see that by using the simple enough express we have got this result which is 4.64 right. Remember the exact solution gave us something like 5.2 even if you recall the graph. So, we are not way off; we are off by about 10 percent or below in this particular case right. So, that is what we get.

So, just by using this approximate analysis we got an answer which is pretty close to the actual value. So, this proves that the integral approximation is not a bad idea after all ok. So, based on this what we have shown, that your delta you have to after doing all these things all we need is ultimately that expression for delta right. Once we determine delta everything right C f and everything are basically dependent on delta right. So, that is the thing that we have found out, next class we are going to go and to do the integral formulation for the energy equation.

Thank you.