Heat Transfer And Combustion in Multiphase Systems
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Lecture 34
Spray / droplet breakup models -II (WAVE model)

Welcome to today's lecture in the previous lecture we talked about the TAB model and we showed that how the TAB model is basically a one degree of freedom model in which the droplet actually oscillates but in a unidirectional manner. And it breaks up when the North Pole and the South Pole of the droplet actually merge.

And you get to dotted droplets and we also found out how you can get the velocity and the sizes of these dotted droplets. But as we said that the tab model was only valid for a low wave numbers range basically between 0 to 11 or 1 to 11 okay. And but in most of the cases for large wave number whenever you have this multimodal kind of a breakup or catastrophic mode of breakup you actually experience waviness or develop on the droplet surface okay.

And the TAB model is not a useful model for that because it is no longer a single degree of freedom system at all right. In this situation one needs to do a linear rigorous stability analysis. And basically through that stability analysis what we try to find out as we explained in the last class that there are waves that have created on the surface.

How fast that these waves grow, which are the waves that actually grow to such an extent that it might actually atomized the liquid okay. So, that is the thing that we actually said. Now if we, if you recall if you just look at the slide.
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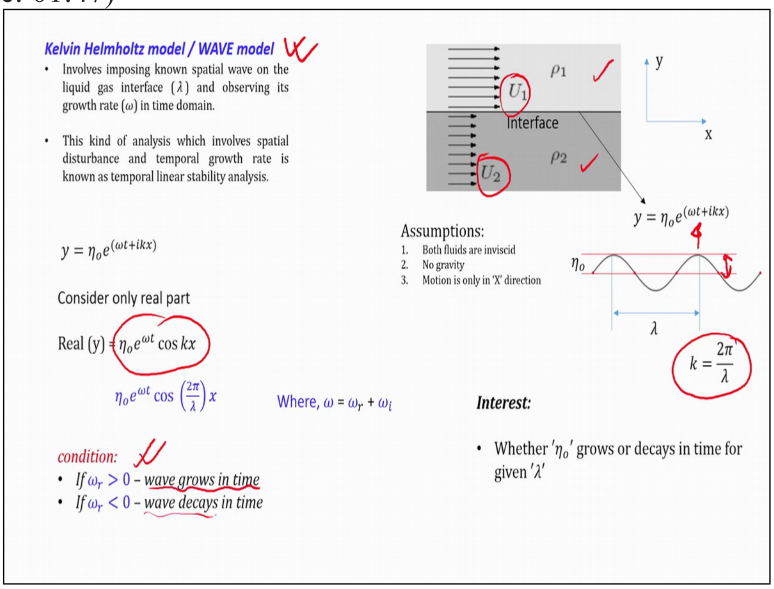

You will find that in this particular case okay the most of the ways we wave model this model basically on the Kelvin Helmholtz type of a model okay. In this particular model okay what we take is that both the fluids were inviscid they had a variation in density. There was a U1 and U2 right. Now any small perturbations like this, what we have created on the droplets surface is modelled like this okay.

And we consider only the real part okay and if the real part this is basically omega is basically the growth rate and lambda is basically the wavelength and k is basically the corresponding wave number. So, if Omega r is greater than 0 okay that means the real part of the growth rate, the wave grows in time.

If it is less than zero it decays with time okay. So, that was what the thing that we covered in the last class.

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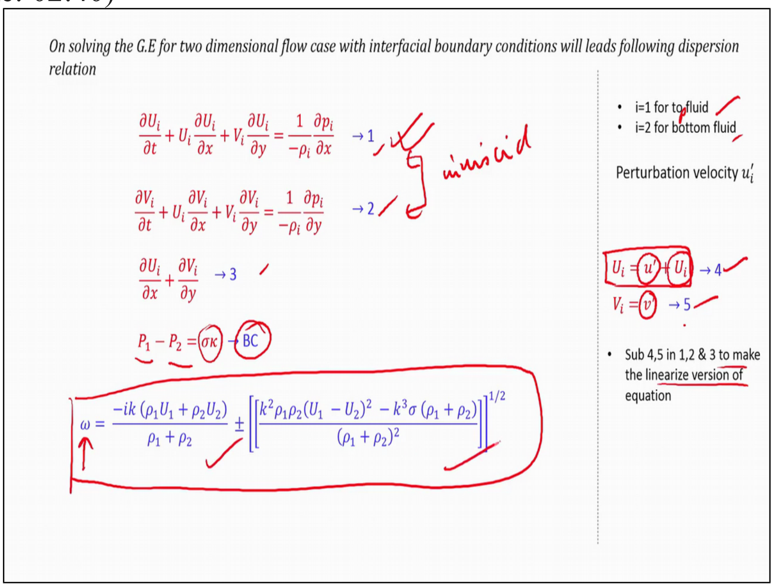

How do I get this basically we do it in two ways one is that if you look at this particular equation over here, on this okay. You will find that we have written basically two equations which is basically nothing but the momentum equations right. But these are inviscid form because you do not see any discuss terms over here anymore right.

So, I = 1 is basically the top fluid and I = 2 is the bottom fluid okay. So, the other thing is that this is the boundary condition this is basically across the interface the pressure differential is given by the surface tension into the curvature. This we already did in the previous classes right.

