Heat Transfer And Combustion in Multiphase Systems

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Lecture 09

Interfacial heat and mass transfer-III - Evaporation from thin films

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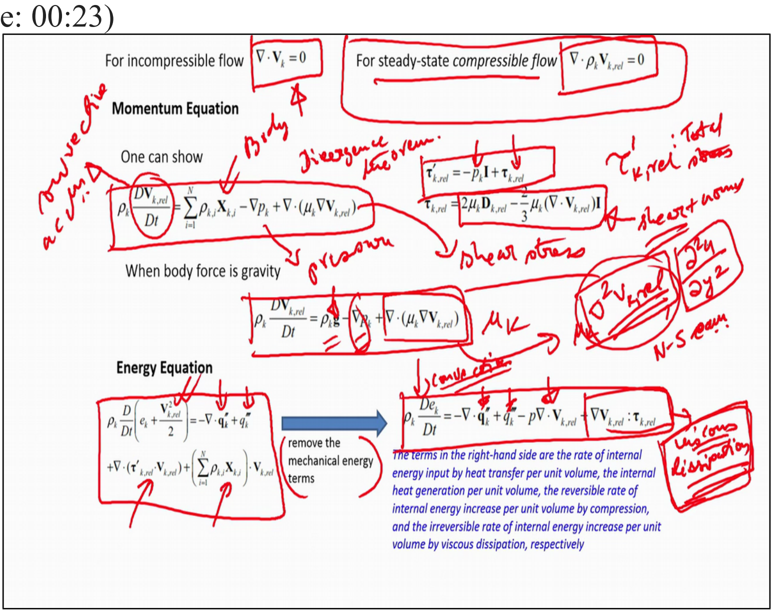

For incompressible flow $\boxed{\nabla \cdot \mathbf{V}_k = 0}$

For steady-state *compressible flow* $\boxed{\nabla \cdot \rho_k \mathbf{V}_{k,rel} = 0}$

Momentum Equation

One can show

$$\rho_k \frac{D\mathbf{V}_{k,rel}}{Dt} = \sum_{i=1}^{N} \rho_{k,i}\mathbf{X}_{k,i} - \nabla p_k + \nabla \cdot (\mu_k \nabla \mathbf{V}_{k,rel})$$

$$\boldsymbol{\tau}'_{k,rel} = -p_k\mathbf{I} + \boldsymbol{\tau}_{k,rel}$$

$$\boldsymbol{\tau}_{k,rel} = 2\mu_k\mathbf{D}_{k,rel} - \frac{2}{3}\mu_k(\nabla \cdot \mathbf{V}_{k,rel})\mathbf{I}$$

When body force is gravity

$$\rho_k \frac{D\mathbf{V}_{k,rel}}{Dt} = \rho_k\mathbf{g} - \nabla p_k + \nabla \cdot (\mu_k \nabla \mathbf{V}_{k,rel})$$

Energy Equation

$$\rho_k \frac{D}{Dt}\left(e_k + \frac{\mathbf{V}_{k,rel}^2}{2}\right) = -\nabla \cdot \mathbf{q}''_k + q'''_k + \nabla \cdot (\boldsymbol{\tau}'_{k,rel} \cdot \mathbf{V}_{k,rel}) + \left(\sum_{i=1}^{N} \rho_{k,i}\mathbf{X}_{k,i}\right) \cdot \mathbf{V}_{k,rel}$$

remove the mechanical energy terms

$$\rho_k \frac{De_k}{Dt} = -\nabla \cdot \mathbf{q}''_k + q'''_k - p\nabla \cdot \mathbf{V}_{k,rel} + \nabla \mathbf{V}_{k,rel} : \boldsymbol{\tau}_{k,rel}$$

The terms in the right-hand side are the rate of internal energy input by heat transfer per unit volume, the internal heat generation per unit volume, the reversible rate of internal energy increase per unit volume by compression, and the irreversible rate of internal energy increase per unit volume by viscous dissipation, respectively

convective acc^n

Body

Divergence theorem.

pressure

shear stress

$\tau'_{k,rel}$: Total stress

shear term

μ_K

$\mu_K \nabla^2 V_{k,rel}$

$\frac{\partial^2 y}{\partial y^2}$

N-S eqn.

convective

viscous dissipation

So, welcome to this lecture, last class what we did if you recall that we went from the integral formulation to the differential formulation for and we did it for the continuity equation just to show that how it actually reduces to the most common form that you are familiar with like the divergence of velocity is equal to 0 for incompressible flow. For steady-state compressible flow it is a little different okay.

Now let us take a look at the momentum equation so I am not going to go into the details of the momentum equation what we can do is that we can apply similarly the divergence theorem okay. And we can show that this is the final form of the momentum equation, right. Now here we have just done two things the momentum equation.

If you look at it this is still the material derivative of the velocity like we explain the material derivative in the last lecture okay. So, this is basically nothing but the convective acceleration okay. This is the body force term that we already know okay. This is that this is the pressure and this is basically nothing but the sheer stress term.

