Heat Transfer And Combustion in Multiphase Systems
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Lecture 28
Comparison of droplet vaporization models

Welcome to today's lecture. So, last time what we did was that we discussed about the complete droplet evaporation model. And we looked at the flow field within the droplet resembles like the Hills Spherical Vortex and then we solved the entire transient energy equation okay including the convective term and we showed that how this case is to be solved numerically essentially. And we will discuss some of the results here.

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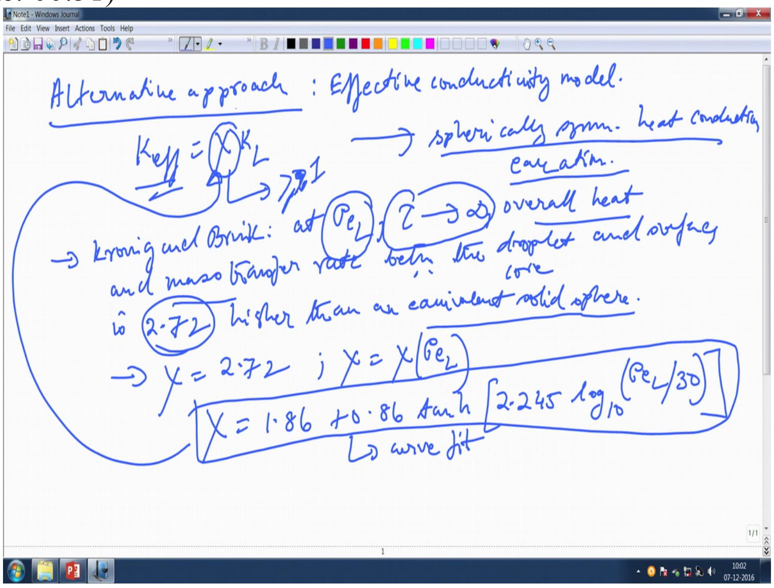

Now there is one more alternative approach, I am sorry, so, there is one more alternative approach and this approach is basically called the effective conductivity model and what does it do essentially okay. So, what it does is that instead of using the actual conductivity, the thermal conductivity.

It uses an effective value of the thermal conductivity. So, your K effective will actually be some factor X or Chi terms the liquid conductivity. This factor is usually much, much greater than 1 or not much, much greater it is actually greater than equal to 1, so to say it is like greater than equal to one, okay. So, the; what approach essentially entails is that what of convection essentially means?

Convection means that it enhances right the energy transport within the droplet okay. Now in this particular case the energy transport within the droplet because it is enhanced by convection as we saw the Peclet number is high and things like that. But the essential spirit of the whole thing is that if we can somehow designate that convection effect by enhanced thermal conductivity.

Then perhaps we can get around solving this complicated transient equation that we did in the last lecture okay which involves the Peclet number and the convection terms right can something like that be done in this particular context. So, that was what the main and added main spirit of this thing was.

So, what Kroenig and Brink did was the following they found that at high a Peclet number okay and at Tau approaching infinity that means that long time scales okay. The overall heat and mass transfer and this is an important statement, the overall heat and mass transfer rate okay between the droplet and the surface.

Between the droplet core and the surface okay is about 2.72 times higher than equal that an equivalent solid sphere understood. So, that means is it basically implies that this factor Chi that we talked about is about 2.72 right. So, what did they find they found that at high Peclet number and high time scales the heat and mass transfer rate is basically 2.72 times higher than an equivalent solid sphere, okay.

So, this triggered the thought process that if we could use a thermal conductivity right which is about 2.72 times when we are kind of okay right. So, ideally this 2.72 is like a misnomer it actually should be a function of the liquid phase Piclet number right. So, if the Peclet number is high that means you will have more thermal conductivity.

You have to give them value of the thermal conductivity. So, the overall form of formulation that the gay it was about 1.86 + .86 once again nothing but a sophisticated curve fit 2.245 log base 10 Peclet number divided 30. So, this comes basically from curve fit, so that you should not be under any misconception okay. So, if we evaluate this factor depending on the liquid phase Peclet number.

