Heat Transfer And Combustion in Multiphase Systems
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Lecture 35
Droplet combustion-I

Welcome, today we are going to start with droplet combustion we have studied vaporization we have studied atomization and now we are going to study the droplet combustion part. And as you will find that the droplet combustion is very similar to the droplet vaporization okay except now that instead of the ambient you now have a flame that surrounds it okay.

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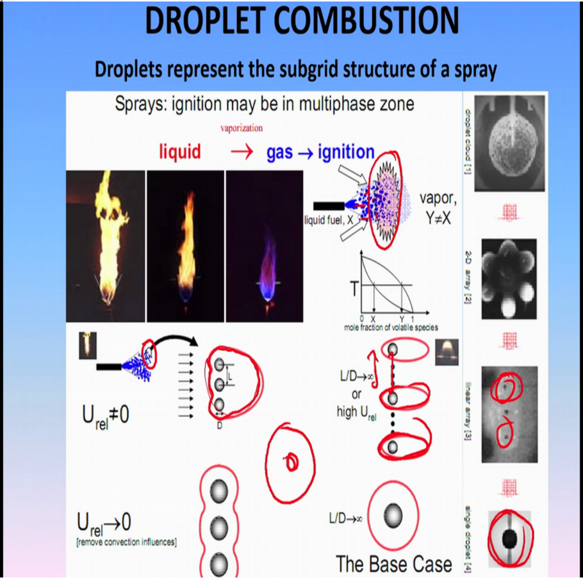

DROPLET COMBUSTION

Droplets represent the subgrid structure of a spray

So, if you look at the PPT there are certain and some of this these are work taken from multiple sources I will give you the references. Many of these works are done by Professor Tom Avedisian of Cornell University. So, what happens is that you would see that if you look at it closely okay the there is a flame front that actually surrounds the droplets.

If you look at this the flame right so it is a droplet and then there is a flame around it if it is a round flame it is under microgravity condition. If it is a flame like this then this is under the normal buoyant flame that you will have around the droplet right. And the droplet basically; why droplet combustion is important because the droplets represent the sub-grid structure of a spray okay.

The spray is actually millions of droplets okay. So, in the sprays the ignition happens that means of fuel has to be ignited in the multi phase region right. So, if you once again look that here for example there is a spray that is coming out which contains millions of droplets. This is ignited right over there right and then you get this flame, flame structure.

If you look closely at the flame that means if you take a region zoom out from that region you will find that there are multiple droplets with an encompassing flame around it like that. Now if it is a very less dense spray right dilute spray you will find that the droplets are further apart and you get separate flames around the droplets okay.

Like for example here or here something like separated flames that. You get in many cases you can get a composite flame also. This depends on the separation distance between the droplets. The dilute spray means to the droplets are well separated that means L by D is basically approaches infinity where L is the separation distance D is basically the diameter of each of the droplet okay.

So, the droplet combustion basically means that there is a droplet and then there is a flame.
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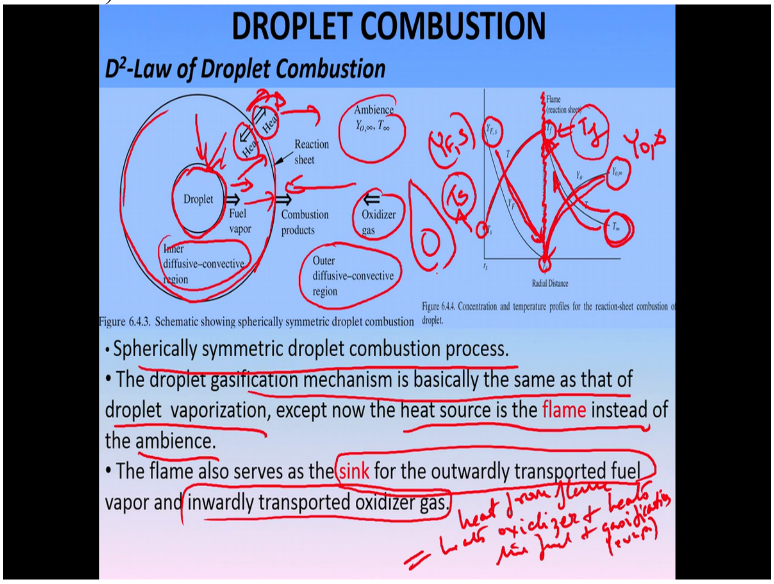

DROPLET COMBUSTION

D^2-Law of Droplet Combustion

Figure 6.4.3. Schematic showing spherically symmetric droplet combustion

Figure 6.4.4. Concentration and temperature profiles for the reaction-sheet combustion of droplet.

