Heat Transfer And Combustion in Multiphase Systems
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Lecture 36
Droplet combustion-II

Welcome to today's Lecture! So, as we said in the last class, we actually did that how a droplet actually combust and we show that the D square law is still valid.
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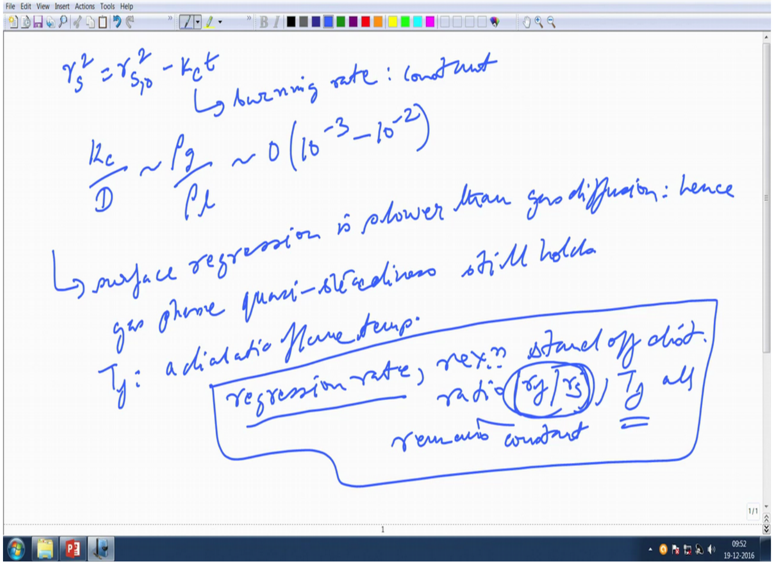

$r_s^2 = r_{s,0}^2 - K_c t$

↳ burning rate : constant

$\frac{K_c}{D} \sim \frac{\rho_g}{\rho_l} \sim O(10^{-3} - 10^{-2})$

↳ surface regression is slower than gas diffusion : hence gas phase quasi-steadiness still holds

T_f : adiabatic flame temp.

regression rate, rexn standoff dist. ratio $\left(r_f / r_s\right)$, T_f all remains constant

If you recall, this was the equation that we wrote where Kc is basically the burning rate which is constant in the current case, okay. Now, remember in previous D square law derivation we always said that the gas phase is always considered quasi steady, right. So, here also, if you take just a ratio of Kc divided by the diffusion, okay you will get, it comes out to be Row g by Row l which is of the order of 10 to the power of minus 3 to 10 to the power of minus 2.

So, it essentially implies that the surface regression which is given by KC, okay is slower than gas diffusion which is given by D, okay. So, hence, gas phase quasi steadiness still holds and that was the same of what we got in the droplet vaporization case, okay.And remember also that Tf was the adiabatic flame temperature. This we wrote in the last class also.

Flame temperature for this statiometric flame, okay and during this period of droplet combustion, the regression rate and this; you should remember, the regression rate, the reaction standoff distance ratio, standoff distance ratio. Ratio is given by rf by rs. So, r f actually decreases r s also decreases by the same amount.

So, these and the flame temperature that is Tf, all remains constant. Please do remember that rf by rs remains constant, not rf by itself, okay. So, as the droplet shrinks the flame also shrinks, okay. But this relative ratio is kept the same, okay. So, and Tf is the flame temperature that also remains the same. The regression rate which is given by Kc also remains the same.

So, all these things are the hallmark of this particular analysis which is basically the D square law, right okay.

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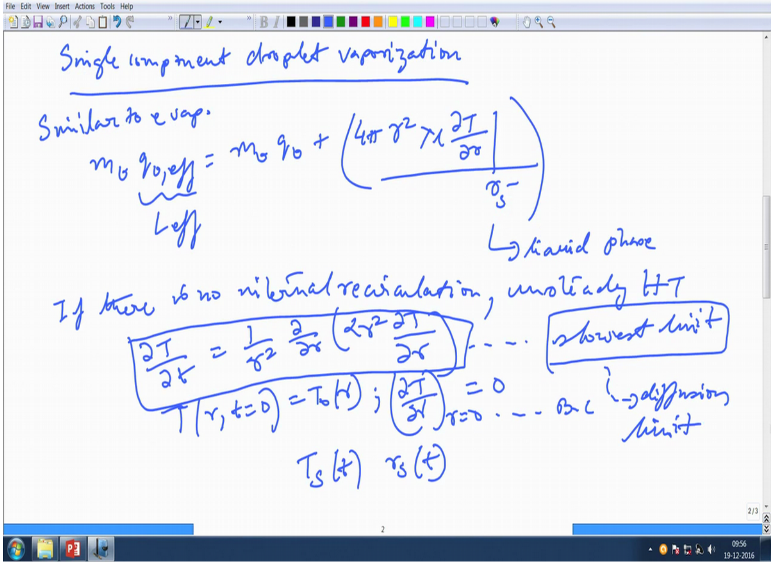

Single component droplet vaporization

Similar to evap.

$$\underbrace{m_v q_{v,eff}}_{L_{eff}} = m_v q_v + \left(4\pi r^2 \lambda_l \frac{\partial T}{\partial r} \Big|_{r_s^-} \right)$$

↳ liquid phase

If there is no internal recirculation, unsteady HT

$$\boxed{\frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\alpha r^2 \frac{\partial T}{\partial r}\right)} \text{ - - - } \boxed{\text{slowest limit}}$$

↳ diffusion limit

$$T(r, t=0) = T_0(r) \; ; \; \left(\frac{\partial T}{\partial r}\right)_{r=0} = 0 \text{ - - - B.C}$$

$$T_s(t) \quad r_s(t)$$

Now, we take a look at single droplet single component droplet vaporization component droplet vaporization, okay. Now, it is very similar to the evaporation case evaporation. So, here what is given as mv, qv effective, this is like your L effective in Sirignano’s notation is mv into qv + 4 PI r square lambda l, dt by dr at r s minus. rs - that is because it is in liquid phase, in the liquid phase, got it, okay.