Now this you did not see earlier because we will only define something called Tau prime k relative, right. So, that we say it was the total stress if you recall, right. So, this total stress is nothing but a combination of the pressure okay that is an identity and the corresponding shear stress component, right.

So, the shear stress component is given by this expression okay. So, you can show, so, this is nothing but the sheer or the sheer and the normal stresses okay and this is the pressure okay. So, when the body force is gravity this xk becomes g okay. So this is the most familiar thing that you are that you have seen.

So, nothing but the convective derivative of the velocity is equal to the weight or the gravity term the Laplace of the pressure and the corresponding shear stress term if mu k is constant okay, then this act this come actually becomes okay, so, sorry this is not a Laplace ,this is the corresponding Laplace of the velocity. That is the grad of the pressure field okay.

So, the pressure is a grad of the pressure field here it is the Laplace if mu k is constant. So, mu k you can actually take it out, got it. So, this is very simple, this is the corresponding Navier-Stokes equation that you are familiar with, right, okay. Similarly let us take a look at the energy equation now. So, the energy equation is something like this if we apply the similar divergence theorem okay.

So, once again what terms we have? We have the corresponding conduction term the heat generation term okay and the corresponding work that is done by the body forces and the work that is done by the sheer or the total stress, right. This if you remove the mechanical energy term from this energy equation that means you just take a dot product with the momentum equation.

A velocity dot product to the momentum equation and subtract it out because we are not interested in the kinetic energy component that much, right. So, you can end up with this particular expression okay where this looks exactly similar to this all the terms are still the same okay, except that if you look at the right hand side, this is this is nothing but once again the convection, it is always the conviction okay.

So, now the can or the convection, advection whatever you call it. So, that is the left hand side of the expression. The right hand side term however if we look at each and every term one is the heat transfer by conduction or diffusion mechanism. Once again this is the total energy that is actually internal energy generation in many cases it can be actually equal to 0.

And then the reversible rate of internal energy increased by compression that is given by the pressure term okay. So, if the divergence is equal to 0 this term will also be equal to 0 okay.

So, it is like the work that is done by the pressure essentially by compression okay. And the last term is basically nothing but this particular term is nothing but something called viscous dissipation.

In many cases this will be also equal to 0 but viscous dissipation is an important quantity that you may have may not have encountered, so far. It is basically, so, take an example like this and this is an experiment that you should not do okay, well it is just a thought experiment okay. Say if you have your fingers, if you can just look at my fingers.

So let us take our like an air hose okay. So, and let us put that airflow through these two fingers okay like this okay you would not feel anything on your fingers, right. Here you might feel a little bit of a tingling and along the edges but nothing bad happens okay. If the fingers are far apart and if the pipe diameter is small enough you do not feel many things okay.

But if I make the finger sufficiently close like this okay and I take a hose pipe and I try to purge the flow through that gap between these two fingers okay. You will feel the burn, you immediately or your region over here is going to be burnt okay. Why is that burn? That is that is happening because of this viscous dissipation. So, why viscous dissipation is important?

This is basically the work done by the shear stress, correct. So, the work done by the shear stress is a lot higher okay, when these two fingers which basically represent say if you think in the analogy of two parallel plates. These two fingers are like two parallel plates. So, naturally the heat that is generated when the flow goes through this finger, these two fingers of mine it actually generates a lot of heat.

But when I spread the fingers out okay, the shear stress is very low okay. So, the viscous dissipation is important especially if we have a very high velocity flow going through a constrained space okay. So, it is a function of the shear stress and the shear stress what it is a function we just saw in this particular thing. It is just basically the derivative, right, of the velocity, right.

So, how do we write it? If you something like this, correct, okay. So, if you look at it; look like a typical component. So, this is basically the change of velocity with some distance, right. So, if this distance is very small and the change of velocity is high you can generate a lot of shear. If you generate a lot of shear your viscous dissipation can be a lot higher as well, right.

So, the work done by viscous dissipation can be high so this can become a formidable component as the example that I showed you with my finger but do not try to do this experiment. You will actually burn your finger okay.

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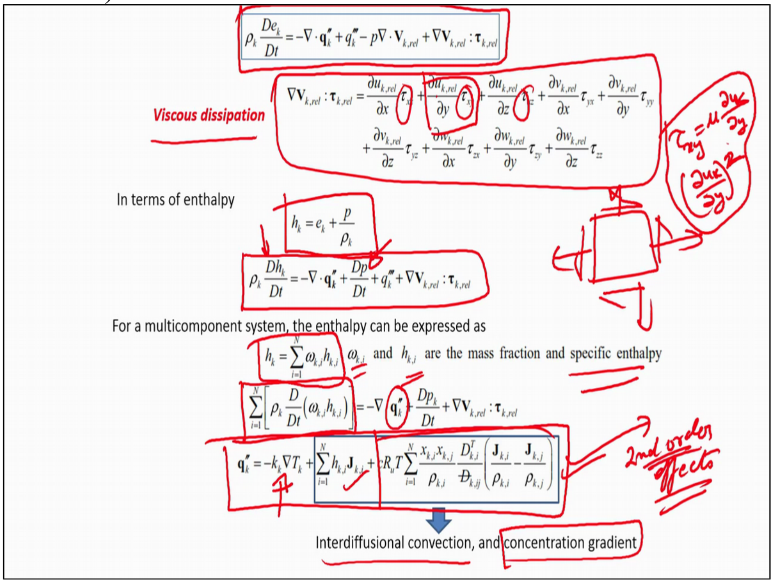

$$\rho_k \frac{De_k}{Dt} = -\nabla \cdot \mathbf{q}_k'' + q_k''' - p\nabla \cdot \mathbf{V}_{k,rel} + \nabla \mathbf{V}_{k,rel} : \boldsymbol{\tau}_{k,rel}$$