And you use that factor to enhance the thermal conductivity the effective thermal conductivity then you can get away just by solving the spherically symmetric heat conduction equation right that you do for a normal solid sphere. For a solid sphere there is no convection, so, inside the solid sphere it has just use the spherically symmetric heat conduction equation.

Here also you can do the same okay but using an effectively higher value of the thermal conductivity okay. But this is a function of the Peclet number okay.
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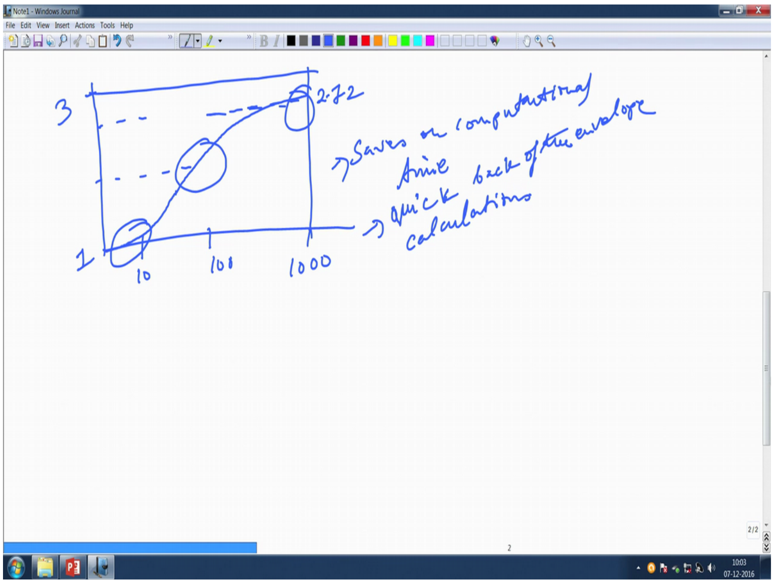

And if you look at now a small plot for the same if I try to draw this you say the two axis. Let us put two marks over here this is 10 this is 100 the log scale 1000, this is 1 and this is 3 okay. So, it shows something like this, this mark is about 2.72, got it, okay. So, you can see initially for low enough Piclet number this is almost bare right.

For high enough Peclet number it lies somewhere in that intermediate region and for very high Peclet number limit it actually goes somewhere to around 3 okay, close to 3 that is 2.72 that is how this particular thing actually work okay. So, the effective thermal conductivity model and we will see some examples.

This usually is formulated to avoid any computational overhead and things like that, so that we can get around the problem right pretty simply okay. And that is the whole criteria for doing this kind of an analysis okay. So, the effective thermal conductivity models saves on computational time essentially.

Computational time it can also help you to do quick back of the envelope calculation envelope calculation okay. So, those things could become vitally important okay. Now let us look at the presentation now.

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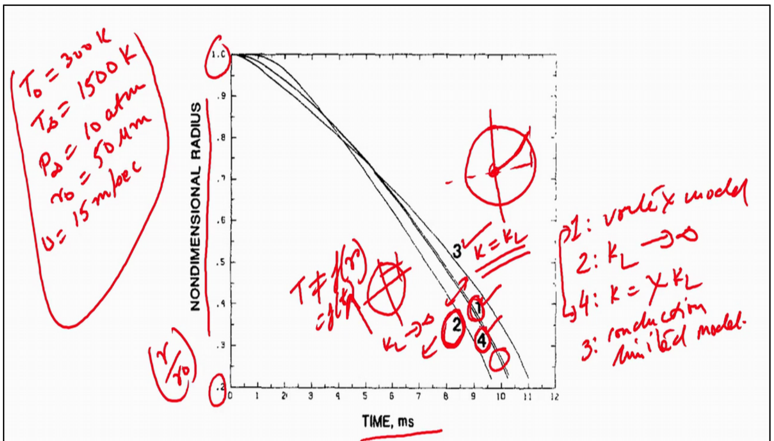

FIG. 2(a). Non-dimensional droplet radius vs time: extended model (curve 1), infinite conductivity model (curve 2), conduction limit model (curve 3), and effective conductivity model (curve 4).