Now what we do is that the simple things as we learned earlier that your Ui is, has got a base component right a base component and there is a fluctuating component on the top of that.

Your Vi only has a fluctuating component so if this is 4 this is 5 if we substitute them in these three equations that we have over here okay.

And in order to make a linearized version one can show and you can do the map yourself in your spare time that the growth rate is basically given by this particular form right. There are the two densities there is the surface tension the wave number okay the difference in the velocities okay.

So, this is the complete form that you will get if you work from this equation. This set of equations okay with the corresponding interfacial boundary conditions.

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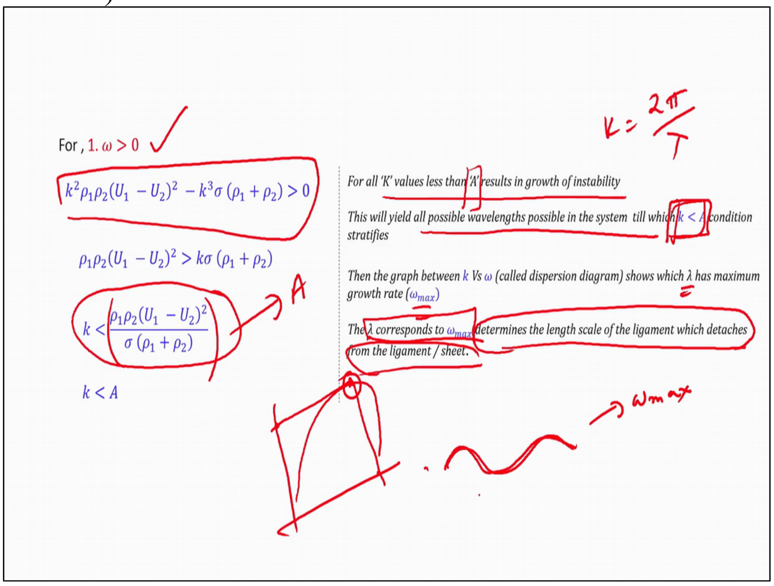

Now as we said when we Omega is greater than 0 okay or the real part of the Omega when it is greater than 0 okay that means the second part of it right when it is greater than 0 okay you will have unstable situation right. So, in this particular case the unstable situation can be written as this. This is greater than zero okay.

So, if you do the algebra you will find that the essential relationship boils down to this where this part is basically represented as A, right. So, it means that when the k values okay are less than A okay where k if you recall was 2 pi by lambda right. When this is less than a there is always a growth of instability okay.

So, any wavelength all possible wavelengths which have value left at k less than A okay well basically intensify right okay. So, this is basically the situation that k less than A, okay. So, and there of course will be a lambda for which the growth rate is maximum that is called the maximum Omega x or the maximum growth rate.

The lambda corresponding to the maximum wavelength determines the length of the ligament which detaches from the sheet.

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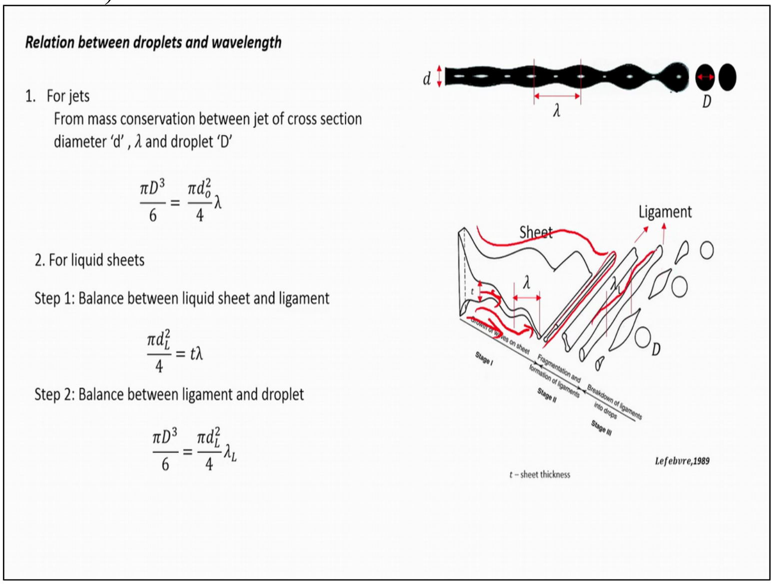

I will show that in this particular plot what happens if you look at it here. So, this is for example a liquid sheet like this right okay. So, the liquid sheet is moving with a velocity and then there is air that is moving also with a velocity. So, you get this ligament right you get this waviness on the surface and the ligaments detach and they subsequently break up with showing all these oscillations.

