**Heat Transfer And Combustion in Multiphase Systems**
**Prof. Saptarshi Basu**
**Department of Mechanical Engineering**
**Indian Institute of Science-Bangalore**

# Lecture 33
# Spray / droplet breakup models - I (TAB model)

Welcome, so in the previous lecture what we talked about that we talked about that there are different types of breaker vibrational, back tied, multimode. Out of that we pick up more simpler of the break-up models.

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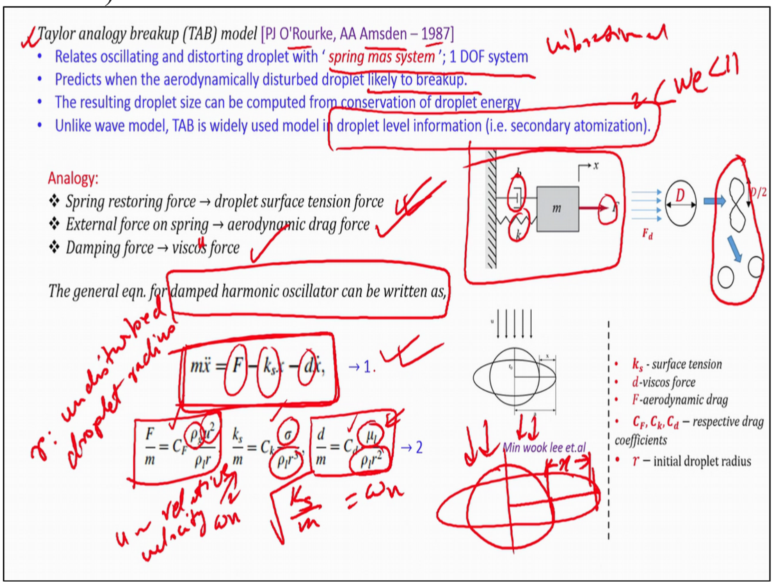


And it is basically vibrational breakup and this comes from the Tailor analogy breakup model okay. The courtesy to O'Rourke and Amsden is the 1987 paper where the first actually did it. Now as I said that the vibrational breakup is like a single degree of freedom system. So, what they are what O'Rourke and Amsden did was that okay.

They compare this oscillating droplet like this right this oscillating droplet and distorting droplet also with a spring mass system like what we have shown over here. Like a spring mass system, this is a damper this is a spring and this is whatever is the forcing. Now that force thing here in this case is aerodynamic forcing right.

The predicts when the aerodynamically disturbed droplet is likely to undergo breakup right, so that is a whole objective. And it uses the droplet level information it is very useful in the secondary atomization in the Weber number range that we said 11 to 2 right, in that particular range it is particularly useful okay.

So, what did what was the equation of O'Rourke and Amsden right what kind of an equation did they actually write. So, they wrote the equation for a simple damped harmonic oscillator, damped harmonic oscillator that mean harmonic oscillator with a damping right. So, they were kind of engineers and so the harmonic oscillators the equation is this right.

So, this is the forcing function okay this is basically what we call the spring constant or the other the restoring force okay this is the damping component for the spring restoring force here okay is by common sense it dictates it has to be surface tension right surface tension okay. Then external forces aerodynamic force and the damping force is nothing but the viscous force.

We missed a u there, so damping force is nothing but the viscous force right so based on this equation okay. Basically interface that this is a spring mass system which has got surface tension as the restoring force, damping; viscosity as the damping force and aerodynamic force as the forcing function okay.

Now if you try to write it, so write it in this particular form and you will find that this is the equation that we; the individual terms will be like this. Now in this individual terms if you look at it carefully okay what is Ks by m, Ks by m root over is usually given as the natural frequency of the system right.

So, this term over here is nothing but Omega n square so to say. This is the natural frequency of the system as you know from your vibration that is given by root over Ks by m right okay. Similarly d by m that is given by that damping okay and F by m is basically the forcing. As you see we have carefully chosen our units over here.

This is Sigma divided by Rho l r cube this is given by Mu l because this is viscosity dependent divided by Rho l r square and forcing of course involves that Rho g because it is aerodynamic forcing okay and this is u square is nothing you is the relative velocity in case you have not guessed it already, it is the relative velocity okay u and Mu l is basically the fluid viscosity, r that you see over here, r is the undisturbed droplet radius.