Because we are supposed to discuss some of the results okay, all these results are from Abramson and Sirignano and in fact the presentation that the material that we covered it mostly from Bill Sirignano book and Abramson and Sirignano model. This is a very famous model and it was widely used okay in the literature okay.

People to this day actually use this model with slight variations here and there but mostly the basic framework is still the same. They still use the Hill Spherical Vortex still solve for the external flow field okay. And we will see that this is actually a very effective thing it matches with experimental data though it is an approximate model because the liquid flow field is actually not solved okay completely inside.

But still it gives you a very good idea and if you know that the liquid flow field has been evolved through the calculations right, involving the Faulkner Skan type of solutions okay. Now let us look at some of the results which will try to give you an idea that what is exactly happening.

So, this comes generally we said that the model can be solved numerically this is basically the numerical solutions that you see over here okay, so, this should be interesting. So, here some of the basic parameters are T naught. The initial temperature is 300 Kelvin this can be anything you can actually do your own model okay.

But this is just to give you an idea the external temperature is about 1500 Kelvin. So, that is more like a flame and things like that. External pressure is about ten atmospheres, so, it is a high structure chamber. The initial droplet radius is about 50 micrometers, is roughly half of a human hair size of a human hair.

And the initial velocity when is of the gas phase flow field is about 15 meter per second okay. So, these are the conditions that are actually given to you okay now let us look at each of the figures very carefully and try to see that what the things are. And if you watch it now that bird will be now if you will see here there are curved like 1, 2, 3, 4 right.

So, one is basically the current model that we are talking about right this is this model that is the Sirignano and Abramson's model okay. Infinite conductivity model we already told you that is 2.2 is basically in finite conductivity model which we already said KL approaches infinity right, so that is the infinite conductivity model okay.

An effective conductivity model is for which we just now say that K is now equal to Chi into KL right. So, that is the effective conductivity model that we just now said okay. The extended model infinite conductivity and then the third is basically number 3 is basically the conduction limited model okay.

The conduction limited model we already covered this that what is the conduction limited model right, got it, okay. And what it says is that it shows the non-dimensional radius versus this time right. This time is the raw time okay for that 50 micron droplet that was injected. Non-dimensional radius is basically r by r naught that is why it starts from 1 and goes all the way up to .2 and things like that okay.

So, if you see now okay 1 and 4 comes very close to each other okay, 1 is basically the current model, the vortex model basically right, okay. And 4 is the infinite conductivity model, so, they pretty much predict the same thing right. While on the other hand 3 which is a conduction limited model okay, slightly underestimate the decrease in the droplet diameter.

Unless means, it takes a little bit of longer time right that is 3, conduction limited model and 2 is basically where it is the other side of the spectrum that means it is a very high level of thermal conductivity right. So, it is natural that our results will fall somewhere in between these two limits.

Why this number 3 will be slow that is because conduct it is conduction limited right. So, the only way that heat is being transferred within the liquid phase is through conduction right through conduction it is happening through the liquid phase right. So, this is basically only conduction, so K is basically equal to KL and here you have KL is approaching infinity right.

So, our conviction or the infinite conductivity model falls, has to fall somewhere in between these two limits right. So, that makes physical sense right it cannot be anything else it has to fall within the two limits. Because one is predicting very high thermal conductivity that means whatever is the temperature on the surface of the droplet that is the same temperature inside the droplet that is your effective thermal conductivity model right.

Because this should imply that T okay is not a function of space anymore it is not a function of r right. It is not a function of the droplet okay. It is only a function of time right. So, this would naturally imply that you are assuming a very high level of heat transfer that is happening within the droplet because the entire droplet is getting to the same temperature right.

Whereas in the conduction limited model it completely ignores the convection effect right. So, here you will have a temperature which is higher at the surface and lower at the center. So, it will be more like this right higher of the surface more at the center right. So, that is this model right. So, you have a temperature gradient in built right.