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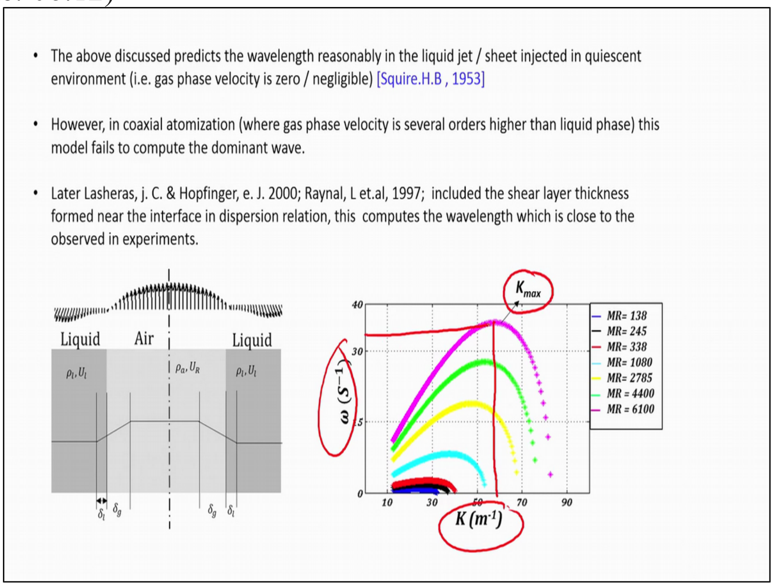

Now if you look at this situation if you plot that Omega versus that k that we were talking about right. You will find that this is the maximum k for which the growth rate is maximum right got it. So, this is what we meant when we said that over here the lambda corresponding to the maximum Omega max right determines the length scale of the ligament which detaches from that sheet.

So, that sheet that we showed what is the length scale of the ligament that actually detaches from the sheet is determined by what is that lambda or what is the wavelength. This wavelength corresponding to the maximum Omega max got it. So there are; so, this that wavelength is the strongest or the most dominant right as we saw in that particular dispersion curve right which looks something like that right.

So, in that particular dispersion curve so you saw this right. So, this is that Omega and the corresponding lambda gives you that what will be the detachment of the ligament that you will get out of it that means the length scale of the ligament let me get out of it okay. So, it is a simple linear stability analysis inviscid conditions okay.

By which you can actually predict what will be the growth rate of the disturbances and what will result in, what will be the resultant ligament diameter okay.

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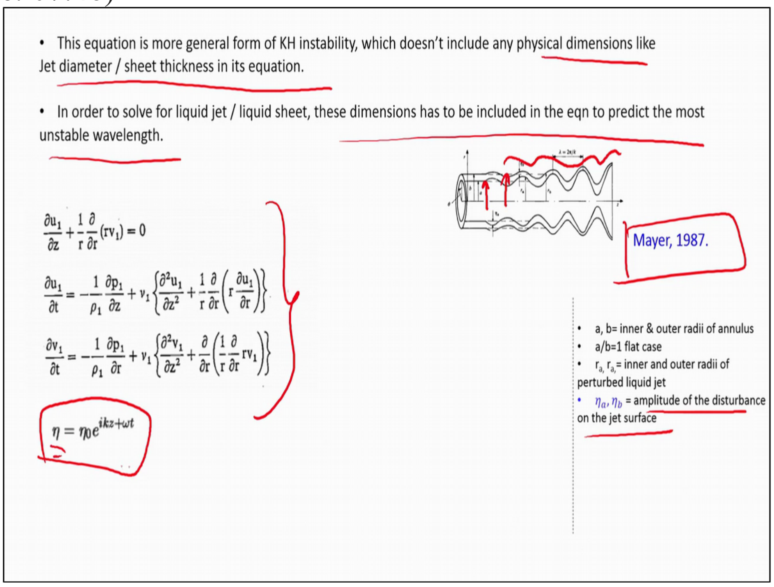

Now the previous equation was a more general form of the KH instability does not include physical dimensions like for example jet diameter and jet thickness. Now if you want to solve for a liquid jet or a liquid sheet okay whatever is the case may be like in the previous case we showed a liquid sheet okay.

So, in this case all the dimensions have to be included to predict the most unstable wavelength correct okay. So, like for example in this case if you look at it there is an inner diameter, there is an outer diameter and there is some perturbations that are created on the surface like this okay.

The jet has gotten or this cylinder has got an undisturbed okay. Liquid jet have got an undisturbed or unperturbed diameter of ra and an rb okay. So, there are amplitudes of the disturbances on the droplet surface on the on jet on the surface. If this is a situation like this which Mayer did in 1987.

You can write basically the governing equations for this right in a similar way that we did earlier right. And you can also; this is the same thing okay you model the perturbation the infinitesimal perturbation that is created in this particular form where k is basically the wave number Omega is basically the growth rate this is the same thing that we did in the previous one okay.

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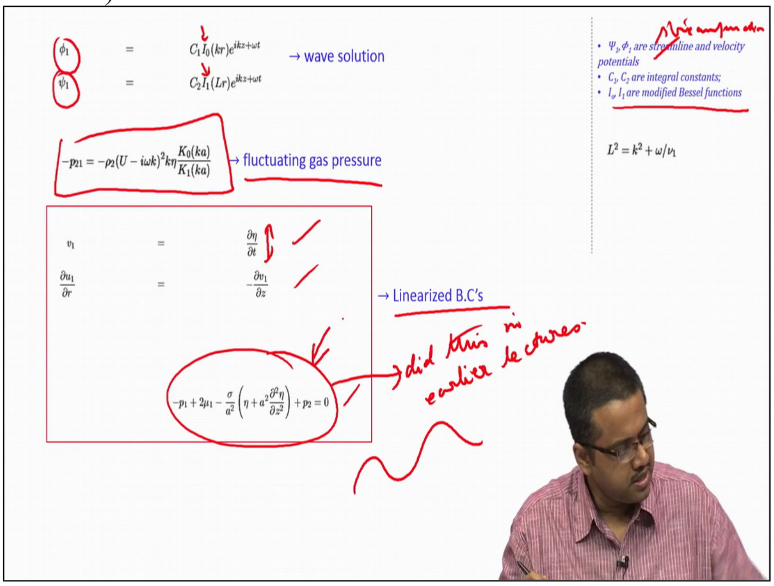