There is no evaporation here, is the undisturbed droplet radius okay. So, these are the quantities that you will have over here got it, okay. So, what is x now, now that we have said that this is the spring mass system. X is nothing but if you look at this particular picture over here and let me draw it a little bigger right.

So, it is basically the displacement of the droplet equator okay it is the displacement of the droplet equator. So, this is basically your x, right. It is a displacement of the droplet equator okay. So, based on an aerodynamic load, so the aerodynamic force is coming this way right, so that is the displacement of the droplet equator okay.

So, when the displacement becomes very high right, what would happen when the displacement becomes very high the droplet will undergo the break up, okay. And we will see when it undergoes actually break up. Now there are few other things that you see there is something like CF, Ck, Cd.

These are all constants about which we do not have an idea over here this requires experiments and other things okay to fill it to plug in those constants okay because we do not have an idea that what those constant values will be right okay.

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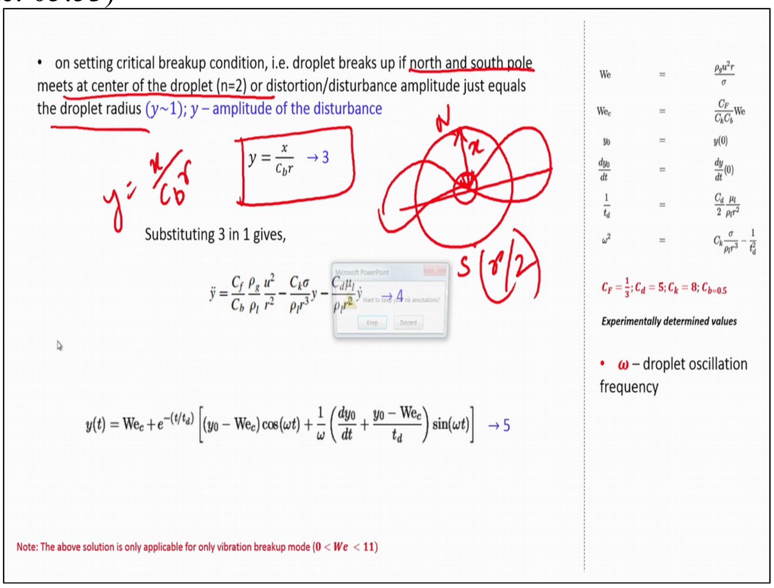


Now the break-up condition happens if the North and the South Pole of the droplets meets at the droplet center right and since this is a single degree of freedom kind of a system say you consider this particular case. So, this is the original droplet right and this is the droplet at the point of break up drawn it like this forgive the drawing part.

So, this is x, this is the deformation of the undisturbed droplet equator right. So, this x if you look at the deformation this is exactly half of the droplet size right. So, in x is basically half of the droplet size right it is supposed to undergo breakup right because then when the north pole and the; this how; this is not this is how this north pole and south poles actually meet.

And the deformation extent that is the deformation of the equator of the droplet that becomes equal to the droplet diameter of the radius of the droplet in this particular case okay. So, what we can say over here is that you can define a quantity which is called y = x into Cb by r, Cb is another constant. So, based on that when Cbs value = 80 equal to half the droplet actually is said to have undergone the break up okay.

So, substitution so now what we do is that so you know all this all these parameters now. Now let us write let us get away from this and write the equation in the journal thing.
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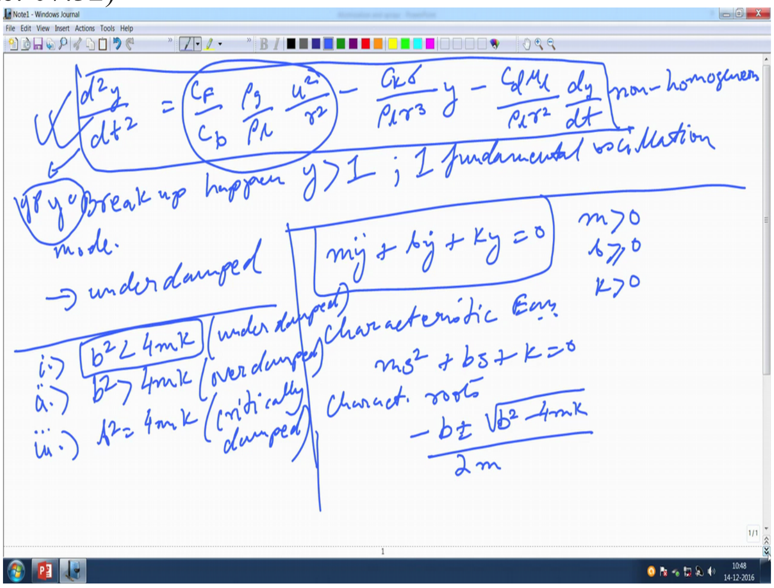