In this case the temperature is the same everywhere right same everywhere correct okay. And deep to model conviction and the effective conductivity because we are pitched against each other they pretty much do the same thing right. They try to bring in the actual effect into the picture okay. So, they should lie in between these two extreme limits. I think that part is clear now correct okay.

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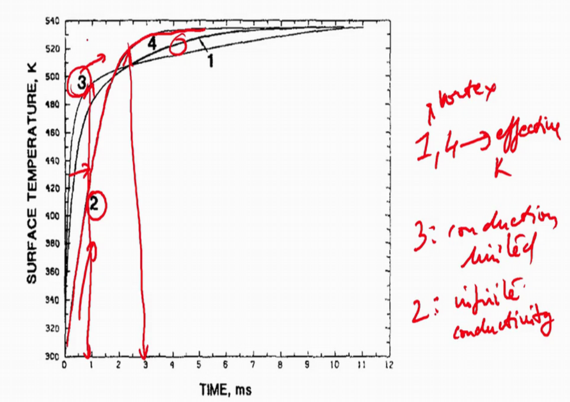

FIG. 2(b). Surface temperature (K) vs time: various models.

So, let us look at the surface temperature on the other hand based on the different types of models that we have over here okay. Now if you look at the surface temperature once again 1 and 4. You should recall 1 and 4, 1 is a vortex on the extended model okay, 4 is the effective conductivity model, K model.

So, once again you see 1 and 4 okay they are very close to each other you see that they are very close to each other, 1 and 4 right, 2 show that kind of a profile, 3 shows that kind of a profile okay. Now once again here what you see if you go back and just see what is 3, 3 was the conduction limited model right.

So, 3 is conduction limited okay and 2 obviously is the infinite conductivity, got it. Now the question is that when there is an infinite conductivity you can see that this rise of the droplet surface temperature is rather slow right. And it is going up to a value okay at a much later time.

If you plot it by the time axis it goes at a much later time right, got it, okay. Whereas on the other hand 3 which is the conduction limited model goes to that surface temperature at a much smaller time, this is also obvious, think about it because the entire droplet is now at the same temperature right.

But the heat that you are supplying is still the same it was not so it is still Ts - T infinity whatever is coming right Ts - T infinity on the gas phase right whatever is conducted into the liquid right. So, Ts - T infinity that so, if your surface temperature and your liquid temperature in core temperature at the same, naturally it will take more time for this entire thing to pick up right, okay.

Because you are heating the whole droplet, in this case you are heating basically the surface and some heat of it is basically getting conducted inside okay. So, that is the reason why you have this slight disparity okay. But all of them as you can see more or less shows a flattening of temperature way around 520 to 500 okay.

So, that is called basically the wet-bulb limit as you already know by now that that is what we have explained earlier.
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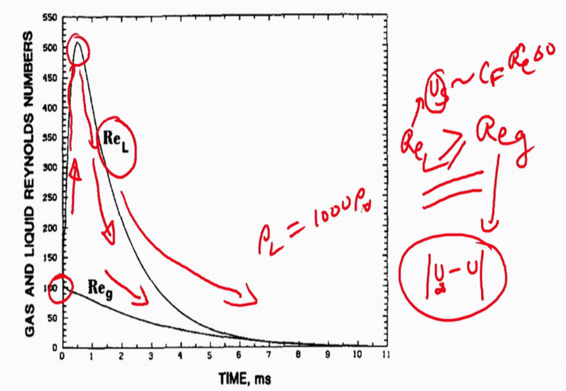

FIG. 2(f). Gas and liquid Reynolds numbers vs time.

But 1 and 4 there is nothing much to choose between the two that also shows that these two models are very effective to each other okay. Let us look at the transfer numbers now okay. So, what are the two transfer numbers we have we have BT and we have BM right. These two transfer numbers that the spalding heat transfer number and spalding mass transfer number okay.