So, in this case, if there is no internal, so, this is the heat that is transferred to the liquid phase essentially, right okay. So, if there is no internal recirculation, okay, the unsteady or the transient heat transfer is HT is given as: this we are writing it within the droplet, okay. This is basically nothing but the slowest limit because, why it is the slowest limit? Because the heat is transferred by conduction essentially in this particular case, right.

And you have the boundary conditions and initial conditions that t=0= T naught everywhere. And at the center of the droplet you know that and r = 0 this is equal to 0. This is the boundary condition that is the initial condition, okay. So, the source of unsteadiness in this particular problem comes from the decreasing value of r, okay or the rs rather.

And the continuously changing profile of Ts, so, Ts is a function of time rs is a function of time; two main sources of unsteadiness. And this has to be the slowest limit. This is because we have taken only the diffusion to be the carrier. So, the droplet surface gets heated slowly the heat goes to the center of the droplet and that's how the heat is actually transferred.

There is no recirculation. It is only a plain diffusion or a conduction problem over here, right okay.

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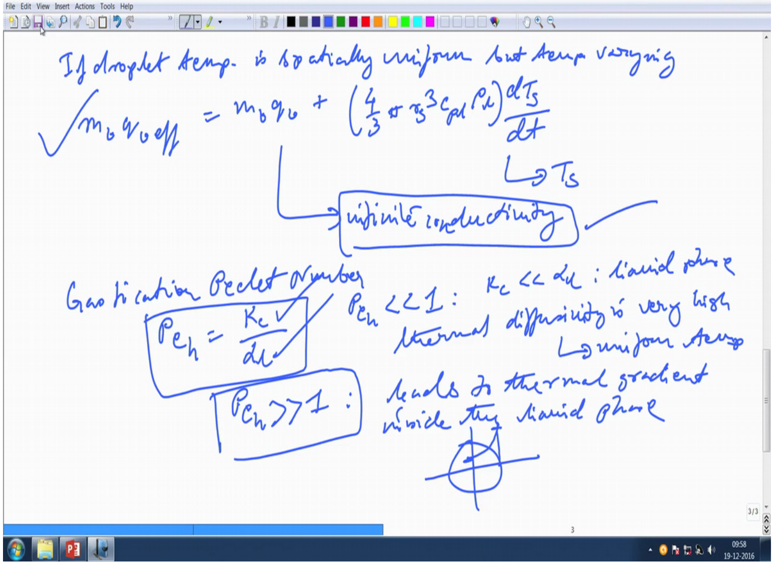

If droplet temp. is spatially uniform but temp varying

$$\checkmark\ m_v q_{v\,eff} = m_v q_v + \left(\frac{4}{3}\pi r_s^3 c_{pl}\rho_l\right)\frac{dT_s}{dt}$$

$\hookrightarrow T_s$

$\boxed{\text{infinite conductivity}}$

Gasification Peclet Number

$$\boxed{Pe_h = \frac{K_c}{\alpha_l}}$$

$Pe_h << 1$: $K_c << \alpha_l$: liquid phase thermal diffusivity is very high

$\hookrightarrow$ uniform temp

$\boxed{Pe_h >> 1 :}$ leads to thermal gradient inside the liquid phase

Now, we already know from our droplet come a droplet vaporization counterpart. That if droplet temperature, is spatially uniform, which is one of the assumption okay. If it is like that but temporarily vary. This I am just reminding you, right temporarily varying. You have this mv qv effective is equal to mv qv.

If you recall your old notes, you will find that this is what we did. This is rs cube, Cpl. Then, row l into dTs by dt, right. This is frequently used for determining Ts. And this is roughly what we call the infinite conductivity model also, right. That means that there is no temperature variation within the droplet, okay.

So, that is the diffusion limit. This is that in finite conductivity counterpart of the same, right okay. Now, if you, that is the diffusion limit. So, like vaporization, you have to solve for all these equations, okay. Now, we can define the gasification Peclet number. You already know the definition of Peclet number.

So, this gasification Peclet number is given by the burning rate kc divided by alpha l or the liquid thermal diffusivity, got it. Now, if this Peclet number, Ph is much, much less than 1, this implies a very high value of the thermal diffusivity, right because if this is much, much less than 1, that means kc is much, much less than alpha l which implies that the liquid phase thermal diffusivity, diffusivity is very high, right.

So, this would normally lead to your uniform temperature which is basically this case, right. If the Peclet number is much, much greater than 1, okay this should lead to a thermal gradient inside the liquid phase, okay. So, in other words, this is the droplet, okay the temperature will go like this, right; little heist at the surface and lowest at the court right, okay.

So, the difference comes from the Peclet number argument that we have put forward over here. So, infinite conductivity is the fastest, okay. Diffusion limit is the slowest, right. So, both depend on what is your gasification Peclet number which is nothing but the ratio of the burning rate divided by the thermal diffusivity. So, keeping this in mind, okay let us now move to the presentation mode.

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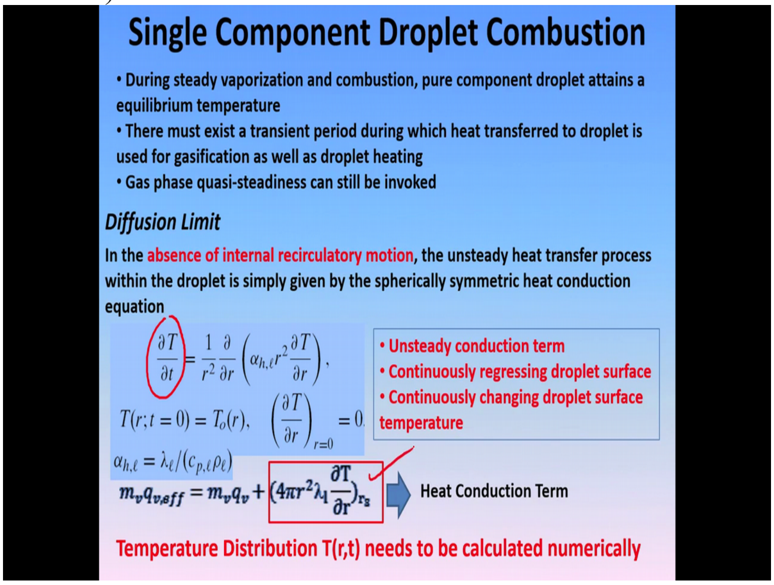