So, when you put all these things, so Cb is also a constant. So, using the expression d square y by dt square, now become CF by Cb Rho g by Rho l u squared, u remember was a relative velocity minus Ck Sigma divided by a Rho l r cube into y - Cd into Mu l divided by Rho l into r squared dy by dt right that is the full equation that you have right.

So, breakup happens when y is greater than one right that means when North Pole and South Pole of the droplets actually meet at the droplet center okay. And there is only one fundamental oscillation mode right one fundamental oscillation mode only got it that part should be very clear to all of you okay.

Now how to solve this particular equation right, so if you take this system to the under damped okay. So, basic assumption is under damped it is not critically damped it is not over them so if you look at what is what is that if you recall. If you have a generalized equation like this is a generalized free vibration equation.

The free vibration equation is not forced okay what you have m should be greater than 0, b should be greater than equal to 0, k should be greater than 0. Now the characteristic equation for this the characteristic equation for this will be ms squared + bs + k = 0. So, the characteristic roots would be roots of this particular equation will be - b + - b square - 4 mk divided by 2m right.

So, these are the characteristic root of this particular equation. Now there are three possible cases, one is when b square is less than 4 mk this is called under damped which is what we are dealing with here. Remember your Weber number versus Ohnsergo number plot we showed that most of the systems are under damped usually okay.

So then two is basically be square is greater than 4 mk that is over damped okay and three, b squared = 4 mk which is critically damped right okay. So, these are the three outcomes that are possible right, we are concerned with this outcome mostly okay and this is from the free vibration equation right.

And but what equation that we have over here is a non-homogeneous equation right that is our non homogenous equation right. So, this equation the basic vibration equation that we wrote right usually will have two solutions one will be a particular solution right and one will be the complementary solution correct okay.

So, out of that the complementary solution will be the same as a free vibration equation it will be equivalent to a free vibration equation minus this particular term right. So, if you solve a free vibration equation minus this particular term which is basically you just remove the forcing away okay.

You will get whatever is going to be the complementary solution right. So, that is what you know from your vibrations already okay.

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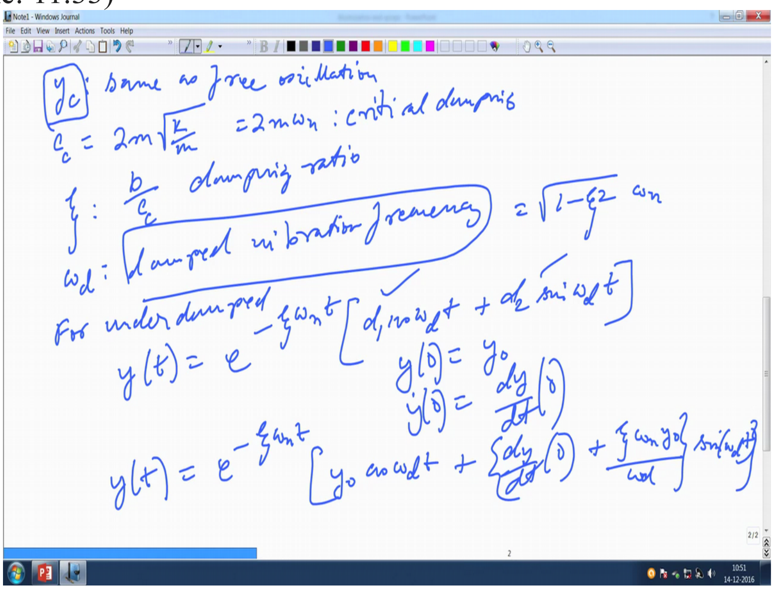

So, complementary solution is nothing but the same as free oscillation okay in this case it is under damped okay. So, for under damped cases okay we normally there are several other parameters that we define there is something called the; called critical damping, it is basically this, this is called critical damping.