Normally people use the potential function or the stream line of the stream function okay stream function is not streamlined this is actually stream function okay. So, stream function and the velocity potential okay to basically propose the solution. Now if you see these I naught but the math is a little bit complicated I naught and II are basically the modified Bessel function okay.

And this particular relationship gives you the fluctuating gas pressure. We do not have to understand everything but this is just a mathematical procedure okay and these are the corresponding linearized boundary condition okay. So, this is basically the interface oscillation v1 is basically the interface oscillation right okay the rate of change of that interface oscillation or the change in the distance of that interface okay.

And so based on this these are the equations that you get this is once again the similar type of equation that you would get when you actually have the interface across the interface which we recall that we already did this in earlier classes this in earlier lectures right. If you recall this was done right when we wrote the genetic condition across interface.

And we wrote the pressure and we wrote the surface tension and all those things viscosity everything we took care right. So, that part should be well understood if you have forgotten you can go back and study that portion okay.

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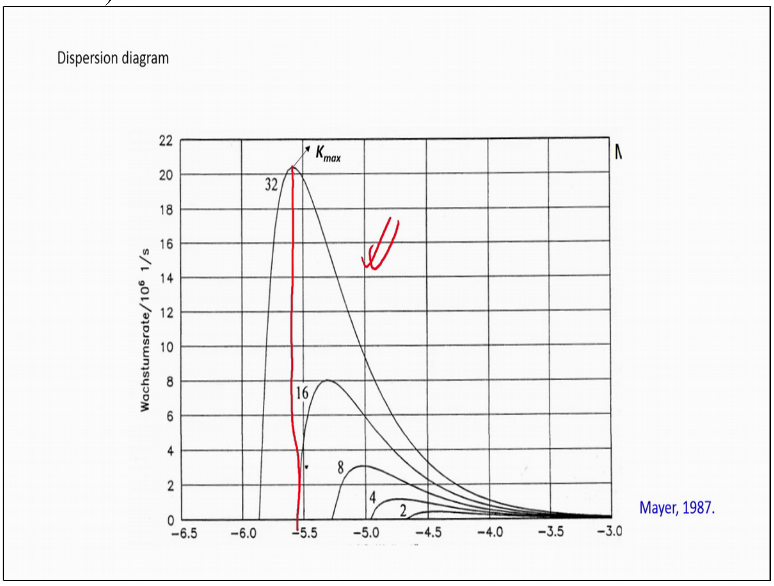

So, this is what the typical dispersion curve actually looks like if you look at this dispersion curve once again it is the same thing okay. So, this is corresponding K max okay the maximum growth rate that you will have okay. And once again the logic is essentially the same that that we showed earlier okay.

So, using this equation one can find this is the most important part so this can predict what is the maximum growth rate that you are going to have okay.

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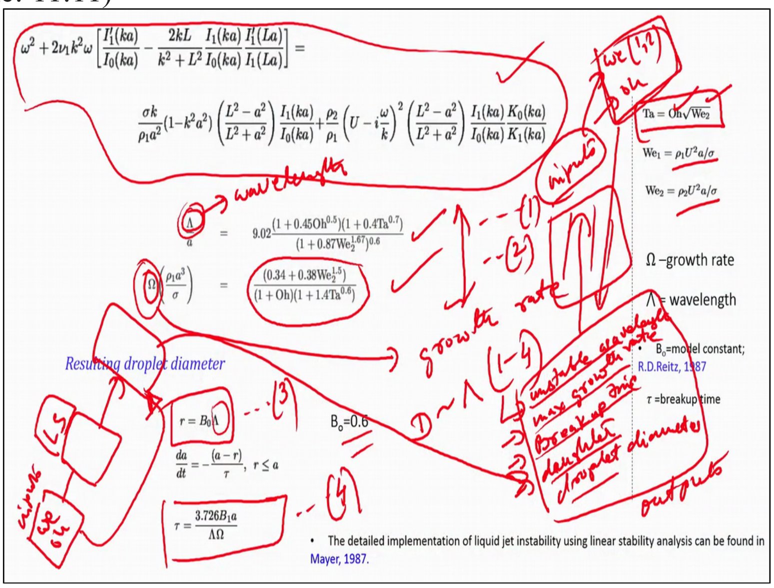

Now this is very complicated but you do not have to go through all the math what is what is most important is that we can show that this relationship is valid. You can try it out okay and then ultimately these two relationships are the most important one. This is basically the wavelength. And this is the basically the growth rate, now you can write it as Omega, you can write it as lambda whatever does not really matter okay.