Single Component Droplet Combustion

- During steady vaporization and combustion, pure component droplet attains a equilibrium temperature
- There must exist a transient period during which heat transferred to droplet is used for gasification as well as droplet heating
- Gas phase quasi-steadiness can still be invoked

Diffusion Limit

In the absence of internal recirculatory motion, the unsteady heat transfer process within the droplet is simply given by the spherically symmetric heat conduction equation

$$\frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\alpha_{h,\ell} r^2 \frac{\partial T}{\partial r}\right),$$

$$T(r; t=0) = T_o(r), \quad \left(\frac{\partial T}{\partial r}\right)_{r=0} = 0.$$

$$\alpha_{h,\ell} = \lambda_\ell / (c_{p,\ell}\rho_\ell)$$

$$m_v q_{v,eff} = m_v q_v + \boxed{(4\pi r^2 \lambda_l \frac{\partial T}{\partial r})_{r_s}}$$

Heat Conduction Term

- Unsteady conduction term
- Continuously regressing droplet surface
- Continuously changing droplet surface temperature

Temperature Distribution T(r,t) needs to be calculated numerically

So, this is what we did once again this comes from C K Laws material. So, this is we already put forward in writing that this is basically the unsteady, sorry, let us do so. This was basically the unsteady diffusion an unsteady conduction term, okay. And this assumes a continuously regressing droplet layer.

And this is the corresponding conduction term of the heat that is actually transferred within the droplet core. So, that is what we did in the write up that we just did, right now.

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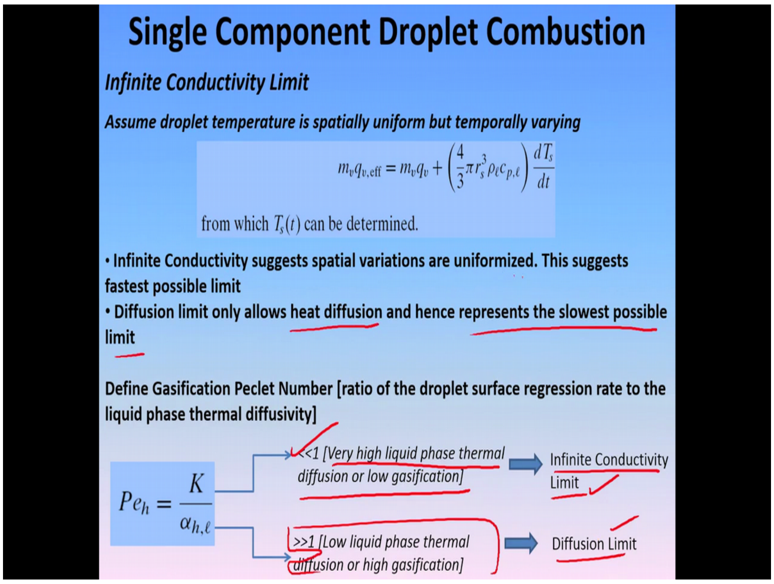


Single Component Droplet Combustion

Infinite Conductivity Limit

Assume droplet temperature is spatially uniform but temporally varying

$$m_v q_{v,\mathrm{eff}} = m_v q_v + \left(\frac{4}{3}\pi r_s^3 \rho_\ell c_{p,\ell}\right)\frac{dT_s}{dt}$$

from which $T_s(t)$ can be determined.

- Infinite Conductivity suggests spatial variations are uniformized. This suggests fastest possible limit
- Diffusion limit only allows heat diffusion and hence represents the slowest possible limit

Define Gasification Peclet Number [ratio of the droplet surface regression rate to the liquid phase thermal diffusivity]

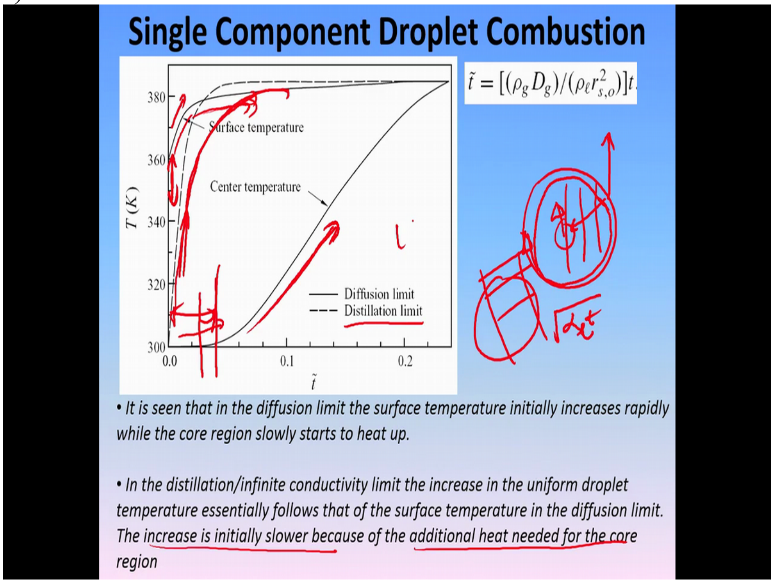
$$Pe_h = \frac{K}{\alpha_{h,\ell}}$$

<<1 *[Very high liquid phase thermal diffusion or low gasification]* → Infinite Conductivity Limit

>>1 *[Low liquid phase thermal diffusion or high gasification]* → Diffusion Limit

And this also we did that if the Peclet number is much, much less than 1, very high liquid phase thermal diffusion or low gasification. This is an infinite conductivity limit. On the other hand, when Peclet number is much, much greater than 1, this is a diffusion limit. So, both of these two limits actually determines whether we can take the droplet to be a in the infinite conductivity or the diffusion.