- Spherically symmetric droplet combustion process.
- The droplet gasification mechanism is basically the same as that of droplet vaporization, except now the heat source is the flame instead of the ambience.
- The flame also serves as the sink for the outwardly transported fuel vapor and inwardly transported oxidizer gas.

heat from flame = heats oxidizer + heats the fuel & gasification (evap)

So, if you look at this particular graph this will be very clear, so we will first do that D square law for corresponding to droplet combustion. So, what you see over here is basically this is a nice and round droplet right. There is a flame front which sits around it around the drop that over here you can see it right okay.

So, there is evaporation of the droplet is basically fuel droplet okay. So, it can be any real okay. This will vapour and it is a single component droplet that means one single component of fuel is present this well vaporizes get diffused to the droplet, to the flame surface right where the flame front is established right.

The flame actually gives out heat in both the directions this heat goes towards heating the ambient or the oxidizer. This heat actually goes to the droplet and aids in the vaporization process okay. So, this flux of heat which comes from the flame into the droplet is like heating the droplet from ambient.

This situation is equivalent if you put the droplet in a heated flow essentially right okay. But the heat that goes towards the air side what the oxidizer site is very important because the air also needs to be oxidized needs. The temperature needs to be brought up for flame or a reaction to be established right.

So, this is called the outer diffusive convective region this is the inner diffusive convective region. This is the reaction sheet of the flame front this is the ambient. This is oxidizer gas which approaches the flame front and this is a fuel vapour which also approaches the flame front from the two opposite direction right.

If you look at this flame front very carefully, so this is the flame front that we have okay this is some interesting things that you will observe over here. One particular case is that YFs is basically the concentration of the fuel vapour near the droplet surface right what happens is that this will vapour concentration okay.

As you go towards the interface it slowly decreases that is because here it becomes absolutely equal to zero because the fuel vapour is completely eaten up by the flame front right. Similar thing you see for the oxidizer at the far-field oxidizer has got some concentration which is Y0 infinity.

Oxidizer concentration also comes down and gets depleted over here right. So at the flame front both the oxidizer and the fuel get eaten up okay that is the beauty right. So, you have the flame called the YFs decreasing this Y0 also decreases right. One other thing you have this is the temperature at the droplet surface which is given by Ts.

TF also increases and goes to some flame temperature TF okay. This TF the flame temperature is nothing but the adiabatic flame temperature for stoichiometric concentrations right. If you do not recall stoichiometry and other things please pick up your combustion very preliminary course on combustion and you can read it.

Air temperature also increases okay at the flame front. So, at the flame front both the oxidizer and fuel will actually have the temperature TF. So, it is the flame temperature right the droplet surface temperature is much, much lower than the flame temperature remember this. When there is a burning droplet do not think that the droplet surface temperature is as high as a flame temperature that is wrong right.

The droplet surface temperature is at some temperature Ts which is mostly like the wet bulb limit in most cases it is the boiling point temperature of the fuel concern right. Air temperature far field is the normal air temperature that is far away from the droplet. But it slowly gets heated and it goes to the flame temperature as it approaches the flame.

So, now you can see where this heat goes. Part of this heat goes towards heating the oxidizer; part of the heat goes toward heating the fuel and evaporating it right. So, what happens is that the heat from the flame right it serves varieties of purpose right. First and foremost what does it do it first heat's oxidizer right

It first heat's the oxidizer right, it also heats the fuel right plus it aids in the gasification, gasification is basically nothing but evaporation got it. So, that is the purpose of the heat source got it. And this is a spherically symmetric process usually okay that gasification mechanism is basically the same as droplet vaporization.

The droplet does not know what it is what where the heat is coming from except that the heat source is now the flame than ambience. Because the flame is the heat release source, there is the chemical reaction that is going on. The flame also serves as a sink for the outwardly transported fuel and the inwardly transported oxidizer gas.

So, it basically heat's up both the fuel and oxidizer okay. If they are coming in stoichiometric proportion got it. So, that is the basic premise how the flame is established of course this flame does not include the effect of buoyancy or any other things okay. In most of the cases when you have situations like that there will be the flame that will be created where there would be a lot of buoyancy driven effect right.

The flame will be a flame in which the flame will be like this it will be like a teardrop okay instead of a flame like this okay. So, this is now vitally important now that you understand that how does the droplet combustion actually; how does the droplet combustion actually takes place.