Viscous dissipation

$$\nabla \mathbf{V}_{k,rel} : \boldsymbol{\tau}_{k,rel} = \frac{\partial u_{k,rel}}{\partial x}\tau_{xx} + \frac{\partial u_{k,rel}}{\partial y}\tau_{xy} + \frac{\partial u_{k,rel}}{\partial z}\tau_{xz} + \frac{\partial v_{k,rel}}{\partial x}\tau_{yx} + \frac{\partial v_{k,rel}}{\partial y}\tau_{yy} + \frac{\partial v_{k,rel}}{\partial z}\tau_{yz} + \frac{\partial w_{k,rel}}{\partial x}\tau_{zx} + \frac{\partial w_{k,rel}}{\partial y}\tau_{zy} + \frac{\partial w_{k,rel}}{\partial z}\tau_{zz}$$

$$\tau_{xy} = \mu \frac{\partial u_x}{\partial y}$$

$$\left(\frac{\partial u_x}{\partial y}\right)^2$$

In terms of enthalpy

$$h_k = e_k + \frac{p}{\rho_k}$$

$$\rho_k \frac{Dh_k}{Dt} = -\nabla \cdot \mathbf{q}_k'' + \frac{Dp}{Dt} + q_k''' + \nabla \mathbf{V}_{k,rel} : \boldsymbol{\tau}_{k,rel}$$

For a multicomponent system, the enthalpy can be expressed as

$h_k = \sum_{i=1}^{N} \omega_{k,i} h_{k,i}$ $\omega_{k,i}$ and $h_{k,i}$ are the mass fraction and specific enthalpy

$$\sum_{i=1}^{N}\left[\rho_k \frac{D}{Dt}(\omega_{k,i} h_{k,i})\right] = -\nabla \mathbf{q}_k'' + \frac{Dp_k}{Dt} + \nabla \mathbf{V}_{k,rel} : \boldsymbol{\tau}_{k,rel}$$

$$\mathbf{q}_k'' = -k_k \nabla T_k + \sum_{i=1}^{N} h_{k,i}\mathbf{J}_{k,i} + R_u T \sum_{i=1}^{N} \frac{x_{k,i} x_{k,j}}{\rho_{k,i}} \frac{D_{k,i}^T}{D_{k,ij}} \left(\frac{\mathbf{J}_{k,i}}{\rho_{k,i}} - \frac{\mathbf{J}_{k,j}}{\rho_{k,j}}\right)$$

2nd order effects

Interdiffusional convection, and concentration gradient

So, that essentially then the entire equation boils down to this particular form. This is the differential form all the terms we have explained earlier, viscous dissipation if you can write it in a Cartesian coordinate system, right; cylindrical it will be a little different. You can see, it is basically, these are basically, this is the normal stress, this is the shear stress, this was the shear stress in x, y, z plane.

You all know the these are the basically sheer, right, okay. And these are the basically the normal stress components right, okay. So, this is the total expression that you have. Now if you look at this particular expression tau, say for example let us take tau xy. Now tau xy is what at per Newton's law something like this right, okay.

The constitutive relationship that you all know, right, so, now when it is this is basically uk because that is how we are writing it. If you look at this term and this term okay. This is one of the viscous dissipation components, right. So, what this will give you is basically it will give you uk dy square, right. So, that means it does not depend on what is your gradient it can be a positive gradient, negative gradient whatever it is.

Like velocity can go up or down whatever it is. That it is the square of that particular quantity is always positive, right. So, the viscous dissipation term throughout this expression is always a positive number okay. So, that is what happens because of the square nature of this and this you can show it for the other terms of these always a square okay.

In terms of; now this is the expression that has been written in terms of the internal energy of the system, right. So, the internal energy of the system is ek. In terms of enthalpy and why we are writing in the in terms of enthalpy there is a logic that you will find soon. In terms of enthalpy if we write enthalpy is nothing but this ek + p by Rho k is the definition of enthalpy.

You put that same definition over here you basically have the same material derivative of enthalpy okay. In addition you have a material derivative of pressure rest of the terms do not seem to be get affected. Now for a multi-component system if we do a multi-component system like the one that we did in the last lecture.

You can cast the same enthalpy in terms of the mass fractions and the individual specific enthalpies of each of the components. So, it is basically like a sum total, so, if the total mass is something like 1 omega ki is basically the mass of the ith component in the kth phase okay divided by the total mass okay. So, this is the mass fraction okay.

