Heat Transfer And Combustion in Multiphase Systems
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Lecture 39
Boiling-II (Bubble dynamics)

We have started doing analysis on Boiling. So, last time we saw that how nucleation actually occurs.

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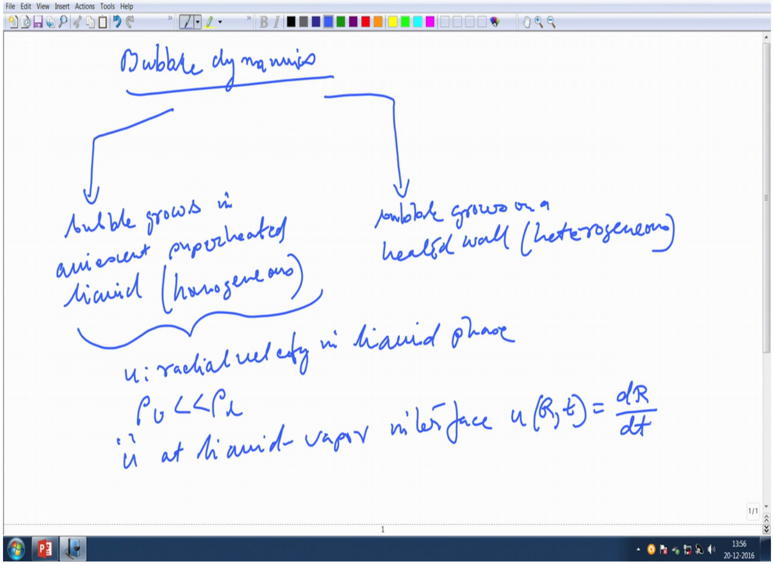

Bubble dynamics

bubble grows in quiescent superheated liquid (homogeneous)

bubble grows on a heated wall (heterogeneous)

u: radial velocity in liquid phase

$\rho_v << \rho_l$

$\therefore$ u at liquid-vapor interface $u(R,t) = \frac{dR}{dt}$

Now, let us now look at the Bubble dynamics. Now, in the case of a Bubble dynamics, what happens is that there are two types of Bubble dynamics that you can see, when the bubble grows, when the bubble grows in quiescent, superheated liquid, it is basically homogeneous, if you recall what is homogenous and what is heterogeneous.

So, this is one type and when the bubble grows on a heated wall, okay. So, this is heterogeneous, okay. So, first we are going to look at what is basically homogeneous bubble growth. Now, if you recall the equation of continuity, what you will find? So, we are going to study this first, okay.

So, u already know what is a radial velocity in the liquid phase, if you recall, the continuity equation that we did last time for a bubble inside a question fluid medium. And we also know that Rho v is much, much less than Rho l, okay. So, at the liquid vapour interface, this u knew at R ,t becomes equal to dR by dt.

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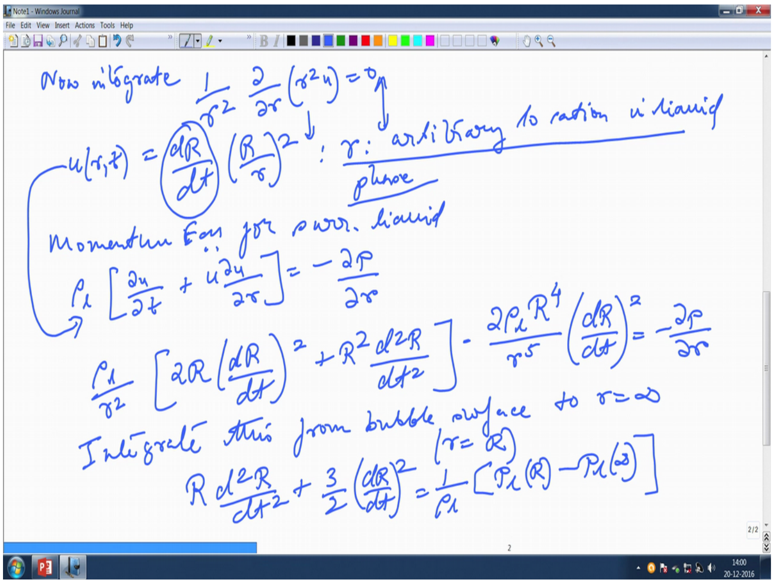