So, there is a flame, slowly the reactants or the fuel in this case diffuses to the flame front. Oxidizer also comes diffusing from the outer from ambience to the flame front okay. There they react and the reaction is actually established okay. Now you can see that the flame here we have taken that there is no leakage.

So this flame across the flame there is no leakage of oxidizer of fuel okay. So, it is like a perfect sync, so this is a flame if you look at it, it is like a flame like this usually the flames are thick look at the board have the PPT, what will happen is that the oxidizer gets diffused to the thing. The reactants also gets diffuses to the flame front.

Now in the flame front if you look at the flame front there is no leakage that means the oxidizer does not go to the other side the reactants does not come to this site. So, the flame is up like a perfect curtain it does not allow, it is like a perfect filter okay. So, that is the basic but in most of the cases they do leak okay.

So, there is; those are basically different issues and also we have not taken into account any accumulation that actually happens in that inner region. If you look at the PPT is in a region is a region here okay.

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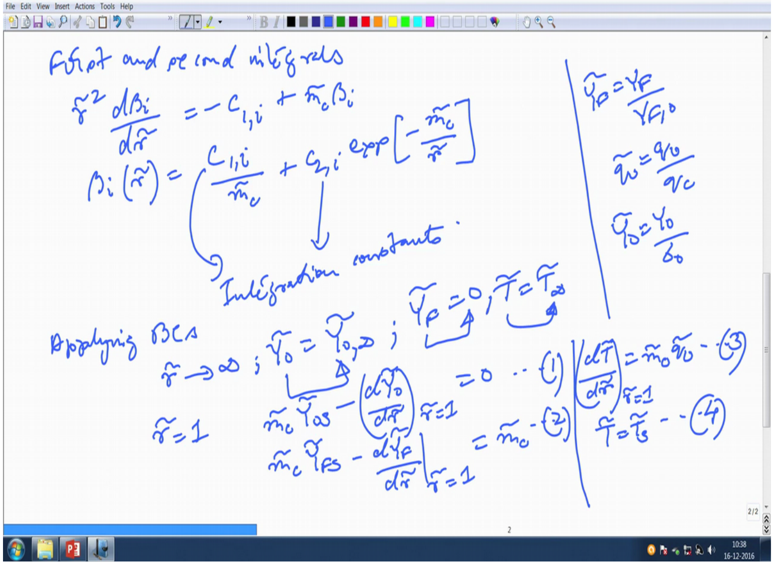

First and second integrals

$$\tilde{r}^2 \frac{dB_i}{d\tilde{r}} = -C_{1,i} + \tilde{m}_c B_i$$

$$B_i(\tilde{r}) = \frac{C_{1,i}}{\tilde{m}_c} + C_{2,i} \exp\left[-\frac{\tilde{m}_c}{\tilde{r}}\right]$$

Integration constants.

$$\tilde{Y}_F = \frac{Y_F}{Y_{F,0}} \qquad \tilde{q}_v = \frac{q_v}{q_c} \qquad \tilde{Y}_O = \frac{Y_O}{\nu_O}$$

Applying BCs

$\tilde{r} \to \infty$; $\tilde{Y}_O = \tilde{Y}_{O,\infty}$; $\tilde{Y}_F = 0$, $\tilde{T} = \tilde{T}_\infty$

$\tilde{r} = 1$

$$\tilde{m}_c \tilde{Y}_{O s} - \left(\frac{d\tilde{Y}_O}{d\tilde{r}}\right)_{\tilde{r}=1} = 0 \quad \cdots (1)$$

$$\tilde{m}_c \tilde{Y}_{Fs} - \frac{d\tilde{Y}_F}{d\tilde{r}}\bigg|_{\tilde{r}=1} = \tilde{m}_c \quad \cdots (2)$$

$$\left(\frac{d\tilde{T}}{d\tilde{r}}\right)_{\tilde{r}=1} = \tilde{m}_c \tilde{q}_v \quad \cdots (3)$$

$$\tilde{T} = \tilde{T}_s \quad \cdots (4)$$

So, let us right now the governing equations for the same and let us see so let me go to the journal and write the governing equation for the flame okay. So, basically we start with the continuity okay. So, the continuity, from the continuity you basically get the burning rate, burning rate of the flame okay.

So, the burning rate of the flame will be actually interesting the burning rate, so the burning rate of the flame is given as mc 4 Pi r square into Rho u, u is basically the radial velocity will go a little fast because you already have seen some of these equations earlier in different forms okay. So, that is the radial velocity that is usually comes because the Stephan’s flow right because of the gasification okay.