And this Ta that you see is basically Ohnsorge number Weber number combination and where the two Weber numbers are given as that Weber number 1 and Weber number 2 right. So, these two relationships enable us to calculate that depending on a particular Weber number and a particular Ohnsergo number right.

What will be the maximum wavelength, what will be the most optimum wavelength right that will lead to this kind of dispersion? Similarly the maximum growth rate can also be predicted if you know the Weber number and Ta and Ohnsorge number right. So, all these things all these numbers; this actually gives you what will be the growth rate and the corresponding wavelength okay.

For the most for the maximum growth rate and the most disturbing wavelength if you add it in that fashion right, so the resultant droplet diameter is more or less half of the maximum wavelength right. The resultant droplet diameter that means the diameter of the droplet will be somewhere around the similar value as this guy okay.

And similarly the time scale for the break-up is given by that particular form right. So, these three equations this is 1, this is 2, this is 3 and this is 4. So, these 4 equations 1 to 4 give you basically many things one it gives you the most unstable wavelength right. It also gives you the max growth rate okay it also gives you the break-up time that is important okay.

It also gives you the daughter droplet the dia right okay. And all these information's you can get these are your output. Your inputs are Weber number basically 1 and 2 right, Ohnsorge number if you know Ohnsorge number Ta is basically a derived quantity okay. So, if you feed these two values into the what we call the black box.

Let us consider this to be a black box right, so it is like this, so your input is like that these are your input. Then this is your linear stability analysis right. So, that is the box and your output is all these things got it, okay. So, the input is basically Weber number and Ohnsorge number, this is linear stability is the black box.

So, we do some operations we solve some equations which is basically nothing but still the governing equations right. And using all those things you get an output which is basically nothing but unstable wavelengths maximum growth rate break up time and daughter droplet diameter. So, even if you do not understand all the math does not really matter okay.

Because but you can use feel free to actually work out the map right. So, but because of the interest of time we are not solving it we are just giving you the key equations okay. And how we actually derived it right the procedure essentially and from there you can easily show that you can; your main question through the linear stability analysis will be to determine what is the growth rate?

What is unstable wavelength? What is the break-up time? What is the droplet diameter right. If you get all this 4 information that then the model serves this purpose okay that is exactly what we have done. And the input you want to cast it in such a way that the input becomes a function of your Weber number and Ohnsorge number, right well not in all situations you will be able to get such clean quantities.

But in most of the situations you can and that is an interesting over here okay. So, based on this there are other models also.

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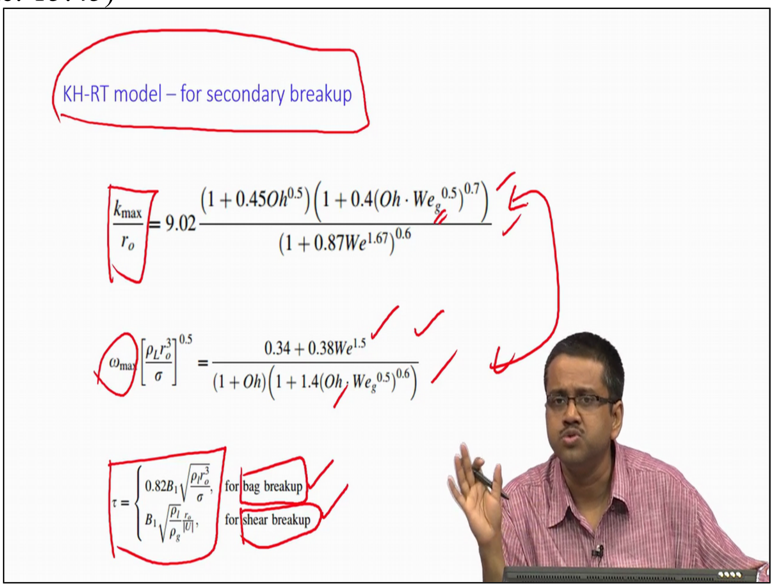

So, if you look at it okay for secondary break up the wave model is particularly useful for primary break up also as you saw okay. And we will just have one more slide on this just to reiterate the Kelvin Helmholtz and Rayleigh Taylor kind of a model for secondary break up okay is it once again let us not go through the math once again.

This is once again the K-max which is a maximum K this is the corresponding maximum growth rate both of these are given in terms of the gas phase Weber number as you can see over there right. So, these are; the once again it is cast in terms of Weber number and the Tau's that means a breakup if you recall what is called a bag breakup and what is called a sheer breakup.