Now integrate $\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u) = 0$

$u(r,t) = \boxed{\frac{dR}{dt}}\left(\frac{R}{r}\right)^2$; r: arbitrary location in liquid phase

Momentum Eqn for surr. liquid

$$\rho_l\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right] = -\frac{\partial p}{\partial r}$$

$$\frac{\rho_l}{r^2}\left[2R\left(\frac{dR}{dt}\right)^2 + R^2\frac{d^2R}{dt^2}\right] - \frac{2\rho_l R^4}{r^5}\left(\frac{dR}{dt}\right)^2 = -\frac{\partial p}{\partial r}$$

Integrate this from bubble surface $(r = R)$ to $r = \infty$

$$R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 = \frac{1}{\rho_l}\left[P_l(R) - P_l(\infty)\right]$$

Now, integrate the continuity equation that we had, equal to zero. Now, once you integrate it, okay. You r, t, so, u r , t will be equal to the growth rate. So, u r,t will be equal to dR dt square, r is any arbitrary location, location in liquid phase, okay. Now, the momentum equation for the surrounding liquid right, that is the momentum equation.

Now, if you substitute this u r, where r is any arbitrary location, as we said, if you use this u r now, in this particular situation, okay what we are going to get? Let us see over here. So, this has been integrated as you know and at the surface the velocity is this, okay. So, it is very easy to show that the velocity progressively changes in that particular fashion.

Once you substitute this into this equation, now, what will happen is that, okay. That is the total momentum equation that you will get, if you integrate this momentum equation, this, okay from bubble surface to r = infinity that has been the far field, okay. Bubble surface is basically small r = R, okay. You will get, got it, okay.

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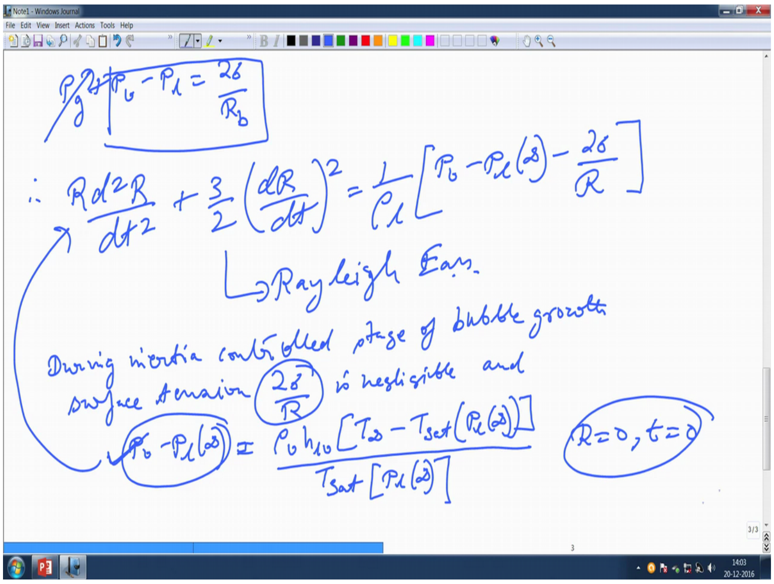

$$\cancel{P_g} + \boxed{P_v - P_l = \frac{2\sigma}{R_b}}$$

$$\therefore \frac{R\,d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 = \frac{1}{\rho_l}\left[P_v - P_l(\infty) - \frac{2\sigma}{R}\right]$$

$\hookrightarrow$ Rayleigh Eqn.