Diffusion is basically like a spherically symmetric droplet evaporation model, okay. So, diffusion limit therefore always allows heat diffusion and hence represents the slowest possible limit while this suggests the fastest possible limit infinite conductivity, okay. (Refer Slide Time: 11:05)



Single Component Droplet Combustion

$\tilde{t} = [(\rho_g D_g)/(\rho_\ell r_{s,o}^2)]t$

- *It is seen that in the diffusion limit the surface temperature initially increases rapidly while the core region slowly starts to heat up.*
- *In the distillation/infinite conductivity limit the increase in the uniform droplet temperature essentially follows that of the surface temperature in the diffusion limit. The increase is initially slower because of the additional heat needed for the core region*

So, now, if we look at the temperature profiles of the droplet, but how the temperature actually evolves with, with non dimensional time? You will find that in the case of yours diffusion limit, that is the curve. And in the case of your distillation of the infinite conductivity limit, that is the curve, right.

So, what you find is that in the diffusion limit the temperature initially increases very rapidly, okay. And then it starts to slow up. That is because the core of the droplet starts to heat. So, here the surface temperature rises is very fast, right. The core is still does not feel the heat, okay. Slowly, as time goes on, the core temperature also starts to rise.

That is also shown in this particular curve. This is a center temperature, only valid for that diffusion limit; because in the case of infinite conductivity limit, there is no difference between surface temperature and core temperature, right because both are the same; it is spatial uniform.

So, here you can see that initially the core temperature hardly rises that is because the diffusion, the diffusion front has to propagate and reach the droplet center, right. As you know, that that is given by something like alpha lt right, okay so, there is a little bit of a time lag, before the core starts to feel the heat essentially. So, as it starts to feel the heat the surface temperature slowly starts to slow down and the core temperature starts to go up, okay.

Now, in the case of an infinite conductivity limit as you can see the increase in the droplet temperature is more like this, right. So, essentially follows, okay. This is initially slower this is initially a little slower compared to this; if you look at the difference between the two curves. That is because here the entire thermal mass of the droplet is being heated, right.

It is dr just not the surface you have to take into account the full mass of the droplet because every where the temperature is the same. It is not like that, that the center temperature is lower, right. Everywhere the droplet temperature is like a horizontal line, right. And it just goes up with time. So, therefore naturally, there is a difference this has got more thermal mass and as a result it takes a lot of time for it to go up.

Whereas this can go up very fast because only the region very close to the surface is actually heated, okay and the heating front actually reaches the center only after this particular prescribed time. So, there is quite a bit of a time line between the two, okay. So, you can clearly see that in the case of a infinite conductivity model, okay. There is an additional the increase is initially slow because of the additional heat requirement for the core region, okay.

So, that should be taken into consideration in this particular plot. So, this is just to give you an idea that what happens when you have the 2 limits, okay.
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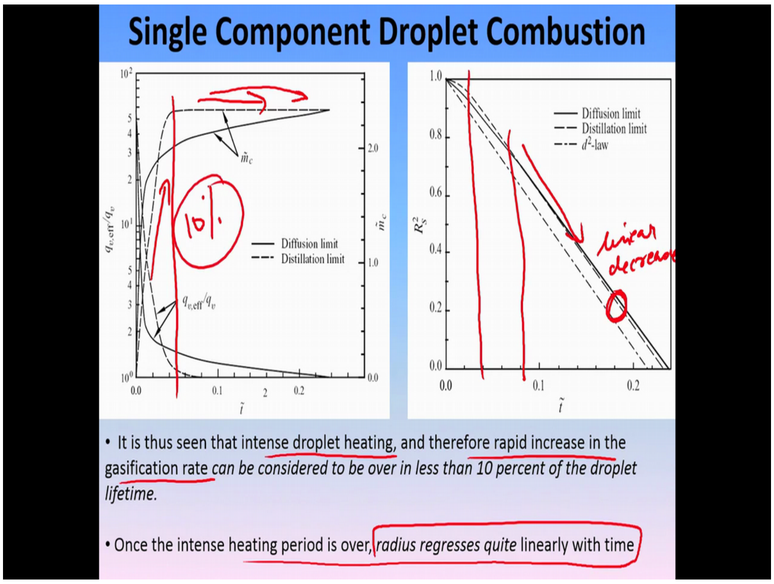

Single Component Droplet Combustion

- It is thus seen that intense droplet heating, and therefore rapid increase in the gasification rate *can be considered to be over in less than 10 percent of the droplet lifetime.*
- Once the intense heating period is over, *radius regresses quite* linearly with time

Now, similar things can be; one other interesting observations and the main observation that I want you to take from this particular thing is that the intense droplet heating that we have and therefore the rapid increase in the gasification rate, right. During that initial part, when the droplet surface temperature is actually growing before it actually flattens out.

This contributes to less than ten percent of the droplet lifetime, right so, for ten percent of the droplet lifetime, okay. There is an increase in the gasification rate after that the gasification way it becomes quite steady, okay. So, this rapid increase in gasification rate due to intense heating of the droplet lasts only for ten percent of the droplet lifetime.

After that the droplet surface temperature kind of flattens out, as you can see, over there, right. And this is also seen in the regression, diameter regression plot you can see that the droplet. Actually this is the line from this point onwards this is a very linear decrease, right in the droplet diameter. Do you see that?

It is a linear decrease in droplet large diameter pretty much after about ten percent of the droplet lifetime after that initial intense heating is over, right. So, and therefore, of course, here also, you can see that the diffusion limit is the slowest the distillation limit or the infinite conductivity limit is that is the kind of the; so, you can see, these two things comes very close to each other, okay.

So, the main concern over here is that during the intense heating period, okay that time the radius regresses linearly with time. And this time period is only ten percent of the droplets total lifetime. That is an important observation, okay.

