Heat Transfer And Combustion in Multiphase Systems

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Lecture 07
Interfacial heat and mass transfer-I
- Interfacial mass, momentum and energy balance,
Surface tension, Wetting

So, this is the seventh lecture okay. So, let us look at the Generalized Governing Equations in a Multi-phase system sorry, just one second okay.

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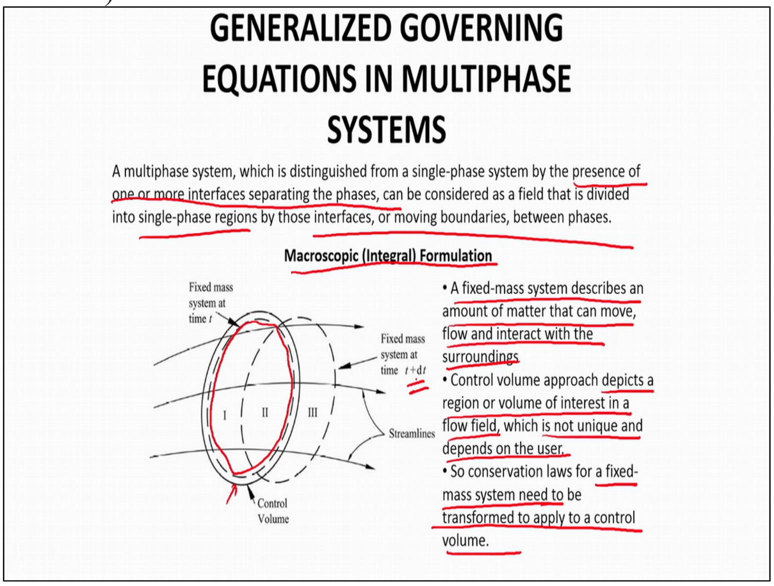

So, the generalized governing equations in multi-phase systems okay, so, you guys have all done single-phase system analysis okay for a long time. And you know that there are three conservation laws essentially conservation of mass, conservation of momentum and conservation of energy right.

So, conservation of momentum is basically nothing but the Navier Stokes Equation okay. Conservation of mass is conservation of mass and conservation of energy comes from the first law is the same as the first law of thermodynamics okay. And in addition people who have taken like reacting flows say for example not necessarily multiphase flows.

They have also dealt with multi-component systems or a multi-species system may also have done the species balance. Species balance and your energy balance looks pretty similar because both are scalar quantities okay. So, the momentum equation of course there are three momentum equations okay.

And there is one conservation of mass equation right. So, that makes it basically a 4, 5, 5 equations including energy and for species as many species you have you have to have for example if you have five species in the system, you need to have four equations because the fifth one is not needed because you already have the mass conservation right.

So, these are the, so, we are going to what we are going to do is that we are trying to see that how these equations can be generalized in the case of multi-phase systems essentially okay. So, what are the multi-phase systems? What are the requirements of the multi-phase system? Right.

So, multiple system is distinguished if we look at the slide is that multi-phase system is distinguished from single phase systems by the presence of basically one or more interfaces separating the phases okay. So, it can be considered as a field that is divided into single phase pockets by this interface.

Or this interface, these interfaces can actually move they may be deformable. There can be transfer of mass, momentum and energy among the phases itself that can also happen okay. It may not happen also because it could be just the two phases may be denying with each other they do not actually interact.

It can be several types of things but it is basically one or more interfaces are separating the individual phases. And so there are like single phase pockets there are regions no, I am not saying what is the extent of the single phase region. It could be very small, it could be very large but they are there are regions okay.

Now if we look at the macroscopic an integral formulation for this kind of a method you find that a fixed mass system. If we take a fixed mass system describes an amount of matter that can move, flow and interact with the surroundings whereas on the other hand control volumes approach.

A fixed mass system is the amount of matter is like an arbitrary if this is the fixed mass system that you have okay. The control volume however depicts a region or volume of interest in a flow field which is not unique and depends on the user. So, I can define my own control volume okay.