During inertia controlled stage of bubble growth surface tension $\left(\frac{2\sigma}{R}\right)$ is negligible and

$$\left(P_v - P_l(\infty)\right) = \frac{\rho_v h_{lv}\left[T_\infty - T_{sat}\left(P_l(\infty)\right)\right]}{T_{sat}\left[P_l(\infty)\right]}$$

$R = 0, t = 0$

Now, so that is the equation that you get. Now, we already know, if you recall the previous class. We already said that for bubbles with non condensable phases, this was the total equation, right if you recall from the last class. If the condensable phases are zero, non condensable phases are zero that goes away. So, what we can do is that we can readily apply this relationship in the, in the other equation.

So, therefore this becomes capital R d square R by dt square, okay. So, this equation is called the Rayleigh equation, got it. So, this equation is called basically the Rayleigh equation, right okay. So, you can see that this is the growth of a bubble inside a superheated liquid, okay. And this is the general form of the equation that you get. But, however, there were some interesting findings that people noted.

One of those findings were that during inertia controlled stage of bubble growth, so, these are the initial stages, okay one can conclude that the surface tension which is given by 2 Sigma by r okay is negligible, okay. And what you have is that your Pv - Pl infinity is given as Rho v hlv T infinity - T sat. That is the expression.

So, the initial condition over here is basically, R = 0 at T = 0 as the initial condition, okay. So, these are the two expressions that we see that is, this is negligible and this is a function like this, okay. So, now what we do is that we substitute, okay. This expression in this particular system, right and we neglect the surface tension term.

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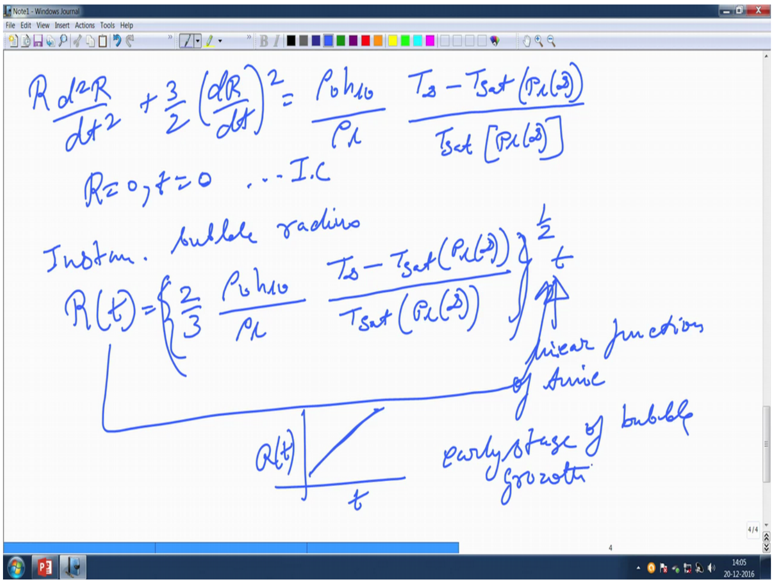

$$R\frac{d^2R}{dt^2}+\frac{3}{2}\left(\frac{dR}{dt}\right)^2=\frac{\rho_v h_{lv}}{\rho_l}\,\frac{T_s-T_{sat}(P_l(\infty))}{T_{sat}[P_l(\infty)]}$$

$R=0, t=0$ ... I.C

Instan. bubble radius

$$R(t)=\left\{\frac{2}{3}\,\frac{\rho_v h_{lv}}{\rho_l}\,\frac{T_s-T_{sat}(P_l(\infty))}{T_{sat}(P_l(\infty))}\right\}^{\frac{1}{2}} t$$

linear function of time

early stage of bubble growth

So, using that what happens is that the instantaneous, when you substitute that and become dR squared dT square + 3 by2. These 2 terms remains the same, okay. Now, if you substitute the initial condition, now, that capital R = 0 at t = 0 + I.C, initial condition, when you substitute that, the instantaneous bubble radius or bubble radius becomes, okay.

So, this actually tells you if you look at it carefully that this is a linear function of time, right. Or, in other words, the early stage of the bubble growth, okay. It will predict you something like this, okay, t versus R where the slope will be determined by all these quantities that, you have in a bracket hmm. But it is basically a linear function of time.