That if you recall that total first Stephan’s flow okay. Now the concert; we introduced something called a coupling function. What is a coupling function over here the coupling function is beta i into Yi okay. So, that is the coupling function, if you do not recall it just read your combustion materials okay.

You will know that, what is this coupling function? What is swabs? All do each variable and things like that okay. So, we do a conservation of the coupling function. This is useful okay. So, the conservation of the coupling function. So, once we do the conservation of the coupling function this is basically my Cp r squared into 0.

Here i is basically can be oxidizer, it can be the fuel okay, so that is the thing okay. We are assuming that this lambda by Cp is basically a constant okay. And we are also defining this m dot c as mc divided by 4 Pi lambda by Cp into rs where rs is basically the droplet diameter of the surface okay, so, you got it.

So, using the non-dimensional variables okay using the non-dimensional variables where r tilde you can define it as r by rs also. So, if we can write d by dr mc tilde beta i - r bar square d beta i by t r delta = 0 where once again i = 0 and O, F and basically the oxidizer and the fuel combined.

So, this is the equation that you get using the coupling function do not be too much bothered once again about the math this is a very simple equation that we have written okay. And this is basically the burning rate okay. The burning rate has been normalized okay and we have just used the normalized variables for the for the next few things okay.

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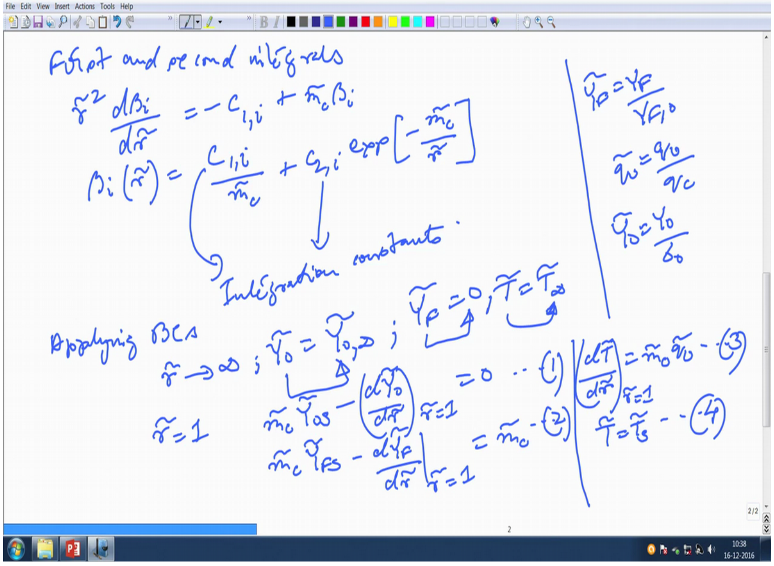

First and second integrals

$$\tilde{r}^2 \frac{d\beta_i}{d\tilde{r}} = -C_{1,i} + \tilde{m}_c \beta_i$$

$$\beta_i(\tilde{r}) = \frac{C_{1,i}}{\tilde{m}_c} + C_{2,i} \exp\left[-\frac{\tilde{m}_c}{\tilde{r}}\right]$$

Integration constants.

$\tilde{Y}_F = \frac{Y_F}{Y_{F,0}}$

$\tilde{q}_0 = \frac{q_0}{q_c}$

$\tilde{Y}_0 = \frac{Y_0}{\delta_0}$

Applying BCs

$\tilde{r} \to \infty$; $\tilde{Y}_0 = \tilde{Y}_{0,\infty}$; $\tilde{Y}_F = 0, \tilde{T} = \tilde{T}_\infty$

$\tilde{r} = 1$ $\quad \tilde{m}_c \tilde{Y}_{0s} - \left(\frac{d\tilde{Y}_0}{d\tilde{r}}\right)_{\tilde{r}=1} = 0 \quad \cdots (1)$

$\tilde{m}_c \tilde{Y}_{Fs} - \frac{d\tilde{Y}_F}{d\tilde{r}}\Big|_{\tilde{r}=1} = \tilde{m}_c \cdots (2)$

$\left(\frac{d\tilde{T}}{d\tilde{r}}\right)_{\tilde{r}=1} = \tilde{m}_0 \tilde{q}_0 \cdots (3)$

$\tilde{T} = \tilde{T}_s \cdots (4)$

Now if we go to the first, if we evaluate the first and second integral okay. So, that is what you get that is the first one and then this is the second one. If you integrate it one more time okay, so that is the second integral that you are actually you are going to get out of this okay. So, how to now the problem remains that how to find out this integration constants right.