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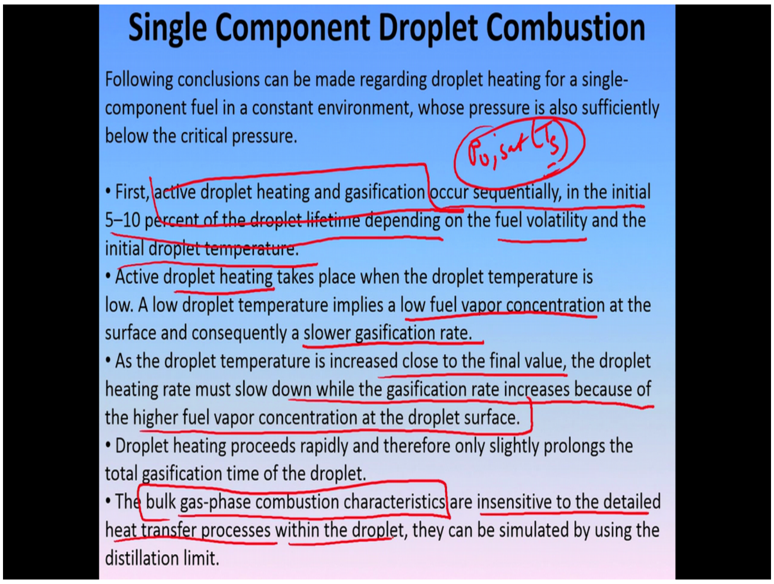

Single Component Droplet Combustion

Following conclusions can be made regarding droplet heating for a single-component fuel in a constant environment, whose pressure is also sufficiently below the critical pressure.

$P_{v,sat}(T_s)$

- First, active droplet heating and gasification occur sequentially, in the initial 5–10 percent of the droplet lifetime depending on the fuel volatility and the initial droplet temperature.
- Active droplet heating takes place when the droplet temperature is low. A low droplet temperature implies a low fuel vapor concentration at the surface and consequently a slower gasification rate.
- As the droplet temperature is increased close to the final value, the droplet heating rate must slow down while the gasification rate increases because of the higher fuel vapor concentration at the droplet surface.
- Droplet heating proceeds rapidly and therefore only slightly prolongs the total gasification time of the droplet.
- The bulk gas-phase combustion characteristics are insensitive to the detailed heat transfer processes within the droplet, they can be simulated by using the distillation limit.

Now, in single component these observations now, kind of compiled over here; that, for active droplet heating, okay and gasification, okay occur sequentially in the initial 5 to 10 percent of the droplet lifetime depending on the fuel volatility of course and the initial droplet temperature. So, the active droplet heating and gasification occurs during that period, okay. So, active droplet heating means a sensible enthalpy transferred to the droplet.

Now, that takes place when the temperature is low. A low temperature implies low fuel vapour concentration and consequently slower gasification rate; because if you recall, okay from the Clausius-Clapeyron, okay the vaporization rate is a function of the Pv saturation, right. And that is a function of the surface temperature essentially, right. So, if the surface temperature is low because the droplet temperature is not initially at the wet bulb limit.

When the surface temperature is low, okay there is a slower gasification rate because there is a lower fuel concentration at the surface, right. So, as you increase the droplet heating or as the droplet surface temperature increases the gasification rate also increases, okay. So, as the temperature is increased close to the final value, the heating rate must slow down while the gasification rate increases because of the higher fuel vapour concentration at the droplet surface, okay.

The bulk gas phase this is also an important statement. The bulk gas phase combustion characteristics are basically insensitive to the detailed heat transfer processes within the droplet, got it. So, in the case of the bulk combustion characteristics their intensity to all this whether, whether you are taking infinite conductivity or diffusion limit etcetera, these are mostly insensitive, okay to the entire thing. This is from the droplet perspective, okay.

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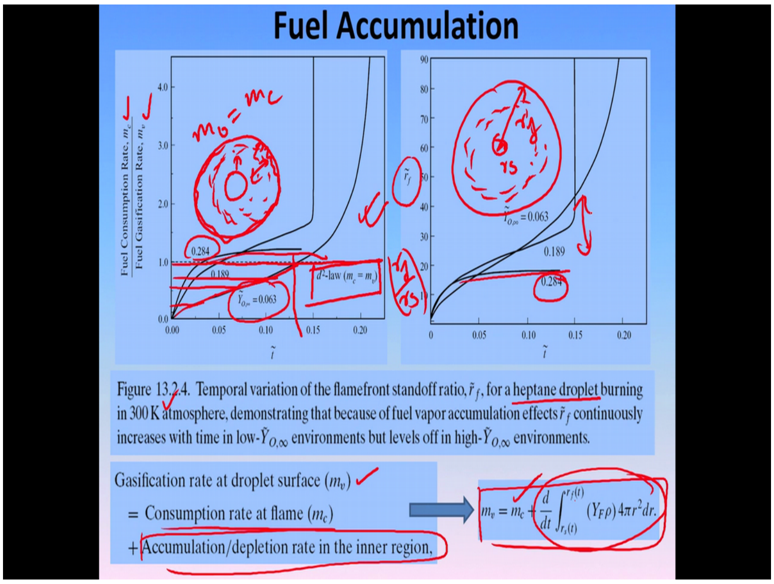

Fuel Accumulation

Figure 13.2.4. Temporal variation of the flamefront standoff ratio, $\tilde{r}_f$, for a heptane droplet burning in 300 K atmosphere, demonstrating that because of fuel vapor accumulation effects $\tilde{r}_f$ continuously increases with time in low-$\tilde{Y}_{O,\infty}$ environments but levels off in high-$\tilde{Y}_{O,\infty}$ environments.

Gasification rate at droplet surface (m_v)
= Consumption rate at flame (m_c)
+ Accumulation/depletion rate in the inner region,

$$m_v = m_c + \frac{d}{dt}\int_{r_s(t)}^{r_f(t)} (Y_F\rho)4\pi r^2 dr.$$

This is another important slide which actually we want to bring forward one key factors, remember, we said that mv =mc, we said that whatever is reprising, it goes towards burning, okay right. That is what we said when we actually derived the d square law. But in reality that is not the case. There is, if you recall, the droplet graph, this is the flame front, right.

This is the inner region, correct. So, this fuel whatever vaporizes from here can actually get accumulated in this region and not all of them are consumed at the flame front, okay. So, there can be an accumulation. So, as you know, that a d square law takes place that there is no accumulation, right. So, if I divide mc by mv, what you will find is, that the straight line is basically your d square law, right at one, right.

But what do you see over here is very interesting, okay. You see over here. Let us not look at this part of the plot. The second one, okay let us look at this plot only the left-hand side, okay. So, the gasification grade or the droplet surface is therefore given as a linear sum of the consumption at the flame front which is basically mc. And the accumulation and depletion rate at the inner region, right.