Then there is something called the damping ratio damping ratio is basically b by Cc is called the damping ratio okay. And then there is something called the damped vibration frequency which is given as 1 over E squared into Omega n right okay. Now for under damped system the solution becomes yt cos Omega dt + d2 time Omega dt right okay.

That is the main form of the equation that you are going to get right okay. And this you remember is a damped vibration frequency got it. Now there are two boundary conditions that are needed to solve these two constants okay. Usually what you are given is y naught = y0, y at 0, time = 0, y dot 0 at zero okay this we can obviously write it as dy by dt at zero something like that right okay.

So, based on that the equation becomes and this is the same this is the complementary equation that we are trying to find out right. It is basically Omega nt into y naught cos omega dt + dy by dt 0 right + Omega ny naught by Omega d, this entire thing becomes associated with sine Omega dt, got it, okay, so that is the total expression, got it. So, that will be the complementary solution right okay.

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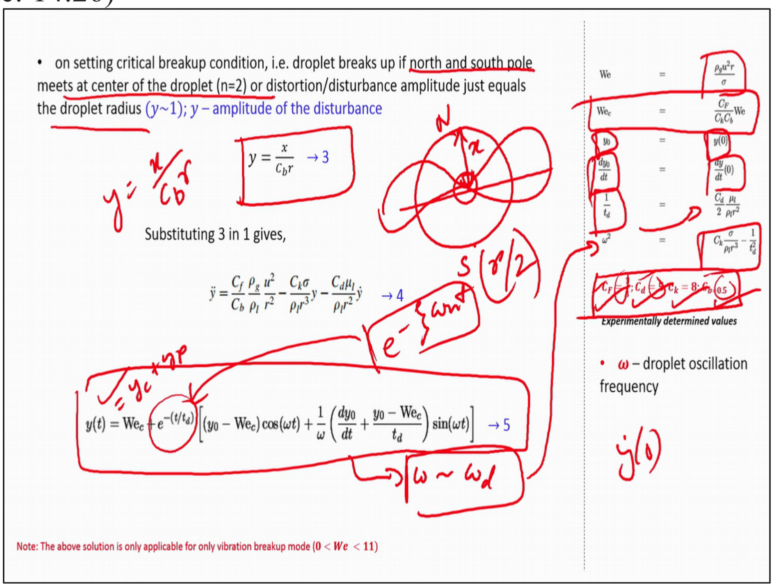


Now in this particular problem if we go back to our PPT now, okay. If you go back to our PPT here okay what we have done is that instead of Omega d okay we have used Omega okay. So, here Omega and Omega d are the same okay it is the same thing right. So, Omega is actually defined over here okay.

And that is basically the damping time scale that you have that is the damping time scale okay dy 0 by dt is basically that y naught 0 right and y0 is the same as y0 okay. And we have defined a critical Weber number kind of a parameter which is nothing but the Weber number multiplied with all those constants that sits in front right.

And the Weber number is given by as Rho g u square r by Sigma, got it. Based on this we get this complete solution for the vibration of the droplet right. As you can see it is very similar to what we did just in the previous journal thing that we did okay. And you can see that this is this exponential term comes from that complementary solution that we just now saw right,e to the power of minus remember that okay.

So, that is what we got. So, that is basically the same as this kind of okay. And so you get that entire set of equation okay in which you get the complete solution which is equal to complimentary plus particular right. So, that is the net solution for a forced vibration kind of a system okay.

Now another thing is that you do not see those constants Cd and all those other things any longer that is because we have substituted them with some values over here like Cd was .5 Ck is 8, Cd is 5, Cf is one-third okay. What is the rationale behind that absolutely no rational it is basically done experiments repeated experiments have been done and people have found out that this could be the possible values right.