This is the early stage of bubble growth for homogeneous nucleation, okay. Now, during the later stages what happens.

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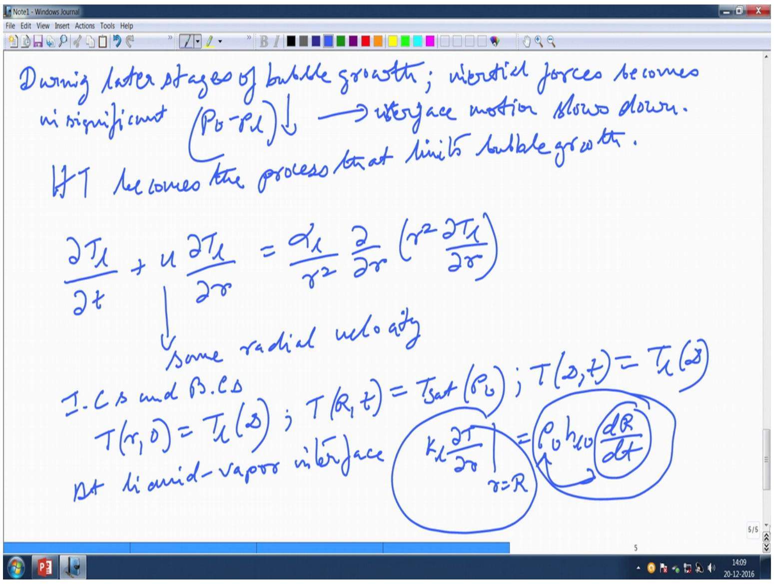

During later stages of bubble growth; inertial forces becomes insignificant $(P_v-P_l)\downarrow \longrightarrow$ interface motion slows down.

HT becomes the process that limits bubble growth.

$$\frac{\partial T_l}{\partial t}+u\frac{\partial T_l}{\partial r}=\frac{\alpha_l}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T_l}{\partial r}\right)$$

some radial velocity

I.Cs and B.Cs

$T(r,0)=T_l(\infty)$; $T(R,t)=T_{sat}(P_v)$; $T(\infty,t)=T_l(\infty)$

At liquid-vapor interface $\left.k_l\frac{\partial T}{\partial r}\right|_{r=R}=\rho_v h_{lv}\frac{dR}{dt}$

So, these are the early stages, during the later stages, we get slightly different situation, okay.

During the later stages or bubble growth so what do you have in the later stages of the bubble growth the inertial forces, okay becomes insignificant, okay. So, as the pressure Pv - Pl actually goes down, okay the interface, this leads to interface, interface motion, okay slows down.

Therefore, heat transfer, okay becomes the process that limits, become the process that limit, okay that limits bubble group, okay. So, it is like the not an inertial dominated framework anymore. It is the heat transfer because everything is becoming a little bit slow now, okay. So, now, if we have to write the energy equation for the heat transfer part, what will be the basic equation?

This will be this, right, u, got it? This u is the same radial velocity that we got earlier, right. So, the initial conditions and boundary condition therefore becomes T at r , 0 is the same as T infinity. T at R ,t equal to the saturation pressure and this is the surface of the bubble and T at infinity, T will be equal to Tl at infinity, got it okay.

So, these are the initial and the corresponding boundary conditions that you have, okay. So, what happens is that, at the liquid vapour interface, at the liquid vapour interface, you should have this being a wet, right. So, this is rather because this is nothing but the mass. This is Rho into v right, okay. And the area cancels from basically from both sides.

So, basically Rho into v area cancels from both sides. So, it is basically Ka into, the into the conduction flux is equal to whatever is the, that heat is being used for the latent heat of vaporization. That is the essential essence of this particular story, right okay.