So, mass fraction a multiplied by the specific enthalpy of that particular component, right. So, it will be just summation okay. The left hand side will be just the summation of the each individual enthalpy components or each individual component okay, the sum total of across all the components over there, right.

So, for that this qk double prime that you see over here which we say is a conduction term. It is basically a little bit more complicated than that. If you look at this expression over here the first term you recognize immediately that is like the Fourier's law, right. That is a Fourier's law of heat conduction, correct.

But you do not know much about the second and the third term, right. The second and the third term is basically the heat transfer due to inter diffusional convection that the first term over there and also the heat transferred that due to the concentration effect okay, as you know that concentration can be transferred by temperature gradient.

Similarly heat can also be transferred by concentration gradient but these are strictly, these two components are very strictly second order effects. And most of the cases they are negligible, right, okay. But however many of the equipment that we use actually work on the second order effect, right.

One example of that is basically your thermoelectric okay thermoelectric that is a second order effect essentially you also have like thermoelectric like healthier cooling and things like that these are all second-order effects that also thermocouple one of the most important things that you use that is also a second order effect, right.

So, you have actually second order effects in situations where there may be very important but most of the time they are very negligible we are only concerned with that first one that is heat transferred due to the temperature gradient not due to concentration gradient or any other thing okay.

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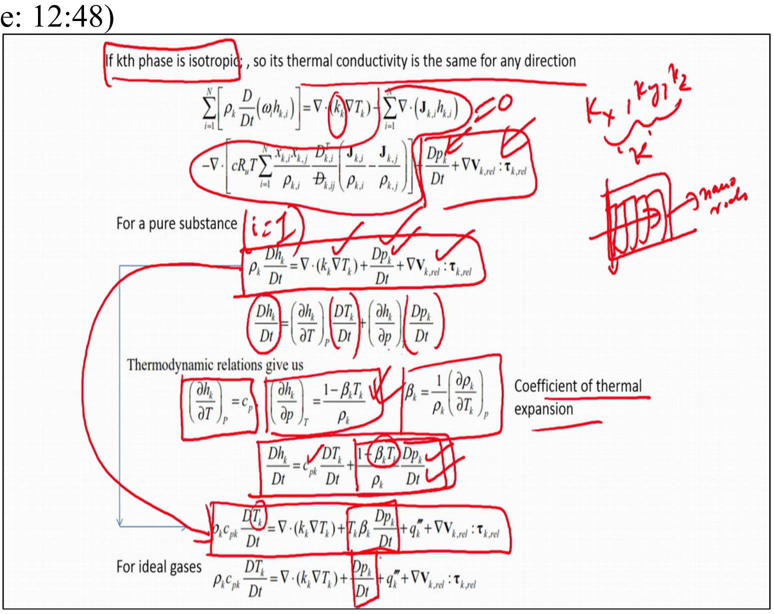

If kth phase is isotropic; , so its thermal conductivity is the same for any direction

$$\sum_{i=1}^{N}\left[\rho_k \frac{D}{Dt}(\omega_i h_{k,i})\right] = \nabla\cdot(k_k\nabla T_k) - \sum_{i=1}^{N}\nabla\cdot(\mathbf{J}_{k,i}h_{k,i})$$

$$-\nabla\cdot\left[cR_uT\sum_{i=1}^{N}\frac{x_{k,i}x_{k,j}}{\rho_{k,i}}\frac{D^T_{k,i}}{\mathcal{D}_{k,ij}}\left(\frac{\mathbf{J}_{k,i}}{\rho_{k,i}}-\frac{\mathbf{J}_{k,j}}{\rho_{k,j}}\right)\right] + \frac{Dp_k}{Dt} + \nabla\mathbf{V}_{k,rel}:\boldsymbol{\tau}_{k,rel}$$

For a pure substance

$$\rho_k\frac{Dh_k}{Dt} = \nabla\cdot(k_k\nabla T_k) + \frac{Dp_k}{Dt} + \nabla\mathbf{V}_{k,rel}:\boldsymbol{\tau}_{k,rel}$$

$$\frac{Dh_k}{Dt} = \left(\frac{\partial h_k}{\partial T}\right)_p\left(\frac{DT_k}{Dt}\right) + \left(\frac{\partial h_k}{\partial p}\right)_T\left(\frac{Dp_k}{Dt}\right)$$

Thermodynamic relations give us

$$\left(\frac{\partial h_k}{\partial T}\right)_P = c_p \qquad \left(\frac{\partial h_k}{\partial p}\right)_T = \frac{1-\beta_k T_k}{\rho_k} \qquad \beta_k = \frac{1}{\rho_k}\left(\frac{\partial \rho_k}{\partial T_k}\right)_p$$