Now we said that they have to be less than 20, right for this model basically to work right. So, can see that both of them actually goes up okay and they actually flatten out towards the end of the cycle. So, BM actually flattened somewhere around 8. BT flattened somewhere around 15 okay. For that all the heat and mass transfer numbers actually okay go up okay with the flow, with time okay.

Now this is the interesting part where we look at the Reynolds number right previously we said, what if you recall that this Reynolds number is greater than equal to the Reynolds number of the gas phase. Recall that is what we said right. The gas phase Reynolds number uniformly comes down it has to.

That is because this Reynolds number is dependent on U infinity - U right and this is the relative velocity between the gas and the droplet. So, naturally which should come down with time right because the gap and the droplet okay, so they will; so, it is like if you inject a droplet in a stream right what happens to the droplet? The droplet is accelerated and ultimately approaches the velocity of the gas phase right.

So, their relative velocity kind of comes down as their relative velocity comes down what will happen is that this Reynolds number of the gas phase should also come down right. So, this Reynolds number of the gas phase therefore comes down with time. So, it starts with a value of around 100 it comes to very close to 10, 5 etcetera with time right.

The liquid phase Reynolds number however shows are different characteristics okay slightly different characteristics. So, it initially goes up and then it kind of comes down can you imagine why because the liquid phase, Reynolds number will go up that is because when the droplet is actually accelerated right.

Because liquid phase this is a function of Us right the surface velocity right. And if you recall the surface velocity was a function of CF it was a function of the Reynolds number and all those things right. It was a function of the delta U right, if you recall. So, this Us therefore okay initially when the droplet is accelerated this goes up and then after relative velocity comes down this slowly comes down in profile like that, right, okay.

So, that is the nature as you can see this Reynolds number is at least 5 times higher than the gas phase Reynolds number. Mainly because your Rho L is 1000 times Rho V essentially Rho gas, okay whatever you call it, clear. On this particular part okay.

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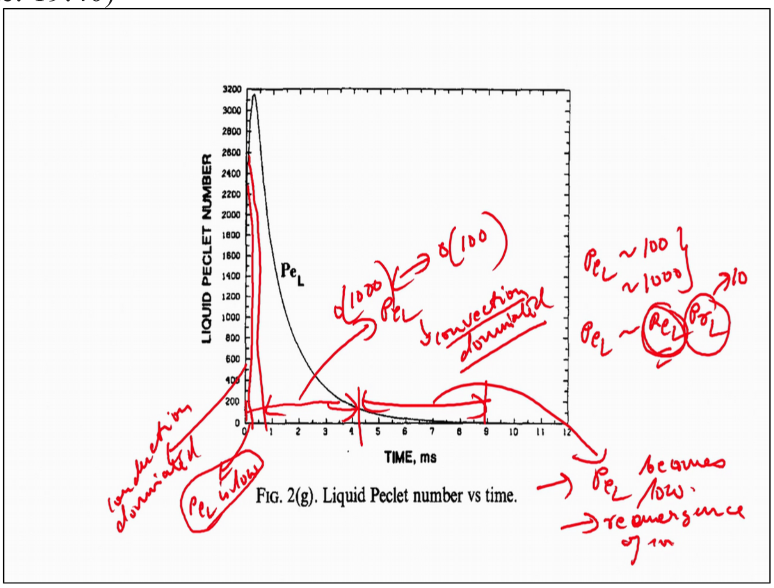

FIG. 2(g). Liquid Peclet number vs time.

Now other important part is a Peclet number right see we used the vortex model mainly because of our Peclet number issue right. So, the Peclet number we said that because the Peclet number is of the order of 100 and sometimes even more sometimes of the order of 1000 okay we have to use a convection right that was the whole logic right.

And we also said that the Peclet number is the one that actually determines what kind of a mode of heat transfer one would have right, is it convection dominated or it is conduction dominated right. So, if you look at the profile of this Peclet number this is very interesting to say the least right.

Peclet number as you know is a function of Reynolds number and Prandtl number right. So, liquid phase Prandtl number is of the order 10. So, naturally whatever is the value of the liquid phase Reynolds number it is multiplied 10, right. So, if the liquid phase Reynolds number is 300 this will be like 3,000 of that order right.