So, conservation laws for a fixed mass system needs to be transformed to apply to a control volume framework right. So, if you look at this nice little picture over here you will find that that is the fixed mass system right. And that is the control volume that we have considered okay. So, what happens is that this control volume of the fixed mass system basically moves. So, this is at time t = t, this is the fixed mass system at time t + delta t okay.

So, the control volume can be made to be fixed this fixed mass system is actually moving ok around it.

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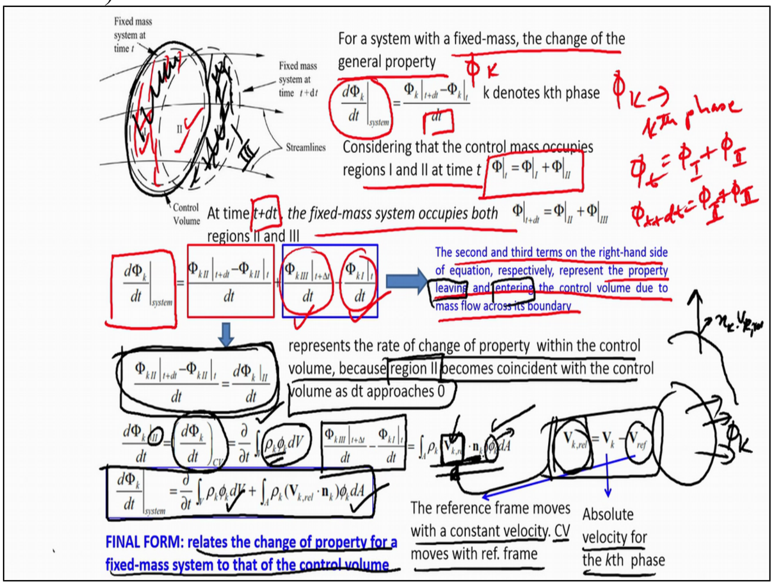

So, let us look at now the situation over here okay. So, it is a little bit of a busy slide but we will walk through the steps okay. Let us for the time being consider that there is only one phase present okay and that is the kth phase okay. And so, for a system with fixed mass, the change in general property right that the change in the general property is basically nothing but any property this property is like Phi k, this property can be anything right.

So, we will see later it can be momentum, energy, whatever that you can think of. For the time being it is some property, some generalized property which is given by Phi k. This Phi k of the system change in Phi k of the system is basically nothing but the change in Phi k at time t + delta t minus whatever was a Phi k at time t and this is divided by delta t okay.

And this is k have written at Phi k where k actually basically denotes the kth phase okay. Do not be bothered about the phase right now. Just take that here k is almost like the single phase okay. So, considering the control mass occupies region 1 and region 2 at a time t right. So, at a time t if you consider this there is a region 1, there is a region 2, right.

So, in region 1 and 2 this Phi is basically Phi at time t = t that means Phi at t is basically nothing but Phi 1 + Phi 2 is not that so. It is sum total of this plus this is not that so okay. Now at time t + delta t what happens that is after some time the fixed mass system now occupies 2 & 3, do you see that it has moved from 1, it is now occupying 2 and it is occupying 3 okay.

So, it is almost like Phi t + delta t, now is basically Phi 2 + Phi 3 correct, okay. Therefore now the rate of change of whatever that property of the system is. Is given by basically two quantities right, so, it is the change in region 2 at time t + delta t - dt right. And it is the change of between region 3 and region 1 right that is what actually happens.

Now if we look at the third and the second and the third term of this expression this and this very closely what does it show you. It shows basically the rate of change of property within the control volume right is the represent sorry is the second and the third term on the right-hand side of the equation represents the property that is leaving and entering the control volume using due to mass flow across its boundary.

Look at it very carefully, if you look at the control volume okay you can find that 3 is the mass that has, so 1 is the mass, what is this particular shaded region right okay. So, if I change the colour of my pen. Let us say I you black now okay, black will be, so, and this is the region 3 correct okay. So, 1 represents mass in 1 represents a mass out okay.