So, there is no theoretical basis for this right. So, this basically these values basically match the data as simple as that right so that is how you actually have done all this thing, got it. So, that is very interesting that you now have this entire vibration problem worked out in which you have these are the kind of solutions that we have okay. And these are the kind of values that we have used okay.
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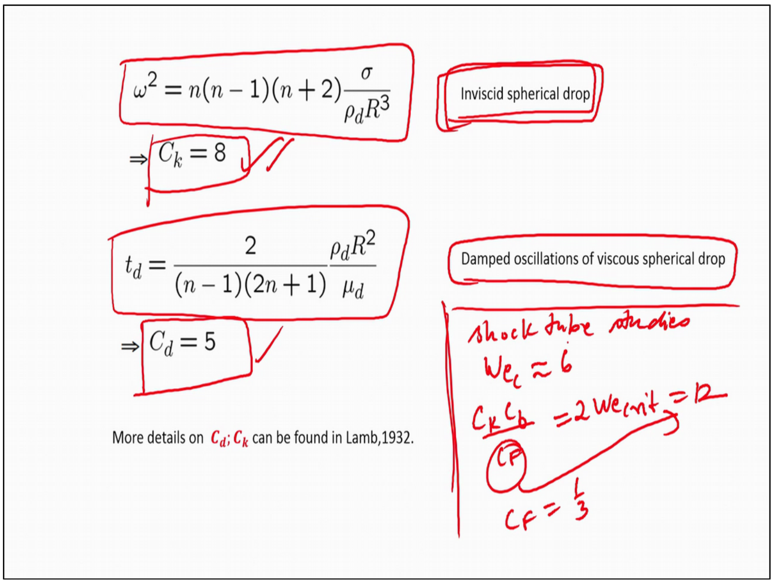


Now as I said how do we find out all these constants? Say for example one constant Ck that you can see over here which is this guy over there right. How that is found out basically it work done by lamp, you considered an inviscid spherical drop okay. And from the frequency of oscillation of an inviscid spherical drop one can find out what will be the value of this Ck.

So Ck is usually 8, so this comes from the inviscid to drop analysis right okay. Then people have done damped oscillation for a viscous spherical drop right and they have found the corresponding time scale okay. And from there you have found out what will be the value of Cd right.

So, like for example Cds then becomes 8, Cd becomes 5 and Ck becomes 8 right, Cd we already establish it has to be .5 right okay. Now there are two other constants that needs to be found out for that what happened was that people did shock tube studies. These are all experimental and you can learn more about the experiments if you go to the individual books okay.

So in the shock tube experiments people found this critical Weber number to be about 6 right okay. So, based on that substituting that value of the Weber number Ck Cb by Cf is approximately equal to 2 times a critical Weber number that gives the value of approximately 12 right. So, from there you can back out what will be the value of your Cf going to be.

Because Ck and all the other things unknown, so Cf usually becomes one third from this exercise right okay. So, that is interesting because you have then you have planted in all this constant values but the formalism I think you guys are very clear. We have just taken a spring mass system we have assumed that it breaks up at y greater than 0 and then we have found out this net solution right.

It is as simple as that okay and these are the constants that we have picked it and plugged in from other theoretical and experimental works right.

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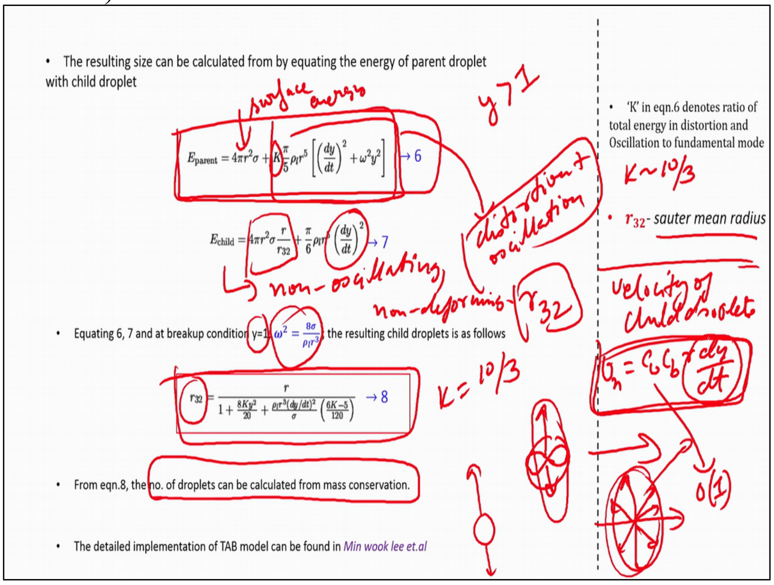


Now one other thing that remains that still remains to be done is that what is the size of the daughter droplets, we know that it breaks up at y greater than 1 but what will be the size of the daughter droplet as an engineer as an experimentalist whatever you say it is very important that you find out what those values are going to be right. So, it is mandated right.