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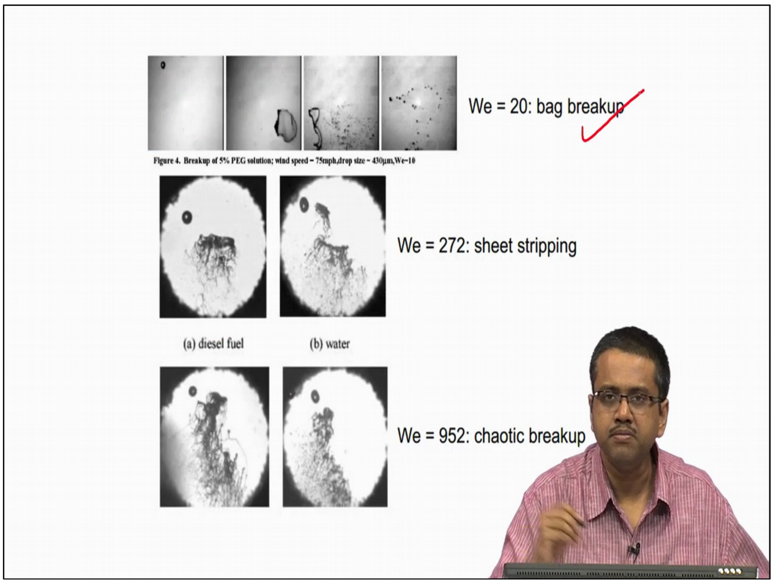

You do not recall let us just go and show you the picture once again to recollect from memory. So, this is like a bag breakup that you saw right okay.

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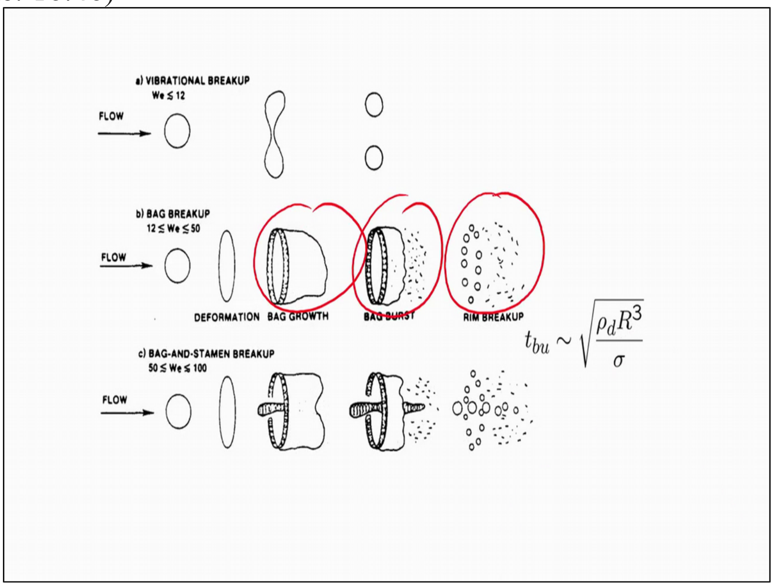

And if you have forgotten even now so this is also how the phenomenological bag breakup actually looks like right okay. So, based on all these things so this is for the bag breakup and this is for the corresponding shear break up that you have okay so both of these are written in terms of Weber number and Ohnsorge number.

And the break up times also you can calculate, so basically this was a lot of purpose okay that you can know from this expression okay. These are let me remind you these are this does not include any effect of swirling flow or any other complicated slow dynamics okay which happens in real life.

This is a pure jet, a liquid jet that is coming out in a pure uniform flow field that is how this is models okay. So, let us not get ahead of ourselves and think that this is something else okay.

So, this is just a pure liquid jet that is coming out in a coaxial liquid flow. So, there is no swirl no tangential component and all of those things have been neglected in coming up with this expression.

But they are surprisingly powerful because they enable you to calculate right that what will be there for example the droplet diameter or the time scales, back up the envelope calculations you can quickly do okay. For your real life situations you might have to do a more sophisticated simulation okay.

These are not sophisticated is a pure linear stability analysis okay so these are workarounds right. Because ideally if you want to really predict the droplet breakup and other things computationally you need to do something like the volume of fluid or level set kind of things okay.

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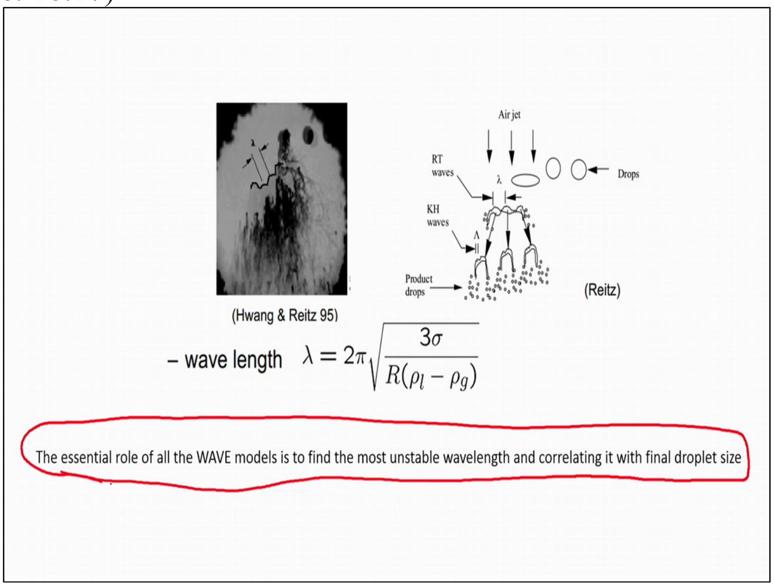

So, essential role of the wave model is to find the most unstable wavelength and correlate it with the droplet size so that is all that is there in this right. So, let us assume that this is all that you have to know about the wave model and it does a really good job about it.