If you consider it in that particular way when the control volume, when this system has actually moved right okay. So, that is the second and the third term represents respectively the mass leaving the system and the mass that is entering the control volume, control volume remember. It is I am not talking about the fixed system, the control volume partr okay. This is the control volume right that we drew right.

So, there has been some mass that has left the system means this is the mass that has left the system right and this is the mass that has entered the system correct is not that so. So, the control volume is fixed, this fixed mass has come and moved right, this fixed path is transiting through the control volume right.

So, what happens is that as the fixed mass transit through the control volume this region 3 over here is basically the mass that is leaving the control volume correct. Whereas 1 is the mass that has entered into the control volume correct so, that is the thing that we have stated over here okay whereas this particular expression over here represents the rate of change of property within the control volume.

Because region 2 becomes coincident with the control volume as dt approaches 0 look at region 2 now okay. Now in region 2 when dt that means the extent of this time becomes very small right this region 2 becomes coincident okay with the control volume that we have drawn over there right.

So, this is nothing but the rate of change of property within the control volume this particular term over here okay. So, now since we have established these two okay. If you now look at the individual term that means the rate of change of this property in the region 2 say for example right okay which is driven by the rate of change of that property within the control volume right.

That is given nothing by the rate of change of property within the volume correct. So, that is why I we have written it as Rho k Phi k into dv that means whatever is the volume and it is integrated over the volume. So, the rate of change of that property within the volume right on the other hand this two term which we said on the flux terms right.

These two terms we already paid by the flux term they are represented by the flux right. The flux is what that property is transported by some velocity okay across the control surface. This is the control volume, so, these are basically the control surface right. So, this is the mass or momentum or energy whatever it is this is transported putting this Phi k across the control surface.

And why we have written it as V relative that is because let the reference frame move with a constant velocity and the control volume may move with a reference frame okay. So, if the reference frame velocity is equal to 0 this becomes the same as absolute velocity for the kth phase. Otherwise if the control volume is moving with the reference frame we have to take that out okay.

So, this is basically nothing but the V relative, the control volume is moving with a reference frame this is what the control, this is the relative velocity with which the mass energy or momentum or whatever that is that that Phi k represent is entering or leaving the control volume okay.

So, if you combine this term and this term now okay. If you look at the representation, if you combine these two terms we find that the system, the rate of change of that property in the system is given by the volumetric change right. The rate of change of that property within the volume plus there is a flux term okay.

Flux term transports whatever it is, that it is transporting across the control surface right, okay. So, this form relates the rate of change of property for a fixed mass system with that of a control volume, got it. So, essentially, this essentially means that there is a control volume the fixed mass is actually moving across the control volume.

So, this region 1 and region 3 basically represents the flux whereas region 2 is basically whatever was there within the control volume always right it is there, right. So, the change in region 2 is essentially is, whatever is the change of property inside the control volume whereas region 1 is whatever is entering the control volume region 3 whatever is leaving the control volume.

So, those are basically two flux terms okay in the limit of dt = 0 of course this is drawing a highly exaggerated way when dt is actually approaches 0 okay, this becomes region 2 becomes the control volume in itself. These are basically the flux term entering into the control volume and leaving the control volume okay.

And whatever the flux terms have been represented by this relative velocity over here accounting for that account the reference frame moves with a constant velocity and the control volume moves with the reference frame okay. We have been able to cast this equation in this particular form and this be relative is what is used over here right okay.

Remember that there is a dot product with nk where nk is nothing but the surface normal. So, the velocity may have a direction okay angular direction. But the surface normal may have this right. So, this is basically nk dot Vk relative right okay. So, it is the dot product between the surface normal and the velocity component okay.

So, that is what it is remember Phi k can be a scalar, can be a vector can be anything else but this particular term that you see Vk relative into nk this was basically a scalar, right. Because it is multiplied with a surface normal Phi k that you see over here can be anything okay. It can be mass, it can be momentum whatever, we will see to that, okay.

But on the other hand Vk relative okay dot nk basically is a flux term which is perpendicular to the surface right that is what we need to take and that is basically a scale, clear on this particular part.