So, here if you look at it as that I know it is mimics the similar profile as Reynolds number if you look at it, it is similar to this Reynolds number kind of a profile right because Prandtl number is kind of uniform, so to say, right. Unless you take into account the property variation it is more or less uniform. So, it should mimic a very similar kind of a profile.

Now here we can see there are several distinct stages that we can identify right, if you look at this particular figure very carefully. Here you will find that initially as a very initial part of the time that is very small time instants right and we will see what happens. Your Peclet number is actually no ready usually when it is just picking up right in that few in less than a millisecond okay.

It is actually going up right for it is going up from some value right. So, it is a very sharp rise. So, here Peclet number is low, if Peclet is low here we can expect that the heat transfer will basically be conduction dominated right, okay. In this region that is from this point all the way up to say about to about 100 up to about this point in this large region okay.

The Peclet number varies between say something like thousand of the order thousand to the order 100 right 100, 10 things like that right. In this part the; it will be a conviction we can expect that it will be convection dominated right. So, that means the isotherm that you are going to get will mimic the stream line right.

Here in conduction dominated the streamline and isotherms will be different to each other right. But here of course because it is convection dominated okay the streamlines should now be the similar to the isotherms okay. But whereas the end towards this last stages this tail of the distribution the Peclet number once again becomes low right.

So, here you can think about that there is a re-emergence of conduction there should be a re-emergence of conduction right. So, how can we validate this just by looking at the temperature profiles inside right. If we look at the temperature profiles then you know what is going on right.

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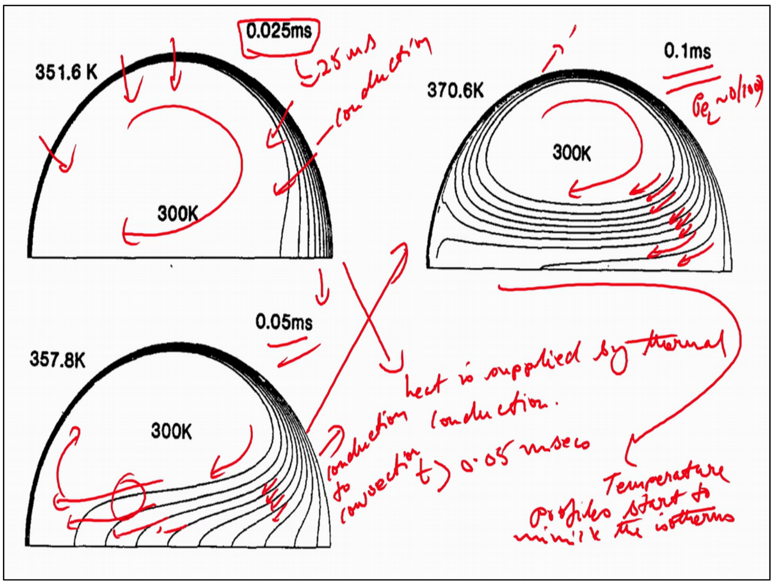

So, let us look at the temperature profiles like this okay. So, this is a half the droplet this upper half is symmetrical, so it is not drawn right. So, what is happening here is the first .025, milliseconds. It is very first new instance, you can look at it here it will be somewhere there right, very first few milliseconds not even seconds, it is like sub milliseconds right.

So this is more like 250 microseconds or 25 microseconds and thinks about it as microsecond's right. So, what is happening over here if you look at it okay the liquid isotherms in this very short time the heat is basically transferred by thermal conduction, right. This is basically heat is supplied by thermal conduction, mainly by thermal conduction right.

So, you can see that that isotherms okay these are the constant temperature contours they do not mimic the flow. Because the flow is like a Hill Spherical Vortex right, it is like that correct it does not show that it is just this kind of a typical diffusion dominated problem right, okay, now when we go to say beyond .05 milliseconds.