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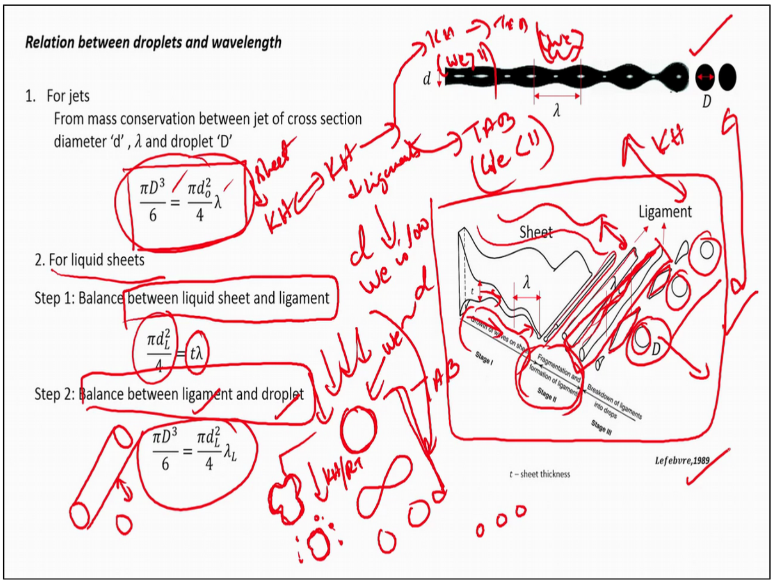

Now before we end this particular discussion on atomisation okay. So, let us revisit this figure that we say it once again and let us look at the different stages. So, what happens is that in most of the applications the droplets do not come out as droplets. So, there is a liquid sheet that comes out okay whether this liquid sheet forms outside the nozzle or inside the nozzle that is subject to the nozzle design.

Whatever nozzle design that you are kind of pursuing right, so one needs to predict, so there is a common misnomer that oh I know what droplet sizes are coming out of the nozzle that is incorrect right. In order to predict what droplet size is coming out of the nozzle okay or after certain distance away from the nozzle right.

One needs to actually know that that droplet atomization mechanism, so to say. So, in this case most of the atomization mechanism will be that you are going to actually have a liquid sheet this liquid sheet you impose some kind of a disturbance or that disturbance naturally happens because of the difference in the flow rate or the other difference in sheer because of the sheer.

And there is a wavelength that actually starts to grow over this liquid sheet if you look at the PPT over here you will find then you have this sheet on which varies disturbance that has been created. This liquid sheet immediately comes out and breaks up into this ligament is called fragmentation and formation of ligaments.

So, the formation on waves of the liquid sheet is what that Kelvin-Helmholtz type of instability is. Then this forms actually this ligaments on the surface this, ligaments actually detach so these are like cylinders. These cylinders further undergo oscillations that can be Rayleigh Tailor it can be Kelvin Helmholtz once again okay.

This cylinder that you see this particular cylinder if you concentrate you are developing these oscillations on the surface right of the cylinder. The cylinder then after that undergoes this kind of break up okay and they form actually this dotted droplets right. For it is breakup from ligaments into droplets.

Up to this that KH instability is particularly powerful right even when this droplet actually forms these are nice and round droplets sometimes okay. They still face this aerodynamic load right around it. So, these droplets can actually undergo that kind of break up which is basically like the TAB.

So, the TAB model is good in the case of secondary breakup okay. This droplet however can also undergo oscillations like this right and then it can actually split. So, that will be more like KH or Rayleigh-Taylor or something like that got it, okay. So, there are two pathways basically so it depends all on the droplet size.

Because your Weber number is proportional to the droplet size right, so if your droplet size is sufficiently low after this stage after the primary breakup stage okay. If it is sufficiently low that is Weber number is actually low okay you can use the TAB model to predict the secondary breakup.

However even after this breakup is over if you see that Weber number is still very high you can get this secondary connotations on the droplet surface and you can get this secondary breakup through KH of the surface active modes right. But there will be a certain size at which the droplet Weber number will be sufficiently low where you can use that TAB model at that point okay.

So, this basically a hierarchical or a sequential modelling first you do KH then you do KH once again right then you can have a bifurcation you can do KH then you can go to TAB or you can directly go to TAB. So, this will be for Weber number greater than actually here. Weber number greater than 11.

This will be for Weber number less than 11, right. This is basically the sheet, this is basically the ligaments got it, that part should be clearly understandable correct okay. So, you can also find out say for example so by simple math that the mass conservation between a jet and the droplet you can find out for jets what will be the droplet diameter.

So, the droplet diameter and the corresponding perturbation, perturb jet, cross section. For liquid sheets you first balance between the liquid sheet and the ligament and then you balance between the ligament and the droplet. So, when it is liquid sheet, liquid sheet is like a sheet right. So, it is given by Phi dl square into the actually this is given by t into the corresponding lambda.

And then you correspondingly get a ligament, ligament is like a cylinder right okay. So, you can get it from the ligament that particular thing okay. So, l cancels from both sides and then you get a balance so this is a balance between the liquid sheet and the ligament right. Then from the ligament and the droplet you get another level of because that also has this wavelength right, on the top of the of the ligaments surface.