Coefficient of thermal expansion

$$\frac{Dh_k}{Dt} = c_{pk}\frac{DT_k}{Dt} + \frac{1-\beta_k T_k}{\rho_k}\frac{Dp_k}{Dt}$$

$$\rho_k c_{pk}\frac{DT_k}{Dt} = \nabla\cdot(k_k\nabla T_k) + T_k\beta_k\frac{Dp_k}{Dt} + q_k''' + \nabla\mathbf{V}_{k,rel}:\boldsymbol{\tau}_{k,rel}$$

For ideal gases

$$\rho_k c_{pk}\frac{DT_k}{Dt} = \nabla\cdot(k_k\nabla T_k) + \frac{Dp_k}{Dt} + q_k''' + \nabla\mathbf{V}_{k,rel}:\boldsymbol{\tau}_{k,rel}$$

Now if the kth phase is isotropic what do you mean by isotropic means that there is no variation in thermal conductivity in any direction. So, there is no thermal conductivity is not like kx, ky, kz right, they are all the same as k, right. But there can be materials which can have directionality like for example if you take a rod.

We take a material which is composed of these rods like say nano rods or something like that okay. What will happen is that the conductivity along this direction and the conductivity along that direction is different okay. So, but in cases where the conductivity is the same all the direction okay, you can replace them by a single conductivity parameter, right, okay.

When these are once again the second-order components that we talked about and these are the terms that we already spend viscous dissipation and the corresponding pressure. This is written in terms of enthalpy. Now if we get a pure substance pure substance means i = 1, right. There is no multi-component system over here.

So, the expression reduces to something like this and we are neglecting this also okay, which we can safely do because these are all second-order effects as we talked about. So, this is basically the; once again the same three terms that you are familiar with. Now we can do a little bit of jugglery, right.

Because this is not a very usable form okay that is not the one that you normally encounter with. So, what we do is that this material derivative of the enthalpy of the kth phase okay, we write it as the differential of hk with respect to T at constant pressure into the corresponding material derivative of the temperature of the kth phase, right.

Similarly, we do the same thing with respect to pressure at constant temperature and we cast this Pk in terms of Dt, right. So, what we have essentially taken is that it is a function of P and T, hk, right, which is a standard thermodynamic definition right. That your property can be a function of you can specify the property in terms of two other properties, right, okay.

Now the thermodynamic relationship; this particular relationship is important that is the change of the enthalpy of the kth phase with respect to temperature for a constant pressure is nothing but CP the specific heat at constant pressure, right, okay. Similarly the change in enthalpy at, with pressure at a constant temperature is given by this expression where this beta k is nothing but the coefficient of thermal expansion.

Which is nothing but the change in density of the kth phase with respect to temperature at constant pressure okay, so, that is beta that is the coefficient of thermal expansion, correct, so we can right now the change the material derivative of the enthalpy in terms of two components the cpk which is the specific heat at constant pressure for a particular phase.

And we can write it in terms of the pressure and the corresponding coefficient of thermal expansion, right, okay. So, not a problem, so, this entire equation therefore now becomes a much more good-looking form I should say, okay. So, it is given in terms of temperature. Temperature is something that you can measure easily.

So, this gives a little bit of latitude to us okay. This pressure term and this pressure term can be combined together, right. Because that one basically drops off okay that 1 - beta k. So, you just get this nice looking expression and the rest of the terms remain the same. Now for an ideal gas if this is an ideal gas okay you can see that this particular expression now reduces to something like this from here to here.

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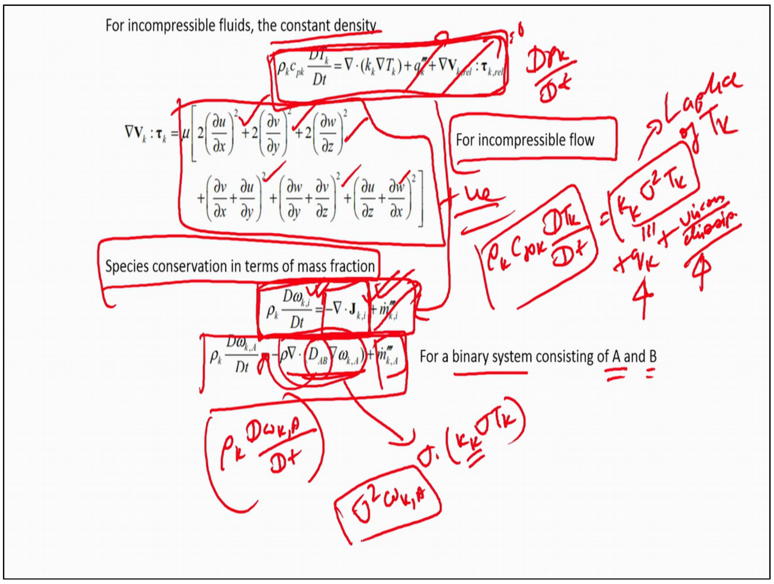

For incompressible fluids, the constant density

$$\rho_k c_{pk}\frac{DT_k}{Dt}=\nabla\cdot(k_k\nabla T_k)+q_k'''+\nabla\mathbf{V}_{k,rel}:\boldsymbol{\tau}_{k,rel}$$

$$\nabla\mathbf{V}_k:\boldsymbol{\tau}_k=\mu\left[2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+2\left(\frac{\partial w}{\partial z}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2\right]$$