So, if you have to put it in this particular form it is basically mc plus this particular term where rs and rf designate the droplet surface and the flame standoff distance, okay. So, it is basically basis to, this is the droplet. So, this is your rs and this is your rf, right okay. So, what you see in this particular plot? That ideally the ratio should be one because that is what D square law actually prescribes.

But do you see that? For example, in oxygen-depleted environment, yo infinity, if the flame is burning, this is a result of our for an N for n-heptane droplet in a 300k atmosphere, okay 300k room temperature burning, okay. So, here if it burns in a low oxygen environment, you see that this ratio approaches 1 but for a significant part of the droplet lifetime, it is not, right. It is quite low. Actually, the consumption rate is only a very small fraction.

If you look at these horizontal lines, right, there are only a small fraction of the gasification rate. That would imply that there is a lot of fuel which is basically accumulated in that inner region, got it, right okay. However, it does go and reach as we increase the oxygen concentration, right.

As you increase the oxygen concentration and at around 2.8 for this basically becomes flat and very close to 1, that implies that you are approaching and behaviour in which there is virtually no accumulation as such, got it. And the flame standoff distance essentially mimics a very similar characteristic, okay.

So, this flame standoff distance has been normalized by rs. So, if you recall, rf by rs is supposed to remain constant, right so, but that it does remain constant, when the oxygen environment is high. Other than that, it actually changes quite dramatically, okay. So, the fuel accumulation is a real problem, okay.

And if you are burning in a low oxygen environment, right the flame the fuel consumption rate comes down drastically, got it, okay.
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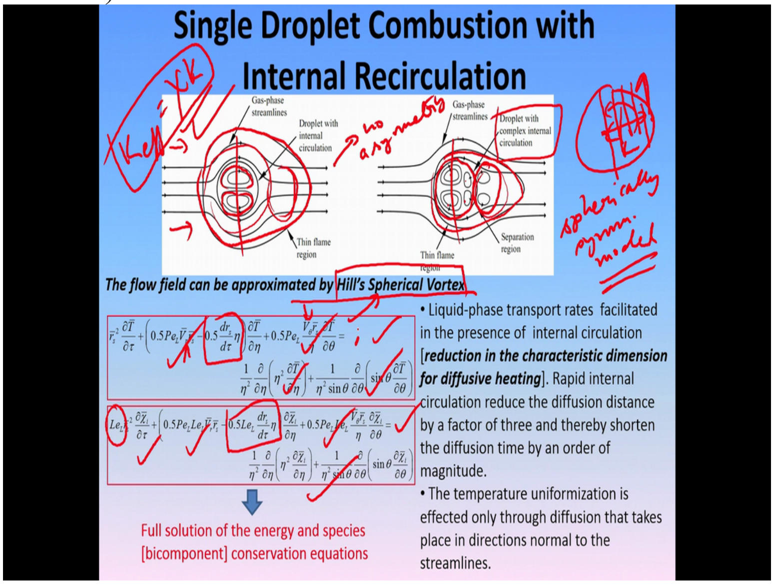

Single Droplet Combustion with Internal Recirculation

The flow field can be approximated by Hill's Spherical Vortex

$$\bar{r}_s^2\frac{\partial \bar{T}}{\partial \tau}+\left(0.5Pe_L\bar{V}_r\bar{r}_s-0.5\frac{dr_s}{d\tau}\eta\right)\frac{\partial \bar{T}}{\partial \eta}+0.5Pe_L\frac{\bar{V}_\theta\bar{r}_s}{\eta}\frac{\partial \bar{T}}{\partial \theta}=\frac{1}{\eta^2}\frac{\partial}{\partial \eta}\left(\eta^2\frac{\partial \bar{T}}{\partial \eta}\right)+\frac{1}{\eta^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial \bar{T}}{\partial \theta}\right)$$

$$Le_L\bar{r}_s^2\frac{\partial \bar{\chi}_i}{\partial \tau}+\left(0.5Pe_LLe_L\bar{V}_r\bar{r}_s-0.5Le_L\frac{dr_s}{d\tau}\eta\right)\frac{\partial \bar{\chi}_i}{\partial \eta}+0.5Pe_LLe_L\frac{\bar{V}_\theta\bar{r}_s}{\eta}\frac{\partial \bar{\chi}_i}{\partial \theta}=\frac{1}{\eta^2}\frac{\partial}{\partial \eta}\left(\eta^2\frac{\partial \bar{\chi}_i}{\partial \eta}\right)+\frac{1}{\eta^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial \bar{\chi}_i}{\partial \theta}\right)$$

Full solution of the energy and species [bicomponent] conservation equations

- Liquid-phase transport rates facilitated in the presence of internal circulation [***reduction in the characteristic dimension for diffusive heating***]. Rapid internal circulation reduce the diffusion distance by a factor of three and thereby shorten the diffusion time by an order of magnitude.
- The temperature uniformization is effected only through diffusion that takes place in directions normal to the streamlines.

Now, we so far have only taken into account a spherically symmetric model, right spherically symmetric model. Now, if we include the internal recirculation like what we did in the droplet vaporization case, correct, okay. Now, in this particular case, if there is a flow around the droplet which is in the normal cases, it is, right. What you will find here is that you can find two kinds of characteristics.

There will be that internal flow that will be created within the droplet which is once again represented by the Hill Spherical vortex, right okay. And this is the flame front. In addition you have this egg-shaped flame front, right. So, lot of the flame actually burns in the wake of the droplet, okay. And there we get a lot of soot and other kind of features, right.

So, this is a very things flame that actually burns but the velocity that is created within the droplet is basically given by the Hill spherical vortex. You can have certain cases. This does not take into account separation. There is no separation; there is no fore-and-aft asymmetry of the droplet, so to say, right. So, there is, no asymmetry. Moment there is a symmetry that you create because of the separation; this is a separation, right.

This is a separation bubble that you create in the aft of the droplet the recirculation structure is a little bit more complicated. Now, you get like four recirculation pockets, right okay and there is a clear asymmetry between the vertical here of course, it is very symmetrical. This is, this is not symmetrical anymore and you still get a flame which closely moves towards the droplet, okay.