So, from there to the drop okay you can add another mass level. So, simple exercises okay. So, whether it is a jet like this right or whether it is a sheet like this, whatever is the configuration most of the falls within these two configurations we can actually show this. The work is from Lefebvre in 1989 particular paper right.

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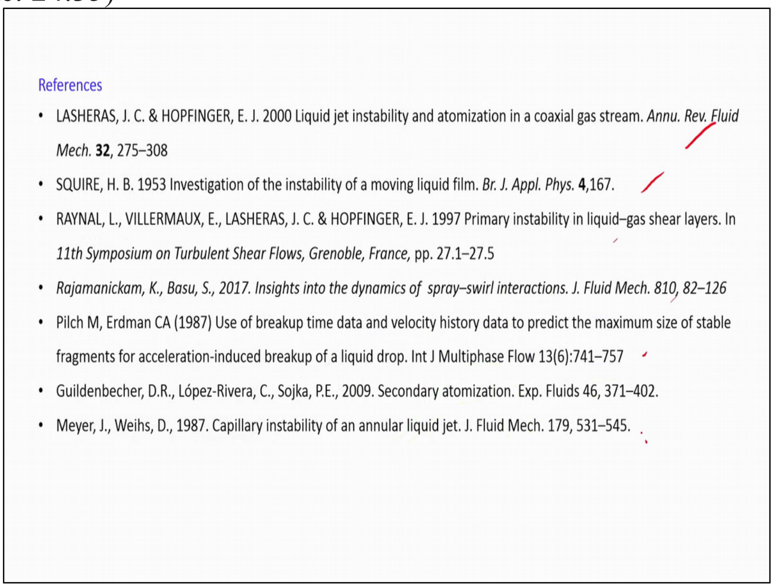

So, this we have already shown how to find it out these cases and these are the references that you should study okay a lot of these are in physics of fluids and experiments in fluids and JSM Journal of Fluid Mechanics. So, it is recommended that you study all of these papers in kind of details okay.

So, I think you have given a nice idea that a jet breaks up a sheet breaks up. A sheet breaks up into ligaments, ligaments breaks up into droplets droplet breaks up into smaller droplets right. This mechanism is either KH or you can predict it by the TAB depending on which end of the spectrum you are. And there are other sophisticated models from the top of that that makes basically fine tuning okay.

In the case of a jet exactly the same thing happens first is KH and then from the drops you can have TAB model or other such models of the enhanced TAB there is enhanced TAB and other types of models okay which basically are more sophisticated in nature okay. How do you want model the KH instability just by doing a linear stability analysis.

You can include the effect of viscosity you may exclude the effect of viscosity right because viscosity is like a damping kind of a system okay. So, as we said some wavelengths can be actually damped okay. What we want to do through the KH instability we want to find out the maximum growth rate, the most unstable wavelength the break-up time scale and the daughter droplet diameter.

If we have all this information then we can predict what will be the sizes that will be coming out. What was the TAB model, TAB model was a vibrational model essentially one degree of freedom which predicted what will be the droplet diameter just by treating the droplet as a spring mass system under damped spring mass system right.

So, that is all that we did these are the two basic classes of models. There are a lot of other models also more sophistication okay. But these are very analytical, so that you can get a feel of what is actually going on right. For example in the KH you know that there is a disturbance that grows on the surface this disturbance just gets amplified.

And as it gets amplified it just actually snatches or just extracts the droplets from the peaks of those waves, from the peaks of those waves okay. So, these are very important for the understanding that why atomization is important, what kind of atomization we can have, what are the different classes of atomization, that also covered.

Bag, sheer, vibration a lot of modes of a atomisation okay. So, we end here in this particular topic okay where we have covered all this whole gamut of activities okay including there is some math which you may want to do in your spare time right. But the basic governing equations we have written.

We have also shown you examples that what are the different types of breakup from our own experimental data. And what will be the corresponding time scales how to determine these constants in the case of a TAB model and things like that right. What kind of a study people actually did okay so all these things will become very helpful when you actually go to the next level okay.

We also showed that way you when Ohnsorge number starts to become important also the number is the viscous effect we saw that it only becomes important kind of after one roughly okay, so to say okay. So, and we define what Weber number is once again there is a d over here okay so that you should remember and this also has a d okay.

So, the Weber number is the key important parameter, always keep that in mind Weber number is the most important parameter right in whatever you do right. An Ohnsorge number basically takes into account Weber number and the viscosity right and the surface tension all of these things combined.

So, and most of the maps will be done by Weber number. We also had a good discussion on how the spray formation actually happens that there is a primary breakup zone; there is a secondary breakup zone. All this is a very complicated dynamics we have not covered for example that droplet like flow interaction cases that would be very interesting to see.

But that is not a part of this course maybe in some advanced course we can look at that particular thing okay. And we also show some videos if you want to play one video once again we will do that what we have to come out of this. So you can see the videos once again okay primary break up essentially and if you look at it this would be the secondary break up okay.

The droplets are not vertical but they do become spherical for the downstream where we can apply all this droplet evaporation model that we covered right okay. So, with this we end this particular lecture we are going to go and look at droplet combustion in some details now, okay, thank you.