For incompressible flow

Species conservation in terms of mass fraction

$$\rho_k\frac{D\omega_{k,i}}{Dt}=-\nabla\cdot\mathbf{J}_{k,i}+\dot{m}_{k,i}'''$$

$$\rho_k\frac{D\omega_{k,A}}{Dt}=-\rho\nabla\cdot(D_{AB}\nabla\omega_{k,A})+\dot{m}_{k,A}'''$$

For a binary system consisting of A and B

Now for incompressible flow with constant density okay, we know that the pressure term will actually vanish that DPk by Dt that will vanish, right. So, you get this particular form, so, you are reducing it slowly, right. The viscous dissipation term, now if you write it in Cartesian coordinate you can see what I talked to you earlier it is always the square, square, square right.

So, it is always positive always the square, right. Now for incompressible flow okay, the species conservation; so, this is the expression that you normally would get. Now this k can be taken out also if it is like a constant thermal conductivity. So, this expression can become, is the material derivative of T can be written as kk Tk + qk triple prime + the viscous dissipation, got it, okay.

So, now this is basically nothing but a Laplace temperature Tk, correct. This is the convective derivative once again heat generation term in many cases it can be equal to 0 and then the viscous dissipation term in many cases can also be equal to 0, right, okay. So, similarly species conservation that we did in the last lecture okay in the integral form can also be reduced to something like this.

We are not going through the details of the math once again, it is exactly looks the same as the energy conservation equation. If in the energy conservation equation this and this becomes equal to 0 okay. So, this is nothing but the same as this expression that we have. Here of course we have internal mass regeneration rate if that can be also equal to 0 then you just have the species balance in it form like this.

Exactly looks the same as a mass conservation as a energy equation. So, there is still the material derivative of the mass fraction there it is temperature here it is mass fraction another scalar and this is basically nothing but the corresponding diffusive flux okay which looks very similar if you consider the next expression for a binary system.

So that if the system composes of A and B that is two components one diffuses through the other. You can see that this is like Rho k D omega kA, this is I am writing it for the A component that is the convective term. This particular term looks exactly the same like your diffusion term in the it looks exactly like this.

If you look at the similarity you can see it, it looks exactly the same as this right okay, looks exactly, exactly the same okay. So, this looks very similar this is also a diffusion term therefore and this is the corresponding generation term in many cases it can be equal to 0. This DA D can also be a constant, so in that case this will come out and you will once again get a Laplace of Omega kA, right.

So this Laplace of omega kA is very similar to this particular Laplace of the temperature, okay. So, this is what we have for compressible; incompressible flow for constant density right. So, we have done all the differential formulations over here okay. So, to say;

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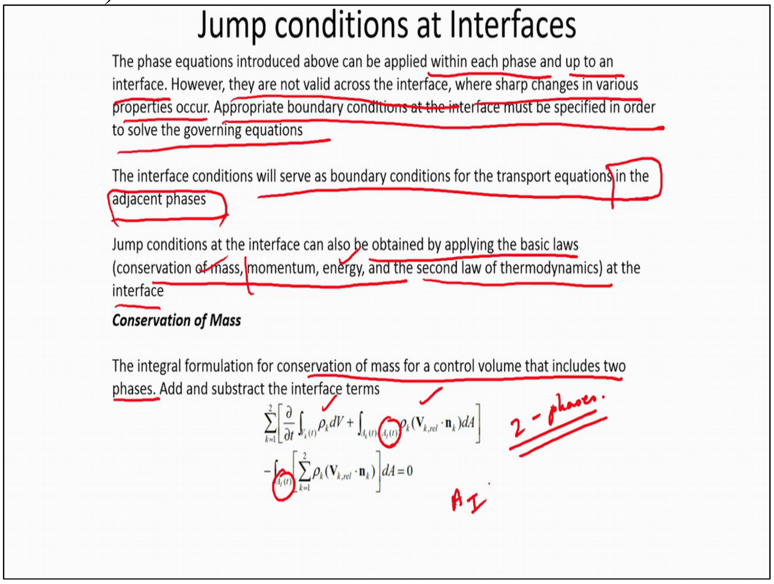

Jump conditions at Interfaces

The phase equations introduced above can be applied within each phase and up to an interface. However, they are not valid across the interface, where sharp changes in various properties occur. Appropriate boundary conditions at the interface must be specified in order to solve the governing equations

The interface conditions will serve as boundary conditions for the transport equations in the adjacent phases

Jump conditions at the interface can also be obtained by applying the basic laws (conservation of mass, momentum, energy, and the second law of thermodynamics) at the interface

Conservation of Mass

The integral formulation for conservation of mass for a control volume that includes two phases. Add and substract the interface terms

$$\sum_{k=1}^{2}\left[\frac{\partial}{\partial t}\int_{V_k(t)}\rho_k dV+\int_{A_k(t)+A_I(t)}\rho_k(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)dA\right]$$
$$-\int_{A_I(t)}\left[\sum_{k=1}^{2}\rho_k(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)\right]dA=0$$

2 - phases.