A lot of the flame burns in the wake so this droplet has got a more complex recirculation structure. As you can see which cannot be represented by a Hill Spherical vortex anymore, got it. But, we are mostly concerned with this kind of flame for the ease of doing analysis, right. So, how to analyze this? What does the liquid phase transport do? It effectively reduces the diffusive, the dimension of the diffusion heating, right.

So, diffusion what it does is that the surface temperature skyrockets, right. And then, this diffusive front takes some time to propagate to the droplet center, right. Within turn and this length scale is basically L or something like that, right. What does this do is, the internal recirculation because of the convection they reduce this length scale the effective length scale of diffusion.

So, because we already, you know, when we consider the Sirignano’s Effective conductivity model, we remember, we did something like K effective which was more than some factor Chi into K, right. So, it is very similar to that. It essentially means that your diffusivity is enhanced, right. And it can be enhanced by an order of magnitude also, okay in certain cases.

But how do you write the equation for within the full solution for the energy and the species you have to take into account, these two equations once again. You have seen these two equations earlier. This is basically the heat transport equation. This is basically the species if it is a by component droplet that means of well and some other component you have to write it for that fuel component over here, right. That we did already earlier.

So, you can see that this entirely think these two are basically the convective derivative. These two are basically the corresponding diffusion terms, right. And this VR and V theta are given by that Hills spherical vortex, right. Similarly, we can write the species. We get the Lewis number over here and which is alpha by d, okay. And once again these two terms are basically given by the by the convective derivative.

And this is basically the diffusion term once again the regression rate of the droplet has been included in this particular analysis over here, okay got it, okay. So, in the case of when you actually have recirculation within the droplet, you just have to follow the exact same methodology as you did for the droplet evaporation, right. And then, you have the liquid phase analysis roughly remains unaltered, got it, okay.

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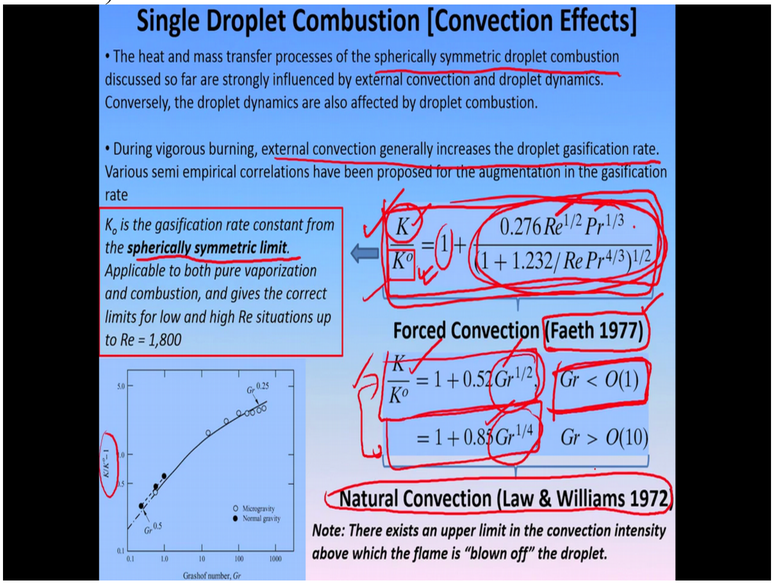

Single Droplet Combustion [Convection Effects]

- The heat and mass transfer processes of the spherically symmetric droplet combustion discussed so far are strongly influenced by external convection and droplet dynamics. Conversely, the droplet dynamics are also affected by droplet combustion.

- During vigorous burning, external convection generally increases the droplet gasification rate. Various semi empirical correlations have been proposed for the augmentation in the gasification rate

K_o is the gasification rate constant from the ***spherically symmetric limit****. Applicable to both pure vaporization and combustion, and gives the correct limits for low and high Re situations up to Re = 1,800*

$$\frac{K}{K^o} = 1 + \frac{0.276\, Re^{1/2} Pr^{1/3}}{(1 + 1.232/Re\, Pr^{4/3})^{1/2}}$$

Forced Convection (Faeth 1977)

$$\frac{K}{K^o} = 1 + 0.52\, Gr^{1/2} \qquad Gr < O(1)$$

$$= 1 + 0.85\, Gr^{1/4} \qquad Gr > O(10)$$

Natural Convection (Law & Williams 1972)

Note: There exists an upper limit in the convection intensity above which the flame is "blown off" the droplet.

We would not cover these, okay. This is also an important thing that when you actually have a signal droplet combustion. We have so far considered that the gas phase does not have in the spherically symmetric model at least. We have considered that a gas phase does not have any external influence from external convection, right when we did the spherically symmetric droplet, right. So, it was burning in an quiescent environment right.

The only question of gasification Stefan flow was that surface glowing effect that you had due to evaporation, right. But as we can see, light in the case of evaporation right as I gave you that example earlier, if you want to say cool a saucer of milk ok how do you do you blow air over it, right. So, in this particular case, when you actually have a strong convective environment around the droplet, right it influences the flame.

It influences the droplet burning, okay. Now, due to vigorous burning external convection generally increases the droplet gasification rate, right. So, the gasification rate because you are removing that boundary layer, right you are removing that fuel vapour. So that more and more soil can actually go in.

So, what people have done? There are a lot of empirical correlations. See, the actual analysis of this is a very time-consuming affair. So, we are just giving you the gist were here k naught is basically considered to be the gasification rate in the spherically symmetric limit. That means in the limit when it is burning in a quiescent environment, okay. And this is applicable for both droplets burning as well as droplet evaporation.

K is the corresponding burning rate or gasification rate of vaporization rate when you actually have convection present, right. So, this is given by this particular ratio and this ratio has been plotted over here, okay. So, this ratio has been plotted over here. And so and this is of course for normal gravity and microgravity.

Let us not go into the details of that. So, for Force convection, this was given, given by Jerry Faeth in 1977 which says that what is the ratio of these two: So one will become, when his factor becomes equal to very small term, then k and k naught will be the same. Other than that k is always greater than k naught by some extent, right because this, if this is a non-zero quantity, right.