So, what do you do about that to do that what you do is that you calculate the energy of the parent droplet. So, what is the energy of the parent droplet it comes from two forms one is the surface energy okay that is the surface energy and the other source of energy comes from the combined action of distortion energy due to distortion plus oscillation got it.

Distortion plus oscillation right and k is basically the ratio of the two, the ratio of distortion and oscillation in the fundamental. Once again we are only concerned with one mode and that is the fundamental mode right okay. So, k is actually gives you a value of around 10 by 3 if you if you consider this.

So, this is the total energy of the parent droplet, what is the total energy of the child droplet, the child droplet is assumed to be non-oscillating, non-oscillating and non deforming okay. The child droplets non-deforming means non distorting essentially okay. So, this particular component comes from the kinetic energy of the droplet right that you have.

Because this atomizes it basically spits of the droplet right and on the other hand this energy comes from the corresponding surface energy right, r32 that you see over here is basically called the Sauter mean radius okay. There is a Sauter mean diameter this is the Sauter of mean radius right.

So, at the break-up condition which is y = 1 and Omega is equal to this the resulting droplet size is given by this equation 8 where k = 10 by 3 that we already mentioned. So, this is done the r32, so from this equation the number of droplets can be also calculated from the mass conservation because the mass of the parent droplet has to be the same as the daughter droplets right.

So, that you can find out but this will be the radius of each of the droplet okay. So, these are the basic things so this actually tells you that what will be the energy of the child droplet okay and what will be the energy of the daughter droplets okay. Now another thing that still remains is basically what will be the velocity of the child droplets right.

So, that is what we mentioned velocity of child droplets okay. As we said earlier this velocity of the child droplet is usually given by, okay, Cv into these are all constants do not worry too much about them dy by dt. So, it is basically the two droplets come the droplet comes and meets at the equator right.

So, that is the velocity with which this actually goes okay so this actually this velocity is normal to the parent drop velocity. This drop is travelling like this right okay so this particular thing now become that right as the droplet spits out, it spits out in those directions perpendicular. Now it can happen in a plane anywhere around that perpendicular plane right.

So, if it is going like this okay the daughter droplets can come anywhere in this plane all the all of these are perpendicular right. Velocity magnitude is given the plane is given but exact direction is completely arbitrary there is a certain random that is because you can never predict exactly. How it is actually happening right okay.

So, this actually gives you that what will be the velocity of the of the daughter droplet okay this is once again this Cv is of the order one. Now there is a lot of empirical things about all this work okay as you can see but it is just to give you an idea that this droplet is travelling it just spits out the daughter droplet in the perpendicular directions, right.

And that velocity is nothing but the velocity with which the deformation is actually moving that means the two sides the deformation is this right this is the velocity of these two sides right. This is the velocity with which this front is actually moving right that velocity is taken out by the daughter droplets okay.
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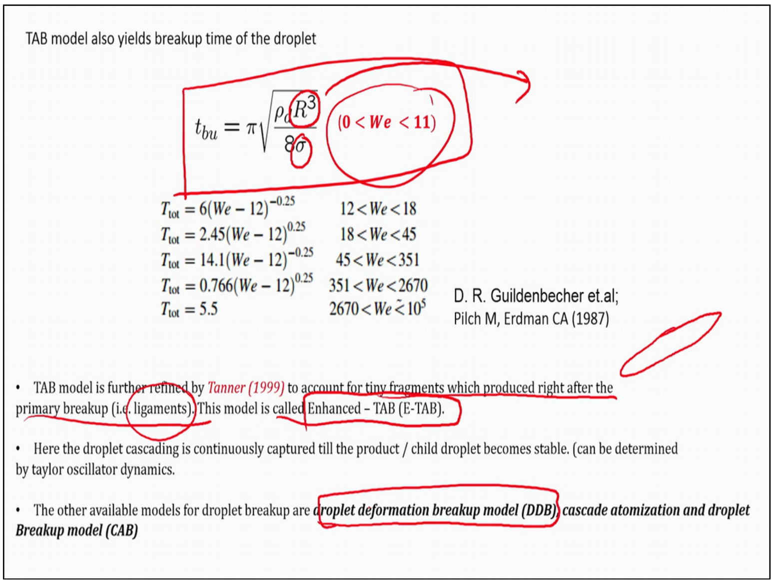