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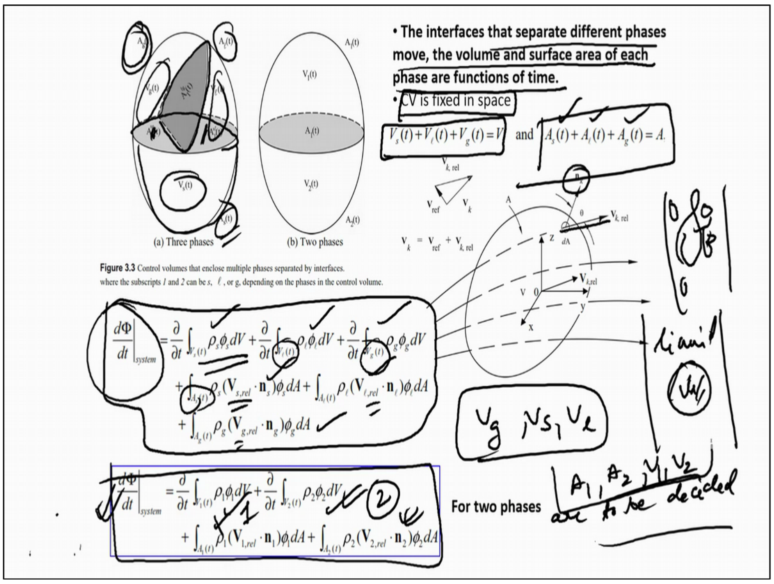

Now if there are multiple interfaces say for example right this is just for one phase that we showed, right. So, let us take the simple thing that this is a solid, this is the corresponding liquid, this is the corresponding gas. So, the interface there are partitions, you can see this partition, you can see this partition.

So, there are partitions across the interface right. These are interfaces and each of this has got their own individual surface area's right. So, this have got a surface area of As, it will got a surface area of Al, this has got a surface area of Ag right. So, the interfaces that separate the different phases the volume and the surface area of each phase are functions of time, got it.

So, let us put pull the, I don't know why the marker is not coming. So, the control volume however is fixed in space okay but this interfaces can move right. So, this interface and the individual surface areas can actually move right. So, what we can say is that the sum total of this volume should be V because the control volume is fixed right.

And the sum total of these areas will be also fixed it does not mean that these areas cannot vary. They can vary individually among themselves but the sum total of the area is A, the sum total of the volume V, got it, okay. And this was the example that we are showing you that this is the surface normal, this is the velocity whatever is angle we take the dot product over there.

Now analyzing we already have analyzed the system right for this. Now we are writing it for each and every phase same term, right. Look at the term this is the rate of change of that property whatever that property is within the volume Vs. Here Vs is not the total volume it is the volume that is reserved for the solid phase, right.

Similarly there is a volume that is reserved for the liquid phase; there is a volume that is reserved for the gaseous phase, correct. These are not the total volume, right. So, these are volumes like V, Vg, Vs, Vl, right. These are not the total volume, this is an important concept that you should remember.

Similarly goes for the area as well that you for example these are the different relative velocity components with respect to the gas phase, with respect to the liquid phase etc. But these areas are once again these areas, right. So, that is that important consideration over here. So, we are basically the total rate of change of that property within the system is nothing but the sum total of all this.

Except that you have to know what are the areas that are occupied by the each individual phase. That means you also have to know what is the volumes that are occupied by each individual phases? Right okay, so, if we write the combined equation which is given over here, right, okay. So, you can say it for two phases there are basically four components, right.

This is for phase one sorry these two are for phase one, these two are for phase two, right. You can have a phase three also but the important consideration here is that A1, A2, V1, V2 are to be decided not that easy right because you do not know the extent right of each of the individual phases. If it is nice droplet in a pool of liquid or a vapour bubble in a pool of liquid then you know the individual things, right.