So, there are two integration constant these and this, these are the two integration constant okay. So, the integration constants can be evaluated by applying the boundary conditions. So, applying boundary conditions, so what will be the boundary condition the boundary conditions will be as r prime goes to infinity, y0 goes to Y0 this is oxidizer that will go to the infinity of its own right.

Its own concentration that is the ambience the fuel will be 0 and the temperature will be the same as T infinity right. Even without the non-dimensional symbols you can make out right the temperature should go to the temperature ambient right. The concentration the fuel vapour will be actually equal to 0 far away from the droplet.

And the concentration of the oxidizer will be whatever its initial concentration was in ambient like the normal ambient right, okay. However at r bar = 1 that is on at the droplet surface. The things are a little bit more complicated equal to 0, as condition 1 and we will look at what these things exactly means.

This is 1, this is 2 a little bit one more that we are going to write over here qv and the T tilda = Ts this is 3, this is like 4 got it. So, these are like four equations that that the 4 boundary conditions and we are going to look at the stature of each of these equations in a little bit okay.

And the non-dimensional parameters can be like this is YF by YF naught okay, qv is basically nothing but qv divided qc, qc is the heat of combustion okay. Then Y naught it is basically Y naught by the stoichiometry coefficient right, that is Sigma naught okay. Once again if you do not know all this stoichiometry coefficient and other things please read up your combustion book okay.

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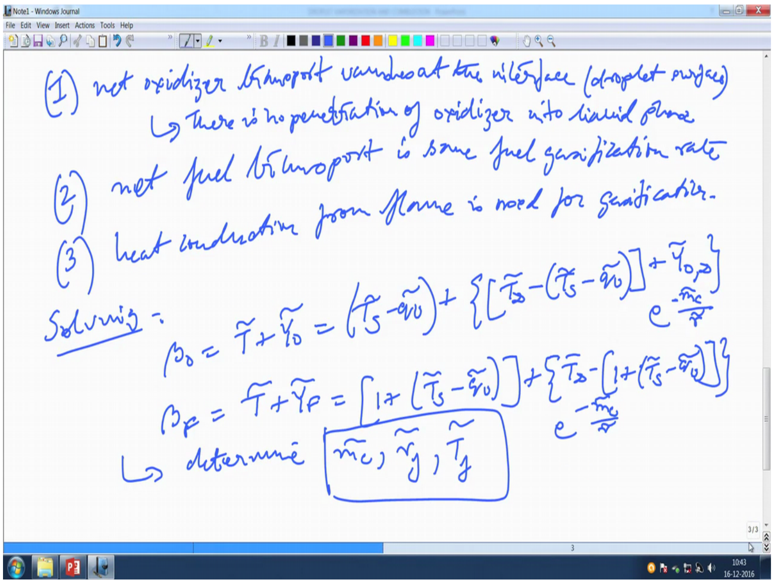

(1) net oxidizer transport vanishes at the interface (droplet surface)
↳ There is no penetration of oxidizer into liquid phase

(2) net fuel transport is same fuel gasification rate

(3) heat conduction from flame is used for gasification.

Solving :

$$\beta_O = \tilde{T} + \tilde{Y}_O = (\tilde{T}_s - \tilde{q}_v) + \left\{\left[\tilde{T}_\infty - (\tilde{T}_s - \tilde{q}_v)\right] + \tilde{Y}_{O,\infty}\right\} e^{-\frac{\tilde{m}_c}{\tilde{r}}}$$

$$\beta_F = \tilde{T} + \tilde{Y}_F = \left[1 + (\tilde{T}_s - \tilde{q}_v)\right] + \left\{\tilde{T}_\infty - \left[1 + (\tilde{T}_s - \tilde{q}_v)\right]\right\} e^{-\frac{\tilde{m}_c}{\tilde{r}}}$$

↳ determine $\boxed{\tilde{m}_c, \tilde{r}_f, \tilde{T}_f}$

So, let us look at the look at the boundary conditions in kind of details. For boundary condition one that we covered over there this essentially means that the net oxidizer transport vanishes okay at the interface. So, net oxidizer transport okay the physical meaning vanishes at interface, interface is basically the liquid or that is the droplet surface okay.

There is there is no penetration of oxidizer either into the liquid phase got it, okay. So, that is the first thing that we should know. The second so this equation was this equation, this particular equation that we had right, this can be also written in a compact Dirac Delta and type of notation.