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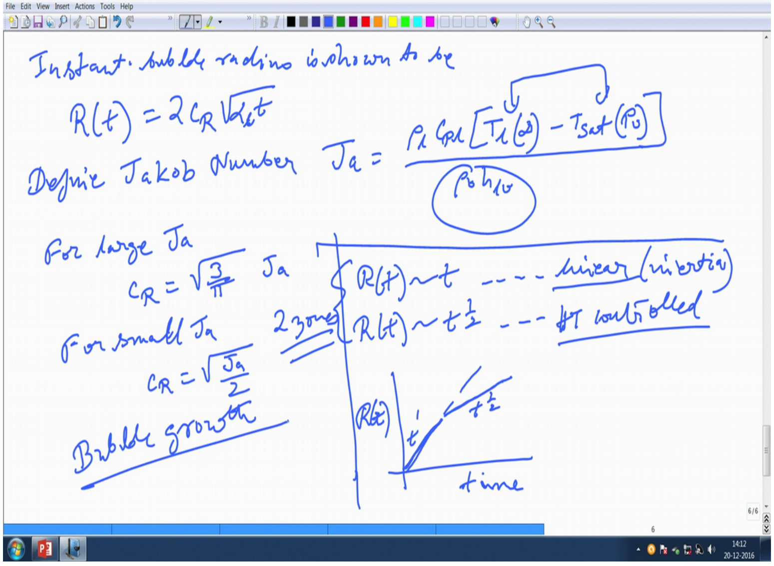

Instant. bubble radius is shown to be

$$R(t) = 2C_R\sqrt{\alpha_l t}$$

Define Jakob Number $J_a = \dfrac{\rho_l C_{pl}\left[T_l(\infty) - T_{sat}(P_v)\right]}{\boxed{\rho_v h_{lv}}}$

For large Ja

$$C_R = \sqrt{\frac{3}{\pi}}\, J_a$$

For small Ja

$$C_R = \sqrt{\frac{J_a}{2}}$$

Bubble growth

2 zones $\begin{cases} R(t) \sim t & \text{---- linear (inertia)} \\ R(t) \sim t^{\frac{1}{2}} & \text{--- HT controlled} \end{cases}$

So, based on these, okay the instantaneous bubble radius, radius, okay is shown to be, okay. Alpha i into t that is the instantaneous bubble radius, okay. Now, we can define something called a Jakob number or Yakob number whatever you call it, okay. So, that Jakob number is given by Rho l CPl, Tl infinity right - T sat into Pv divided by Rho v into hlv, okay.

There is a Yakob number. For large Jakob number, so, you can see what the Jakob number is. It is basically the liquid minus a saturation temperature divided by whatever is the latent heat, okay. So, it is, it is basically that. For large Jakob number, CR is basically 3 PI into Ja. For small J, we have CR = Ja by 2, okay.

So, therefore, there are two stages. In the first stage, Rt is proportional to t which is basically what we call a linear growth regime, right. In the second stage, Rt is proportional to root over t which is basically, this is inertia, okay. This is basically heat transfer controlled, okay. So, you will have basically two slopes: One slope like that, okay; the other slope will be something like that, okay something which is less prominent, okay.

So, this is basically the linear t1 this is t half, okay. So, this is R, the time, got it. So, there are two zones: One is a linear, one is the heat transfer controlled. These two zones basically, two zones that we get over here. So, this clearly establishes the fact that the based on two mechanisms. One is the inertia and one is the heat transfer controlled.

We can actually define and we can actually show that what is exactly happening to the whole process got it. And this is for homogeneous nucleation only, got it, clear, up to this point, okay. So, that is the bubble growth. So, these are all related to bubble growth, right.