So, when the time is now greater than 0.05 milliseconds actually we are showing an instant of .1 second over here okay. You can see that the temperature profiles now start to mimic you see this temperature profiles they start to mimic the stream lines, do you see that, these are the isotherms they are mimicking basically the streamlines right.

So, they are mimicking the as there is a distortion of the spherical symmetry of the temperature field okay. It starts to mimic basically the isotherm the stream lines. So, the temperature profile, start to mimic the isotherms right. So, they start to mimic the isotherms you can see it very clearly isotherms are being mimicked in this particular way, right.

We are starting to mimic the isotherm right, okay. So, from here and here these are the two instances where this has started, this has just started here it is completely. So, these isotherms now extend all the way to that direction right and curve up that is what you see right when you go from here to here this is at a later time instance like .1 second is not that so, right.

So, that is that is obvious because in these cases that Peclet number now is of the order of 1000 okay and that you can see from the previous plot itself right. Piclet number here is of that order it is sharp rise in Peclet them right.
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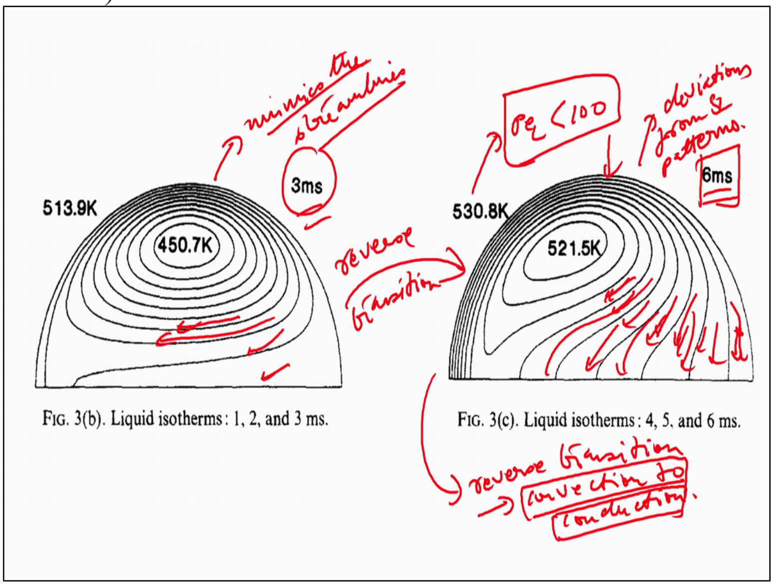

Now say up to at 3 milliseconds you basically see the same thing the liquid isotherms are like that right. So, they start to mimic the flow field okay. After about 5 milliseconds that means this is 3 milliseconds after about 5 million things which is the 6 millisecond is a good enough example okay you can see this at this particular point your Piclet number falls below 100 right that you can see over here also.

So, this is 5 millisecond, right that is the point right after 5 millisecond what happens your Piclet number has already fallen short right. It has already fallen short of that 100 mark right. So, as it starts to fall do you see that these Peclet have now started to show the reverse transition right. So, they have started mimicking the conduction from convection to conduction.

So, they have shown started to show that because you see once again the stream lines are curving down like this right is not that so. It has started to it is no longer like that extending all the way right. So, it is basically a reverse transition that is because the Piclet number here has fallen below 100.

So, naturally that conduction and convection okay can the convection to conduction transition have started to happen and the stream lines of the isotherms have clearly showing deviations from the stream line pattern right. So, this is showing deviation from stream line patterns right, started to show those a deviation right.

Here of course it mimics the stream lines, right, got it. This part should be very clear to everyone okay. So, now in the next lecture we will see what it does to the mass vaporization rate and before we move on to the other topics. But this actually should be clear that there is first, first there is the conduction okay.

Then this shows the transition from conduction to convection. This is full convection and then it starts to show the reverse transition from convection to conduction. So this profile and this profile in a time-limited sense they are kind of similar looking you can see that right is the stream lines are curving like this.

But it is basically like a hysteresis that is the; that is conduction to convection this is convection to conduction migration right that is what is happening in these two cases okay. So, in the next lecture we will go into cover the rest of the topics okay.