Heat Transfer And Combustion in Multiphase Systems
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Lecture 27
Comprehensive droplet vaporization model and correlations-II

So, welcome to today's lecture. So, in the last class we mentioned that what are the modified Nusselt number and Sherwood number in this particular case. So, now let us look at the, what is the form of this F which we already mentioned in the previous lectures. Here we will once again for the sake of completeness we will reiterate that okay.

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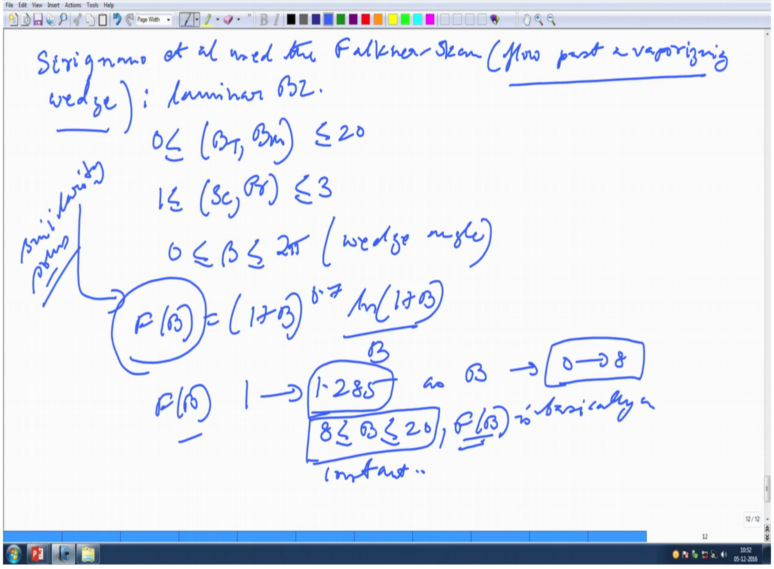

Sirignano et al used the Falkner-Skan (flow past a vaporizing wedge): laminar BL.

$0 \leq (B_T, B_m) \leq 20$

$1 \leq (Sc, Pr) \leq 3$

$0 \leq \beta \leq 2\pi$ (wedge angle)

similarity sol^ns

$$\boxed{F(B)} = (1+B)^{0.7} \frac{\ln(1+B)}{B}$$

$F(B)$ $1 \rightarrow \boxed{1.285}$ as $B \rightarrow \boxed{0 \rightarrow 8}$

$\boxed{8 \leq B \leq 20}$, $F(B)$ is basically a constant..

Sirignano this is; Abramson and Sirignano model essentially so, they as I said that at all used the Falkner-Skan type of solution which with me reiterate is flow over a wedge basically flow past a vaporizing wedge okay. So, it is a laminar boundary layer that is actually formed.

So, in those cases there are some restrictions on the BT and Bm okay. So, they are both less than 20 and this we already mentioned earlier the Schmidt number and the Prandtl number were less than 3. And the beta which is basically the wedge angle is within this right that is what Sirignano used flows faster vaporizing wedge and the corresponding laminar boundary layer right.

So, using similarity solution what he showed this will also narrated earlier that FB was nothing but 1 + B raised to the power of point 7 ln 1 + B divided by B, this we did in the last lecture right. So, using that solution using similarity solution here, this is what is showed right. So, there are some interesting exercises that people did people studied what is the variation of this FB.

This FB has got some variation right so people wanted to study that what is the variation of this FB in the current context. So, they found that FB varies from basically 1 to 1.285 okay as B which is basically done Spalding heat or mass transfer number varies from 0 to 8 okay. And when the heat transfer number was within 28 to 20 this FB is basically a constant, it does not vary at all.

This was an interesting observation right. So, there was about the 30 percent change in FB when B actually increased from 0 to 8, right. But there is a corresponding saturation of FB also that means a near constant value of FB for the major part of B that is from B from 8 to 20, it does not change at all okay. So, that was a good thing that was devised that was observed okay.

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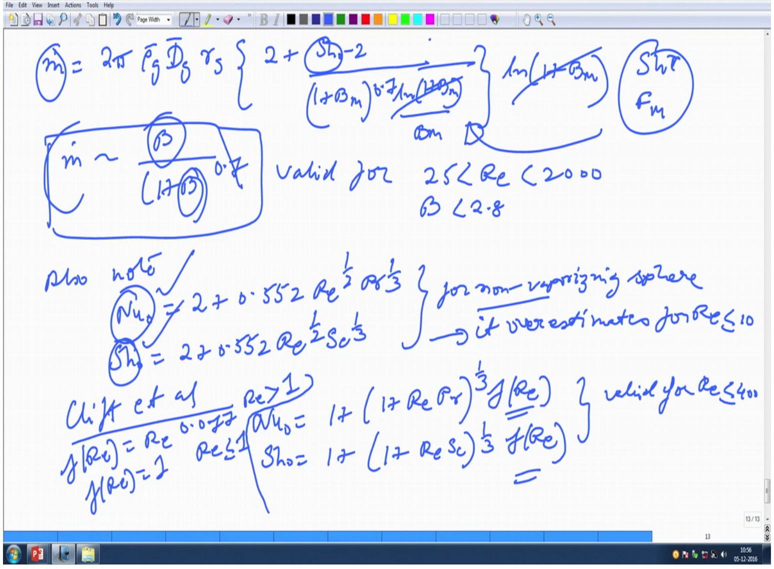

$$\dot{m} = 2\pi \bar{\rho}_g \bar{D}_g r_s \left\{ 2 + \frac{Sh_0 - 2}{(1+B_m)^{0.7} \frac{\cancel{\ln(1+B_m)}}{B_m}} \right\} \cancel{\ln(1+B_m)}$$