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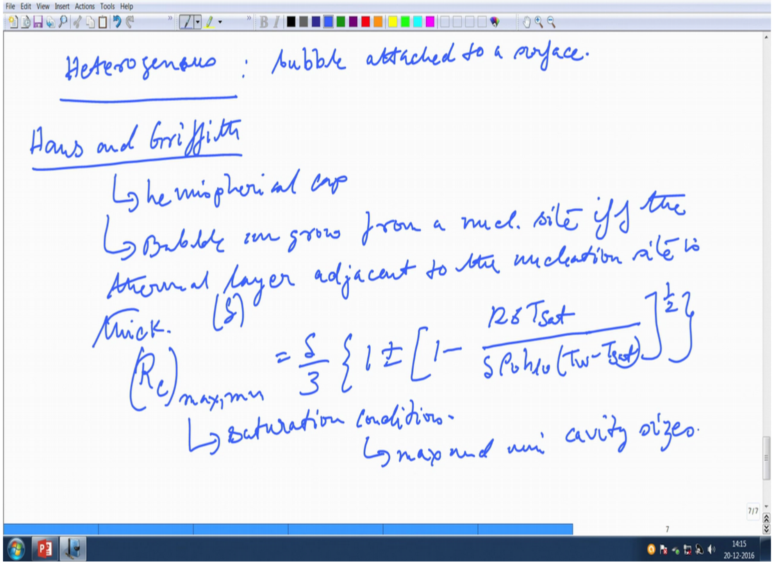

Heterogenous : bubble attached to a surface.

Hans and Griffith

↳ hemispherical cap

↳ Bubble can grow from a nucl. site iff the thermal layer adjacent to the nucleation site is thick. (δ)

$$(R_c)_{\text{maximum}} = \frac{\delta}{3}\left\{1 \pm \left[1 - \frac{12\sigma T_{sat}}{\delta \rho_v h_{lv}(T_w - T_{sat})}\right]^{\frac{1}{2}}\right\}$$

↳ saturation conditions.

↳ max and mini cavity sizes.

Now, in the case of heterogeneous bubble growth, heterogeneous bubble growth, okay. So, this is attached with a surface, right. Bubble attached to surface, okay. Now, the vapour bubbles attached to a heated surface basically, the growth occurs in a non uniform temperature field, okay to say the list, okay.

So, what Hans and Griffith did, what they did was that the bubble can grow from a nucleation site if that thermal layer adjacent to the nucleation site is thick. And they said that the bubble normally grows like a hemispherical cap. So, the bubble is like a hemispherical cap. I will show you the picture in just a second, okay.

And the bubble can grow can grow from a nucleation site. That is, if you recall, it is like a micro crack or a crevice, okay. If the thermal layer, if and only if the thermal layer adjacent to the nucleation site is thick, okay. So, and we will show what that is. So, they found that Rc which is the max and Rc max Rc min is basically given by delta by 3 which will show delta is the thickness of the other thermals bound, thermal layer.

So, that is delta. It is 1 + - 1 minus 12 Sigma T sat divided by Rho v hlv Tw - T sat raised to the power of half, okay. So, this is for saturation conditions, for saturation conditions. Remember, we had saturated boiling and things like that. For saturation conditions, okay that what will be the sizes that will be active, the maximum and minimum cavity sizes that will be active, okay.

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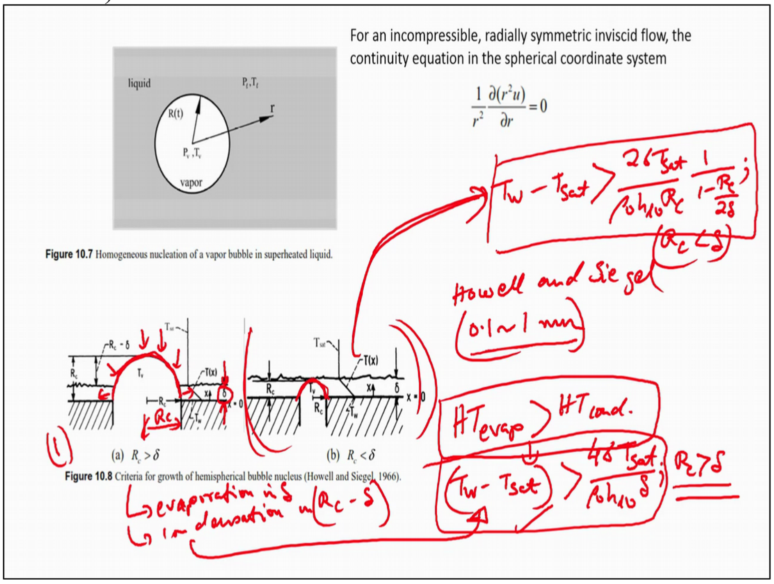

For an incompressible, radially symmetric inviscid flow, the continuity equation in the spherical coordinate system

$$\frac{1}{r^2}\frac{\partial(r^2 u)}{\partial r} = 0$$

Figure 10.7 Homogeneous nucleation of a vapor bubble in superheated liquid.