But for the for the sake, so net fuel transport is nothing but the is the same as fuel gasification okay that is also given over here. This is this expression that you see; on the right hand side you see the gasification or m, the mc tilde right. So, that is the; that is what it actually implies okay.

The third is basically the third actually tells you that the heat conduction okay from the flame is used for gasification, that is about it right okay. So, these are the; so what we have we have now, this equation which has got two constants of integration, these boundary conditions right.

Using all of these things together that means solving, we basically get beta oxygen as the coupling function Ts tilde qv tilde, you already know what this q is our T infinity tilde Ts - qv tilde that is the first expression. Similarly for fuel you can write another one okay. So, these are the two expressions by solving this you get these two expressions over there right okay.

Now we need to determine mc the flames standoff distance and TF the flame temperature. These are the three parameters that one needs to establish. The flame standoff distance we already know, what is the flame standoff distance? If you do not know just take a quick look at the;

so this was the flame standoff distance from here to here. That is the flame standoff of distance right that is rf okay that was the flame standoff distance okay. So, and so, this is the thing, now we can write down the reaction sheet requirement this was all written from a droplet surface perspective right.

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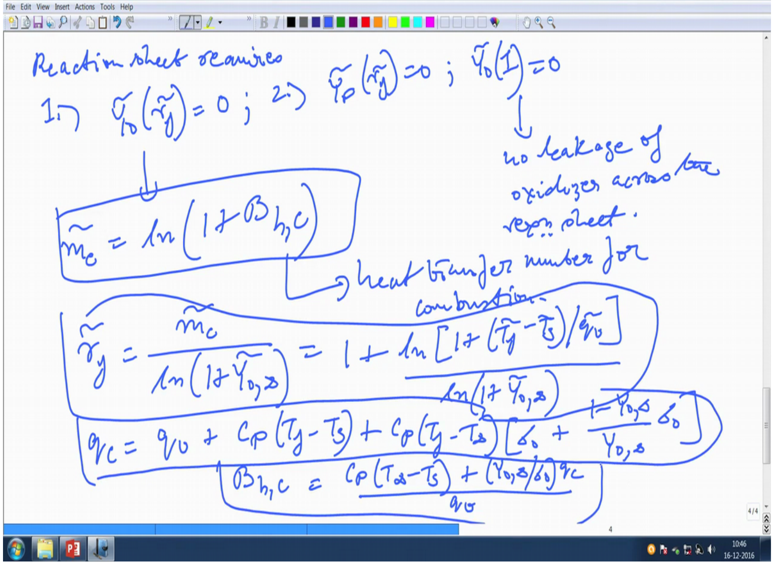

Reaction sheet requires

1.) $\tilde{Y}_O(\tilde{r}_f) = 0$; 2.) $\tilde{Y}_F(\tilde{r}_f) = 0$; $\tilde{Y}_O(1) = 0$

no leakage of oxidizer across the reaction sheet.

$$\tilde{m}_c = \ln(1 + B_{h,c})$$

heat transfer number for combustion

$$\tilde{r}_f = \frac{\tilde{m}_c}{\ln(1+\tilde{Y}_{O,\infty})} = 1 + \frac{\ln[1 + (\tilde{T}_f - \tilde{T}_s)/\tilde{q}_v]}{\ln(1+\tilde{Y}_{O,\infty})}$$

$$q_c = q_v + c_p(T_f - T_s) + c_p(T_f - T_\infty)\left[\nu_O + \frac{1-Y_{O,\infty}}{Y_{O,\infty}}\nu_O\right]$$

$$B_{h,c} = \frac{c_p(T_\infty - T_s) + (Y_{O,\infty}/\nu_O)q_c}{q_v}$$

Now we let us write the expression for the reaction sheet requirements. So, the reaction sheet requires okay number 1 Y naught at the reaction sheet is basically equal to 0 right. Because at the sheet, the oxidizer concentration should be equal to 0. 2, YF at the reaction sheet is also equal to 0 that is also makes sense because the reaction sheet as we say it is a perfect sync for both the terms right.

Y naught at 1 okay that is equal to 0 that is this is at the droplet surface, why this is equal to 0 because there is no leakage, we already said right no leakage of oxidizer across the reaction sheet. There is no leakage of any oxidizer across the reaction sheet that is very, very important right.

So, from these expressions we can first write mc = ln 1 +Bhc, now remember this heat transfer number that we have written is for combustion. It is the spalding heat transfer number for the combustion okay. It is a heat transfer number for combustion right. This is transfer number for combustion got it, okay.