This cannot be negative, right. This is a nonzero quantity because Reynolds number, Prandtl number normally cannot be negative period, right. So, this will be greater than 1 for all practical purposes, right. So, this was given for forced convection. Similarly, for free convection, we already said our natural convection we always already define a number called Grashof number if you did if you recall earlier and we said the grashof number of the order of 1 to 10 is kind of not, it is almost similar.

So, you can say that it is all it is almost natural evaporation may not have much of a much of a say. So, even in that particular case, in these two are the formulation as you can see clearly this is a Grashof number of one-fourth function, this is a Grashof number of half function which once again is an empirical correlation which says that how the burning is augmented, when you actually have natural convection.

How burning is augmented when you have forced convection and this is given in terms of Reynolds number, this is given in terms of Grashof number. The normal quantities, however, it should be noted that, if you increase the rate of convection over a droplet, right there is a limit up to which you can increase its gasification rate. Beyond that what will happen? The flow the flame will actually extinguish. You will have no flame, right.

It is almost like blowing off your candle, right you blow it slowly. What happens is that the candle burns, right. But if you blow very hard at the candle, what will happen? That candle flame will extinguish, right. So, you have to keep that limit into consideration, that the flame may actually gets extinct, if you have too high of a forced convection, got it okay. So, that should be taken into account when you actually do this kind of analysis.

So, these are two empirical correlations which you are given by Lauren Williams and Jerry Faeth, okay which actually tells you that how the burning rate is actually augmented from the spherically symmetric limit, okay when there is a convection quantity present, okay.

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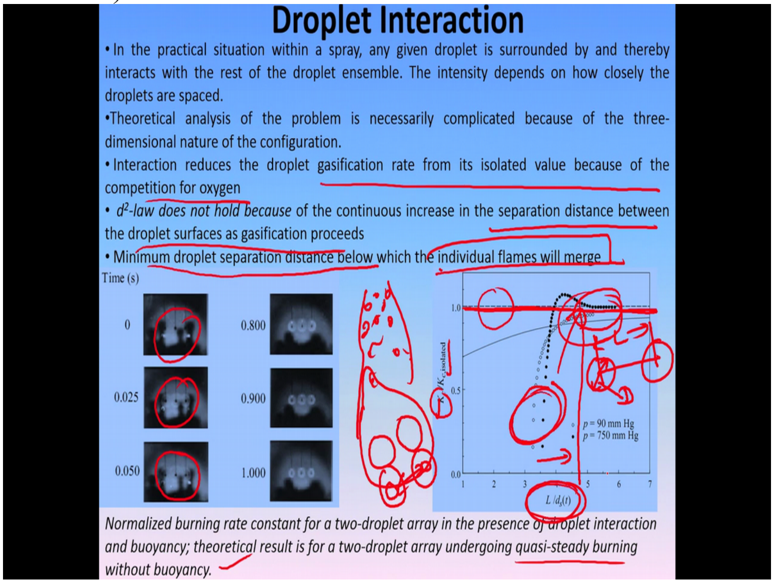

Droplet Interaction

- In the practical situation within a spray, any given droplet is surrounded by and thereby interacts with the rest of the droplet ensemble. The intensity depends on how closely the droplets are spaced.
- Theoretical analysis of the problem is necessarily complicated because of the three-dimensional nature of the configuration.
- Interaction reduces the droplet gasification rate from its isolated value because of the competition for oxygen
- *d^2-law does not hold because* of the continuous increase in the separation distance between the droplet surfaces as gasification proceeds
- Minimum droplet separation distance below which the individual flames will merge

Normalized burning rate constant for a two-droplet array in the presence of droplet interaction and buoyancy; theoretical result is for a two-droplet array undergoing quasi-steady burning without buoyancy.

So, this will be the last slide of this particular lecture. So, in a practical situation however, what you have is that not only you have like a single droplet burning, you actually have a cluster of droplets burning, right that is what is a normal behaviour of a spray. If you look at a spray you will see that there are hundreds and thousands and millions of droplets which are present, right okay and they look like this.

So, in so, you can get you get you can you are not able to distinguish the flames properly, right. They are all kind of very close together they burn as an ensemble, right rather than as individual pieces. Now, obviously this problem is very complicated because it is 3D, the spacing between the droplet actually matters a lot.

And the D square law does not hold here because of separation distance between the droplet surfaces increases as the droplet regresses. So, if you have like two droplets sitting here burning very close to each other so you may have a common flame which enshrouds the two, right. But as time goes on, these two droplets become smaller, right.

And their separation distance now has increased. So, you get individual flames on the two droplets. That is also possible, right. So, the minimum, so there exists a minimum droplet that separation distance below which the individual flames will merge. That means if the droplets are very close to each other the flames will kind of join, so you would not be able to distinguish one claim from another.

So, it is like interacting droplet flames so to say, right. So, in this particular case if I plot kc by kc isolated kc means the burning rate of the ensemble, kc isolated that if we take that single droplet out of that, what will be the burning rate? So, in that particular case, what you have is that, okay.

If you see, look at this particular curve, one is once again when both are the same, right. So, as you increase your L by D this L by D is basically this these are two droplets. So, this is your l and this is your D, right. So, add your L by D ratio increases, right, you approach this value of one that means the individual droplet burning behaviour is re-established, right.

But whenever you are away from where the droplets are very close to each other your burning rate substantially slows down, right because it reduces the gasification rate because of the competition for oxygen. Both the droplets are competing for the same oxygen source, right. So, naturally the burning rate is very low, okay.

When you have this merged flame but as you go towards you but you do proceed towards the value of one as you increase by the L by D ratio all right, okay. So, this is the result for undergoing quasi steady burning. There is no buoyancy effect over here. This actually shows that you do approach the limit of one, okay when the separation distance is about five, got it.

All the droplet seems to be approaching that same value, right it is highly dependent on L by D and the gasification rate is actually substantially lower. You can see it is roughly sometimes 30 percent sometimes 50 percent right okay. So this is important the d square law does not hold interaction reduces the gasification rate and there is a minimum droplet separation distance below which you cannot identify the two flames together.

That two flames gradually de merge, okay. As you increase the L by D ratio depending on whatever is the diameter and the separation distance of the two droplets okay. So, you will see in the next class what happens for that rest of the droplet scenario, okay.