(a) $R_c > \delta$ (b) $R_c < \delta$

Figure 10.8 Criteria for growth of hemispherical bubble nucleus (Howell and Siegel, 1966).

$T_w - T_{sat} > \frac{2\sigma T_{sat}}{\rho_v h_{lv} R_c}\frac{1}{1-\frac{R_c}{2\delta}}$; $(R_c < \delta)$

Howell and Siegel

(0.1 ~ 1 mm)

$HT_{evap} > HT_{cond.}$

$(T_w - T_{sat}) > \frac{4\sigma T_{sat}}{\rho_v h_{lv} \delta}$; $R_c > \delta$

↳ evaporation in δ

↳ condensation in $(R_c - \delta)$

So, now, if we look at this particular presentation, we will get an idea of what we are talking about. So, Howell and Seigel, they did and if you look at figure number ten eight okay they examined that how the bubble from a single nucleation site of known diameter. So, this nucleation sites diameter varied anywhere between 0.1 to 1 mm, okay on a highly polished surface. How do these bubbles actually grow?

So, if you look at it now, there are two scenarios that are possible, okay. When the cavity size, half of the cavity size essentially Rc is so large, that a part of this bubble actually sticks, okay above the thermal boundary layer which is given here. That is the thermal boundary layer thickness, got it.

So, a part of the bubble basically sticks out, okay. And in the second case, most of the bubbles is actually encompassed within this thermal boundary layer, okay. What happens is that, what they say that, if the bubble is extending out, that is case number one, if we consider this as your case number one, then, evaporation happens in this layer and condensation happens in that layer, right.

So, in the first case, you have evaporation in delta and condensation in Rc - delta, okay. So, that is what you are going to have. Now for a bubble to grow that heat transfer due to evaporation must be greater than the heat transfer due to condensation, right. That has to happen. So, part of the bubble is evaporating. Part of the bubble is basically condensing.

So, based on this argument, what they showed was that your T wall - T saturation, this temperature has to be greater than 4 Sigma T sat divided by Rho v hlv into delta when Rc is greater than Delta, got it for a bubble to grow. So, this is for this situation, okay. That Tw - T sat must be greater than this because this has to be satisfied, okay.

Now, if the bubble is only inside this, which is this particular case, the bubble is wholly inside this. This Tw - T sat, okay has to be greater than 2 Sigma T sat divided by Rho v hlv into Rc, okay and 1 - Rc by 2 delta when Rc is less than delta. So, that is for this case. That is the condition for bubble growth, okay.

So, this is what for heterogeneous bubble growth first was given by Hans and Griffith who showed one comprehensive relationship that what kind of cavity sizes will be active, okay depending on the thermal boundary layer thickness. Whereas, on the other hand, Howell and Seigel said we considered two extreme cases.

One in which the bubble is protruding out; in the other case, the bubble is completely confined within the, within the thermal layer. And they showed for a very large variation in a cavity sizes from point 1 to 1 mm that there are two limiting conditions when the cavity size is large in, in order for a bubble to grow, you need to have this relationship.

For the other case, you need to have that relationship, okay. So, these are the two things that Siegel and Howell actually found out, okay. So, we end this lecture with this, okay before we move on to the bubble detachment. So, right now, we have studied the bubble formation and bubble growth, okay. Now, we are going to see bubble detachment, how a bubble actually detaches from a surface.

This is the growth part, right. But ultimately, the bubble detaches and then, it goes into the main fluid, right. So, how to study that? That is, will be the topic of the final lecture that we are going to do on this, thank you.