So, similarly the rf is given by mc into ln 1 + this, this can be also written as ln 1 + Tf - Ts divided qv divided by, if you solve we are once again not going through all the math. We are just showing the basic steps. So, that is rf the qc which is the combustion heat, this is non, this is the dimensional form Tf - T infinity.

This is Sigma naught + 1 – 0 by 0 infinity Sigma naught okay and lastly the heat transfer number is basically Cp into C infinity - Cf okay + Y0 infinity by Sigma naught into qc divided by qv, got it okay. Now if you look at these expressions say for example so we have found out what is the expression for the combustion rate, or rather the gasification rate.

This is the flame standoff distance where the flame will stand and this is the corresponding heat release of the flame right. And this is the corresponding Spalding or the heat transfer number. Just look at one thing if we just deal with pure vaporization what will happen is that that this term if you look at the expression over here Y0 this particular term will actually have no significance right.

We will have no significance as a result of that if this is equal to zero, if this is equal to 0 right this will boil down this Bhc will boil down to the pure vaporization case. All these expressions will boil down to the pure vaporization case. You can look at it and you can see that why that should be the case.

Let us take the first expression if you look at the first expression, this expression boils down to the pure vaporization case because Bch will be the same as the pure vaporization spalding heat transfer number right, rf becomes nonexistent in this particular case. And qc becomes the same as qv + this additional heat okay.

And of course Tf in this case does not make any sense, so this also goes. So, it qc becomes the same as qv in this case okay.
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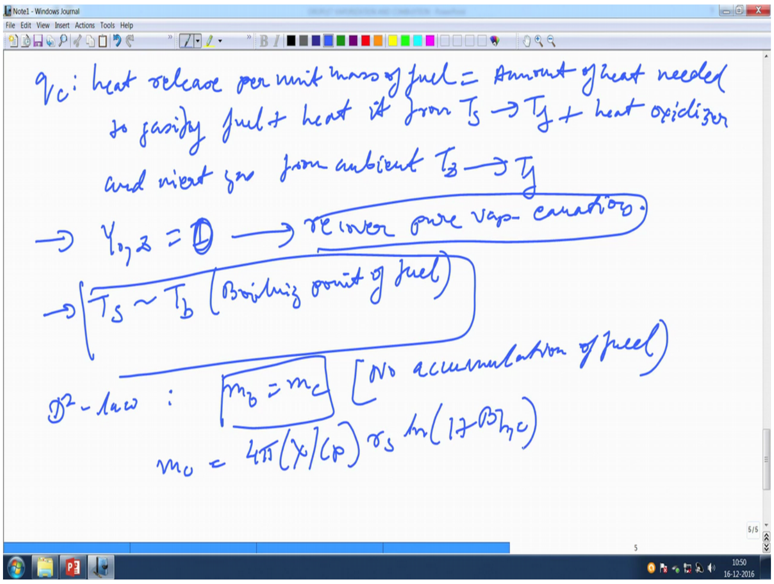

q_c: heat release per unit mass of fuel = Amount of heat needed to gasify fuel + heat it from $T_s \rightarrow T_f$ + heat oxidizer and inert gas from ambient $T_\infty \rightarrow T_f$

→ $Y_{o,\infty} = 0 \longrightarrow$ recover pure vap- equations.

→ $\boxed{T_s \sim T_b}$ (Boiling point of fuel)

D^2-law : $\boxed{m_v = m_c}$ [No accumulation of fuel)

$m_c = 4\pi(\lambda/C_p)\, r_s \ln(1+B_{hc})$

So, this has got some additional significance if you look at it okay say that what is the significance of qc, qc the heat released per unit mass of fuel and this is equal to the amount of heat needed to gasify, the fuel + heat it from Ts to Tf, that is the flame front plus the amount of heat needed to heat oxidizer.

And inert gas from ambient that is T infinity to Tf that is the role of each release right. If we make Y0 infinity = 0 sorry = 0 we recover the pure vaporization equations right that should be what it is right. Ts, is almost like the boiling point temperature that is the boiling of the fuel boiling point of will okay.

And so these are the generalized set of equations that we have in this particular exercise, right. Now in order to derive the D square law for okay, in order to derive the D square law okay what we can do is that we have to equate for D square law what we need to do is that we say that mv = mc that means the vaporization is the same as a gasification.