So, the TAB model also yields what we call the break-up time okay. We would not go into the map of that right. So, the break-up time is right over there in front of you okay. So, as we say it is valid for that Weber number range okay. So, the break-up time as you can see if you have a very low value of surface tension right.

If you have a low value of the surface tension what will happen and what happens when you actually have say a small droplet. So, that can be left more like a homework kind of an exercise where you can kind of see and visualize try to visualize yourself that what will happen to the break-up time okay. What will happen to the total breakup time, okay?

Now we already said a lot of things about droplet breakup okay and we said that what is the role of viscosity, viscosity basically damps out the whole thing for the break-up. As we saw from that charge by Jerry Feith for a large of Ohnsergo number it is still dominated by the Weber number effect okay that is by surface tension essentially okay.

But the TAB model can be further refined okay to take into account the tiny fragments which are produced by that is the ligament like structures. Ligament structures are like this right so they are non-spherical. So, that model is called enhanced tab model or E TAB okay. And there are other break up models also like droplet deformation break up model DDB, cascade atomization and droplet break up model of CAB.

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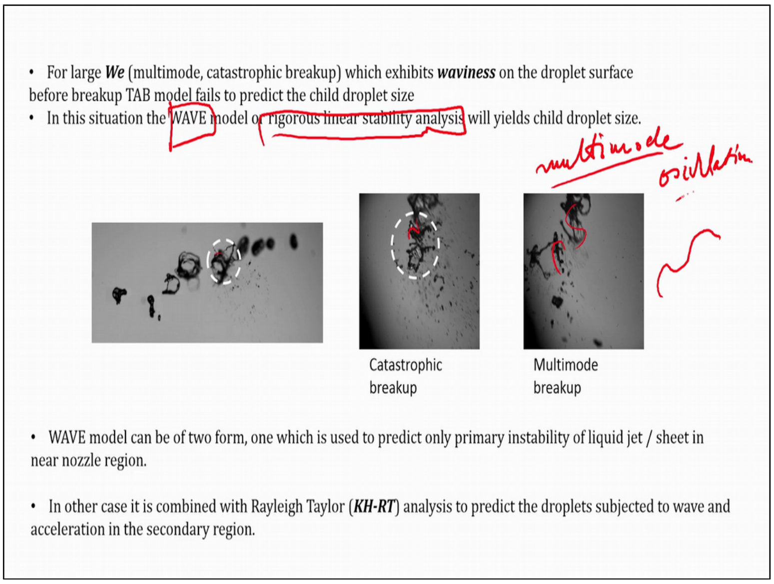


One of the more popular and for example but this models all these are very for low Weber number kind of a spectrum. For a higher Weber number case you will always see that the droplet; if you look at this picture over here there is always some degree of you know oscillations or periodicity within the droplet surface okay.

So, there is some oscillation, so it is basically a multi mode oscillation, so to say right. It is multi mode oscillation okay and that usually happens on the surface waviness on the surface in this situation people normally use what we call the wave model. So, wave model is basically what that is, it is basically a Kelvin-Helmholtz instability type of a model basically that.

It Kelvin-Helmholtz instability type of a model and it has come through the linear stability analysis okay and that is how we get to the KH instability. So, this takes center this is more sophisticated in way okay because it can actually predict multimode kind of an oscillation on the droplet surface.

So, it is very useful in primary break up as well as in secondary break up because the droplets may not break up just by a simple vibration one-dimensional vibration. It is actually multi-modal vibration so to say okay.