It is a very clean interface okay it is a very clean interface we can write this equation very easily over there, right. It is very easy to write but however if you recall the problem that there are say lots of fluid ligaments then there may be interstitial flows right all those things that happens. Determining these areas and volumes becomes a practically a very onerous kind of a task, right, okay.
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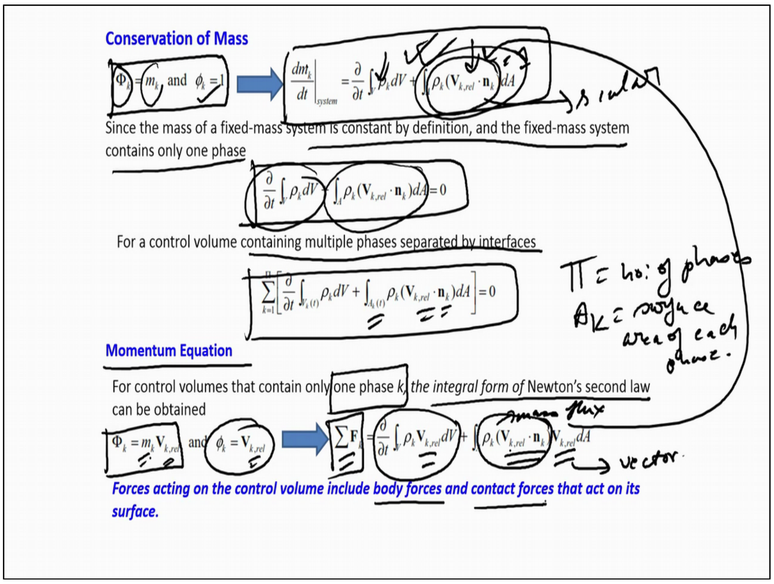

So, let us quickly do the conservation of mass now, okay. For the conservation of mass what we do is that we take small Phi k = 1, right. And the big Phi k right, the small Phi k comes with a V relative, okay. This Phi k is taken as m, whatever is the mass of the system, right. So, we can quickly write this expression which is basically the rate of change of mass within the system.

This is the density of the kth and this is the corresponding flux term, right. So, since the mass of a fixed system is constant by definition and it contains only one phase this is the expression. This is the all the mass conservation equation that you are familiar with, right. The rate the mass conservation is what the rate of change of mass within the system and whatever is the flux term must coming in was going out, right.

Now if it contains multiple phases this is nothing but the sum total over all the phases. That means if see the Phi over here that is nothing but the number of phases, right. And it is a relative velocity and the corresponding surface area of each phase. Ak is nothing but the surface area of each phase is that clear.

So, this is the surface area for each phase right. Similarly when we come across to the momentum equation, now the control volume has say one single phase k for the time being, okay. So, the integral form of Newton's second law as you know the momentum equation is nothing but Newton's second law, okay.

Phi k, capital Phi k is nothing but mk into V relative this is the momentum mass into velocity, right. So, mk is the mass of the system Vk relative is basically the momentum with which the flow is entering or leaving the control volume, right. And Phi k, the small Phi k is nothing but the V relative.

This time it is a vector; previously it was a scalar, right. So, when you write this now the Newton's second law of motion in an integral form what we said the right-hand side once again is very similar to this, right. Except, instead of Phi k we have this Vk relative, here it was 1, here it is Vk relative, right.

It was 1, in here this particular value was equal to 1 is not that so, okay. So, now here also you see Vk relative into mk is nothing but a scalar, is a scalar, right. Now that scalar means this is nothing but the mass flux, right. On the other hand this is a vector, right. So, this entire expression is of course a vectorial representation, right.

So, but in the case of a mass conservation of mass this was entirely a scalar quantity, right, because that was the dot product. Here also this is A is the mass this is nothing but the same as this. These two are exactly identical, that is now what is the momentum that is transferred across the interface okay.

And this is the rate of change of momentum within the control volume, right. And what is the left-hand side that we have written this is the sum total of all the forces that are acting on the control volume. Now the forces that are acting on the control volume include body forces like gravity. And contact forces that act on the surface like your shear stress, okay and things like that. It can be anything though, okay.