A_I

Now we are going to move on to what we call the jump conditions okay. Why are the jump conditions important now the phase equations that we introduced above, okay, it can be only applied within each phase and up to the interface, right. So, it is sum total of each of the phases right that we did. We just summed over all the phases.

However they are not valid across the interface where sharp changes in various properties happens like we said when water, liquid water gets converted to water vapour there is a density change of approximately thousand. So, appropriate boundary conditions at the interface must be specified to solve the governing equations, right, okay.

So, the interface conditions will serve as boundary conditions for the transport equations in the adjacent phases got it. We have to specify when you do the equations these comes as like two phases, this comes as an interface serves as a boundary condition. The jump conditions of the interface can be also obtained by applying the basic laws of thermodynamics momentum, energy.

That energy is basically thermodynamics conservation of mass, momentum. And the second law of thermodynamics at the interface okay. So, let us look at the situation for a conservation of mass over here okay. So, the integral formulation for conservation of mass for a control volume that includes two phases okay is already we have covered that.

Now what we are doing is that we are adding the interface term and we are subtracting the interface term okay. So up to this part it was already there right if you just take this out, this and this was already there, right. Except now that we have added the interface, right and we have subtracted the corresponding interface over here getting my point okay.

So, what we have done this is summed across all the phases okay. So, what we have done is that for each phase we have added the interface and we are taken the interface out. This is for two phases, got it. We have added and subtracted the two phases. The AI is basically the interfacial area, right, whatever it may be right.

So, we have added the interfacial area here we have subtracted the interfacial area there that is what we have done. There is nothing illegal it just have added and subtracted the same term, right.

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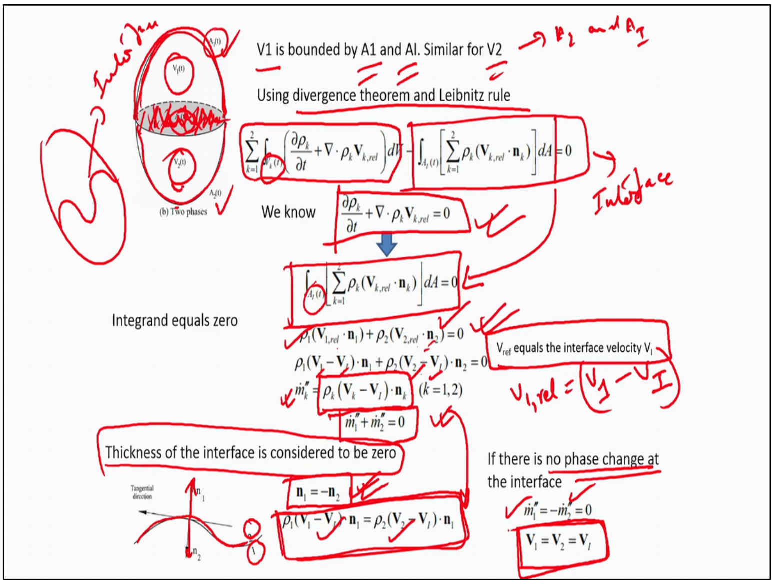

V1 is bounded by A1 and AI. Similar for V2 → A_2 and A_I

Using divergence theorem and Leibnitz rule

$$\sum_{k=1}^{2}\int_{V_k(t)}\left(\frac{\partial \rho_k}{\partial t}+\nabla\cdot\rho_k\mathbf{V}_{k,rel}\right)dV-\int_{A_I(t)}\left[\sum_{k=1}^{2}\rho_k(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)\right]dA=0$$

→ Interface

We know $\frac{\partial \rho_k}{\partial t}+\nabla\cdot\rho_k\mathbf{V}_{k,rel}=0$

$$\int_{A_I(t)}\left[\sum_{k=1}^{2}\rho_k(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)\right]dA=0$$

Integrand equals zero

$$\rho_1(\mathbf{V}_{1,rel}\cdot\mathbf{n}_1)+\rho_2(\mathbf{V}_{2,rel}\cdot\mathbf{n}_2)=0$$

V_{ref} equals the interface velocity V_I

$$\rho_1(\mathbf{V}_1-\mathbf{V}_I)\cdot\mathbf{n}_1+\rho_2(\mathbf{V}_2-\mathbf{V}_I)\cdot\mathbf{n}_2=0$$

$V_{1,rel} = (V_1 - V_I)$

$$\dot{m}''_k=\rho_k(\mathbf{V}_k-\mathbf{V}_I)\cdot\mathbf{n}_k \quad (k=1,2)$$

$$\dot{m}''_1+\dot{m}''_2=0$$

Thickness of the interface is considered to be zero

$$\mathbf{n}_1=-\mathbf{n}_2$$

$$\rho_1(\mathbf{V}_1-\mathbf{V}_I)\cdot\mathbf{n}_1=\rho_2(\mathbf{V}_2-\mathbf{V}_I)\cdot\mathbf{n}_1$$

If there is no phase change at the interface

$$\dot{m}''_1=-\dot{m}''_2=0$$

$$\mathbf{V}_1=\mathbf{V}_2=\mathbf{V}_I$$

Now let us look at this particular expression this particular cartoon that we have over here. This is one phase, this is the other phase this is very simple this is looks like an egg okay. It is actually separated by this interface area which is AI right. So, it is like a membrane which basically is separating the two halves okay.