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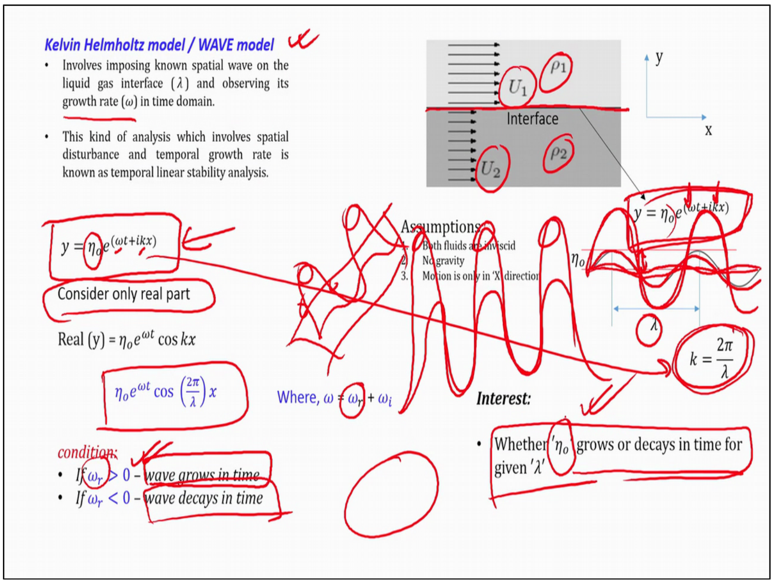


So, if you just do a very quick recap of what we did earlier that even the Kelvin Helmholtz think, if you look at if you remember the first few lectures what were they that gravity is actually not there into the picture right. So, there is an interface sharp interface like this what we have over here.

There are two fluids which are moving with a velocity U1 and U2 with difference intensity. Right now we are not stated anything about liquid and vapour, I mean of course in this case it is one is a liquid phase, one is the gas phase okay. And what happens is that this interface undergoes a degree of small perturbations okay.

Now that perturbations wavelength is given by lambda and the wave number and the corresponding wave number is given by that. Whereas this is an infinitesimal perturbation of the interface right, the idea is to see that if there is a perturbation on the interface whether that perturbation grows on it decays, what happens okay.

If the perturbation; so, if you look at back to the slide once again you will find that the perturbation can be written in this particular form that means the y is an amplitude basically with which this perturbation is actually growing. So, there is an n naught and then there is an Omega there is just k this case are same as this k right, okay.

Now we consider only the real part of this okay and the real part that that now the real part of this Omega which is basically nothing but the growth rate of these perturbations okay. If this Omega is greater than 0 this waviness on the surface basically grows with time, it will grow like this right.

If however this is less than 0 this will attenuate with time okay. So, depending on whether the; so if you imagine this to me like a droplet or like a sheet or like a liquid sheet right okay and you have all these waviness that are created on the surface right. Some waves are going to be damped out immediately because of the effect of surface tension and other player's right.

Some waves are going to grow okay so this will become aggravated right. As it becomes aggravated these points will undergo breakup right. So, you have a aggravated liquid sheet right.  So, it becomes like this then the next instance it become more violent right. So, it will start to detach right the liquid will start to detach from the surface correct okay.

So, the idea is to basically use a stability analysis to see that what is the perturbation that you create on the surface of the sheet right which is a infinitesimal perturbation. This is the growth rate of those perturbations and this is the wavelength of that perturbation. The idea is to see that the real part of the perturbation real part of the growth rate is that grows with time.

Then that means that perturbation will aggravate and become something like this, otherwise it will attenuate. So, there are some criteria space not all wave numbers can grow not all wavelengths can grow some wavelengths will grow some wavelengths will not grow and that depends on the surface tension a lot of other factors okay.

So, in the next class what we are going to do we are going to look at this Kelvin-Helmholtz instability okay. We have already done this earlier right we have to see that whether for a given lambda whether this disturbance how it grows, how it decays and how we can apply it in the droplet scenario.

I have already given you some idea that how it can be applied. So, we will see in the next class that how it is literally applied to basically in a droplet kind of a context. And that would be very important because based on that we will wrap up this atomization part. There are more models, there are more complicated models but these are the two basic models that you should know okay, thank you.