This has got the image here the implication that there is no accumulation of fuel right. So, whatever is being produced is actually that is actually instantaneous rate of the fuel gasification is equal to the rate of the fuel consumption okay at the reaction sheet. So, mv is the same as your mc.

So, mc is basically what 4 Pi lambda by Cp into rs into ln into 1 + Bhc right that we just now we said, that is what it is going to be remember the previous expression was for m tilde. So, if you just dimensionalize once again that is the form that you are going to get.

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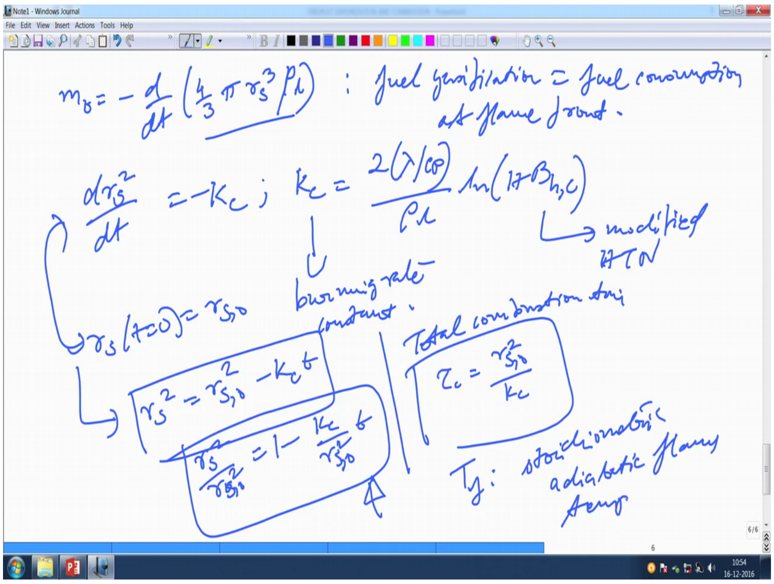

$m_v = -\frac{d}{dt}\left(\frac{4}{3}\pi r_s^3 \rho_l\right)$: fuel gasification = fuel consumption at flame front.

$\frac{dr_s^2}{dt} = -K_c$; $K_c = \frac{2(\lambda/c_p)}{\rho_l}\ln(1+B_{h,c})$

burning rate constant.

modified HTN

$r_s(t=0) = r_{s,0}$

$\boxed{r_s^2 = r_{s,0}^2 - K_c t}$

$\boxed{\frac{r_s^2}{r_{s,0}^2} = 1 - \frac{K_c}{r_{s,0}^2} t}$

Total combustion time

$\boxed{\tau_c = \frac{r_{s,0}^2}{K_c}}$

T_f : stoichiometric adiabatic flame temp

Now the rest of the thing becomes very easy when you have a form like this okay mv on the other hand is given by the rate of change of droplet diameter which is like 4 3rd PI rs cube divided by sorry into Rho l right that is mv right. So, what we have said that the fuel gasification is the same as fuel consumption that was the basic assumption right consumption at flame front got it, okay.

So, now if you just equate this expression with the previous one that we wrote for mc this will give you the same dr s square by dt = -Kc, Kc = 2 lambda by Cp by Rho ln 1 + Bhc right. This you can recall this is the modified heat transfer number HTN okay. So, usually this is the burning rate and it is constant okay and we also have rs at t = 0 = rs0.

So, based on these two expressions if we integrate it you get rs square = rs0 square - Kct right, okay. Kct that looks exactly like the D square law right. So, if you divide rs square by rs0 square it will be 1 - Kc by rs0 square into t, right. So, that is D square law right equivalence of the D square law except K has been replaced by Kc over here right.

And the total combustion time is Tau c = rs0 square divided by Kc right. So, these are the two expressions that you get after all these things. So, what we have got here we have got the same kind of distance, flame temperature, the revised heat transfer number, the chemical heat release rate.

And from that we have shown that it can be also shown that the droplet combustion also follows a similar D square law with whatever assumptions that we made in the D square law right. And we also showed that if you just turn on the oxidizer concentration to be equal to zero you recover the original vaporization equation okay.

So, in the next class we are going to complete this discussion on droplet vaporization and we are going to look at; after that we are going to look at some more phenomena related to this how the droplets burn and how the burners, the cluster and things like that okay. So, that be that the mathematics which you might want to just do a brief review.

And once again Tf just I want to write it down over here, Tf is nothing but the stoichiometric matrix adiabatic flame temperature okay. So, we end the lecture here in the next lecture we are going to pick up and see that what other things we can learn about this droplet combustion, thank you.