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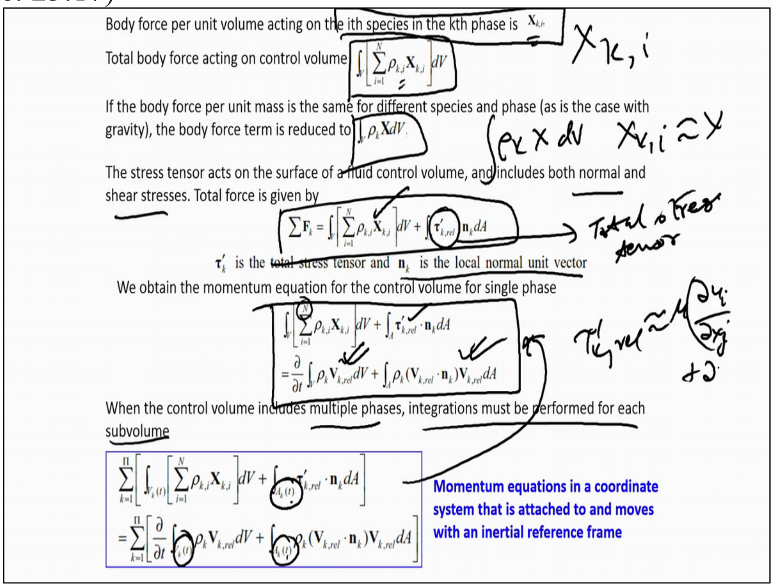

So, the body force it a body force that is acting on the ith species on the kth phase is X ki, if you cannot see it this is X ki okay. Then the total body force expression is given by this, right. So, k is the individual phase i can be the individual components. It can be also a multi-component system if the body force per unit mass is same for all the species and phase as with gravity that is indeed the case, right.

We can just simply ready to write it like this which is nothing but Rho k X dv. So, there is no no X ki's are all kind of X right because this is a gravity, right. And the stress tensor that is acting on the complete control volume includes both normal as well as shear stresses, right. So, the total force is nothing but the body force term which is given by X ki and this term which is nothing but the total stress tensor.

The Nk is a normal local normal unit vector as we stated before. So, the entire equation when it is written is represented like this N is the, N here is the number of components if you have N number of components you can have N number of components in the system not the number of phases though, okay.

Tou k relative is nothing but the stress tensor the total stress tensor and this is basically nothing but the change of momentum within the control volume and the momentum flux that is happening along the control surface, got it. So, when the control volume has multiple phases okay, integrations must be performed for each sub volume.

So, once again we have the same thing these areas come into the picture these volumes come into the picture, right. The total force is summed across all the cases all the all the phases. So, that is the final expression that you get. So, it is no different from this except now we have this additional areas and this additional volume that comes into the picture, right.

Now why it does not represent a Newton's second; normal Navier-stokes equation that you are familiar with. Because this tau k relative that we have, okay. We need to apply the constitutive relationships to get to the form that you are most familiar with right. That is what you are most familiar with, right.

So, those kinds of forms like the shear stress type of forms of course this is only the shear stress component of it. This comes only from the constitutive relationships that we will see later okay. So, what we will do is that we will stop here in this particular lecture and before we take on to the energy equation.

Let us try to digest this fact for the for the time being, what we have done is that we have taken a control volume approach okay and we have shown that in the control volume okay, we can represent it by a general property Phi k that Phi k can be momentum energy, energy we have not covered but we will show that in the next lecture.

But it all shows that how this mass momentum and energy are actually distributed inside the how the transport actually happens we have done a conservation of mass, we have done now conservation of momentum also and we have shown that where the stress tensor comes into the picture it comes from the force the total force.

And we have also stated that this is equal to the rate of change of momentum within the control volume and plus whatever is the momentum flux that is happening across the control surface okay. So, the next class we just will finish the sauce okay and we will we can do and we will move on and see the end this integral formulation.

And then do the differential formulation and then see some of the averaging techniques that we can have okay. So, thank you we will see you next class.