So, if you have seen the toys okay like you will have that you know a separation clear separation between the two components okay. So, it can be other, so it can be anything. So, this is phase one, this is phase two. So, this is the area this particular area is A2 that particular area is AI right. So, VI P1 is bounded by AI and A1 correct.

V1, if you look at V1 what does it if you consider the whole V1 volume it is surrounded by A 1 it is also surrounded by this AI right. If you consider the area that actually encloses V1 you will find that it is AI and A1. Similar for V2 it is basically A2 and sorry A2 and correspondingly AI right, okay.

So, using divergence theorem and Lebnitz's rule like what we did last time okay. We convert the corresponding as we know the area integral to the corresponding volume integral. So, this part is what has been converted to volume, this part is the interface. So, this part is the interfaces part right, not the 0 up to this part, the interface part, right.

Now we already know that this part has to be equal to 0 the kth phase is not that so. You already know that this has to be equal to 0, right. This is summed over Vk nice and easy right. So, we know that this part has to be equal to 0 are we okay with that because this is nothing but the conservation of mass equation.

For each individual sub following we have written it okay. But however this implies that this particular term which is nothing but this written across the interfacial area AI that must also be equal to 0, right, that must be also equal to 0. Except that it is over AI okay. Now since the integral is equal to 0 the integrand should be also equal to 0 because of the, because this area can be very small okay.

So, what we have over here is that it is sum total because it is summed over both the phases right. So, it is a sum total of phase 1 and phase 2 right; what it is it is Rho 1 into V1 relative that is the velocity with respect to phase 1 okay and the velocity with respect to phase 2, right and the corresponding surface normal in both the phases right.

Remember this here of course the perfect normals are equal and opposite because the phase is a nice flat interface but it can be anything. It can be the same thing where the interface is something like that right. So, this can be the interface also does not really matter okay. But whatever it is this is given by that particular expression right now.

If the V reference velocity equals to the interface velocity that is a reference velocity with which it moves the control volume okay if that is equal to the reference velocity okay. So, the relative velocity therefore can be written as V1 relative is basically written as V1 - VI okay. VI, so, this is 1 this is I there is a difference okay and similarly this is V2 - VI right. It is the same thing okay.

Now if you consider this particular term what does this designate this designate some kind of a mass flux right. Is Vk - VI in dot nk into Rho k it definitely designates a kind of a mass flux right. So, it essentially means the sum total of the two mass fluxes considering both the phases has to be equal to 0 okay.

So, m1 double prime, the m2 double prime has to be equal to 0. Now the thickness of the interface here has been considered to be zero. So, it is like a membrane that is what we say it. And if you can take a look at this, this is a typical interface this is like n1, this is n2 okay this is interface, this is the region 2, this is region 1, this is the tangential direction.

So, n1 is basically the negative; one surface normal is the negative of the other okay. So, naturally you can rewrite this equation. This equation can be written in this particular form. If there is no phase change that means that m1 and m2 are basically equal to 0 then you have the velocity V1 is the same as V2 that is the same as VI right.

So, m1 and m2 now you can clearly see the link in one m1 dot double prime and m2 dot double prime is basically the transfer of mass okay between the two phases. That means there is a phase change that is going on so there may be liquid water here vapour here. Some liquid water is getting converted to vapour or some vapour is getting converted to liquid water whatever may be the case.

So, the interface is a thin interface, the surface normal's are basically equal and opposite to each other okay. And this is the expression that we can write if there is no phase change that means this and this basically becomes equal or negative of each other. And we get the velocities to be the same as the interfacial velocity okay.

So, the velocity in the two components is the same as the interfacial velocity, got it. However if these are not the same then of course you have a net transfer of mass then also this expression will be valid, got it, okay. So, this is one of the key things that we can find out in this particular exercise okay.

Now we look at the momentum equation in the next one but you can clearly see that by defining an interface okay what are the conditions across the interface. So, if there is a mass exchange what will be the condition if there is no mass exchange then what will be the condition and it is easy what we have done is that we have basically added and subtracted in our integral formulation that AI.

That is basically nothing but the interfacial area we have added it to each of the phases okay and we have taken it out also right. And then just by using the continuity expression we have set this part to be equal to 0 and then we have worked with this particular part of the right. And that part is also equal to 0 because of the reasons the entire sum is equal to 0 and then we have cast it.

There are two phases we have cast it in this kind of a component and we have taken the V reference velocity to be equal to the interfacial velocity in this particular case. So, you stop the lecture over here okay in the next lecture we will pick up the momentum the corresponding momentum equation and we will see what that implies okay, thank you.