Sh^*, F_M

$$\boxed{\dot{m} \sim \frac{B}{(1+B)^{0.7}}}$$ valid for $25 < Re < 2000$, $B < 2.8$

Also note

$$Nu_0 = 2 + 0.552 Re^{\frac{1}{2}} Pr^{\frac{1}{3}}$$
$$Sh_0 = 2 + 0.552 Re^{\frac{1}{2}} Sc^{\frac{1}{3}}$$
for non-vaporizing sphere → it overestimates for $Re \leq 10$

Clift et al

$f(Re) = Re^{0.077}$ $Re > 1$

$f(Re) = 1$ $Re \leq 1$

$$Nu_0 = 1 + (1 + Re\,Pr)^{\frac{1}{3}} f(Re)$$
$$Sh_0 = 1 + (1 + Re\,Sc)^{\frac{1}{3}} f(Re)$$
valid for $Re \leq 400$

Using that when you actually what is your m dot m dot we are going to substitute all these things into m dot because S, Sh star and all those things were there in m dot. Let us look at the mass transport limited m dot okay. So, this is rs 2 + S naught - 2 divided by 1 + so I am substituting everything over here .7 ln 1 + BM divided BM close it 1 + BM. If you look at this now, this and this gets cancelled okay.

So, your m dot basically scales as sum B, this B goes into the top right sum B divided by 1 + B, let us keep it generic. So, because this will be valid for the other one also right. These two gets cancelled okay this BM goes up, so this is basically B by 1 + B to the power of 0.7 right. So, this is valid for a very large set of Reynolds number actually 2000 and for B less than 2.8 got it, clear.

So, this comes from basically substituting the value of m dot. So, we have just substituted for what we have substituted for Sh star and Fm. These two have been substituted here right that is how we got this expression. Now also note because we still have these two numbers which are basically unknown right Sh naught and Nu naught this is the Nusselt number and Sherwood number.

So, note that those are given by standard correlation; this is for non-vaporizing sphere okay. And it actually has been shown that it overestimates for Reynolds number less than equal to 10 okay. Because this is for non-vaporizing because these are for non-vaporizing right as you have we already stated okay. So, clip at all they actually suggested a modification to this.

All these are very empirical sometimes okay. So, do not be bothered about the nature of the thing that is because as you know in fluid dynamics may most of the problems do not have a clear-cut solution. So, you have to resort to these approximate analyses, so these are not exact analysis. These are all approximate analysis so to say okay.

So, 1 + Reynolds number Schmidt number to the power of one-third f Reynolds number right. So, this is valid for Reynolds number less than equal to 400 up to that limit it is actually valid okay. Now this Fre has got two forms, Fre is basically equal to Reynolds number raised to the power of .077 and Fre = 1 for Reynolds number less than equal to 1.

This is for Reynolds number in greater than one right okay, you put a demarcation mark here there is no confusion right. So, these are that fact that all these are very empirical but I want you to get a feel of that thing that the mass evaporation rate is a function of the heat and mass transfer numbers right okay.

And the normal non-vaporizing Sherwood number and Nusselt number is given by that particular expression right.
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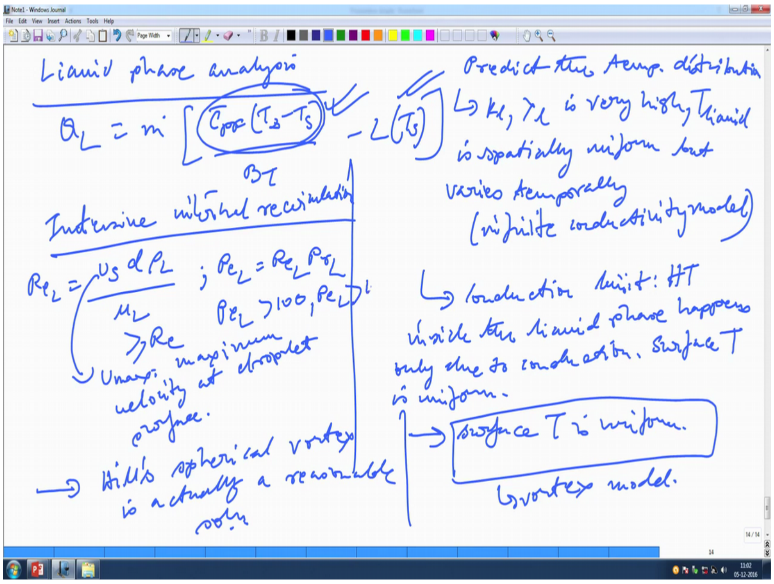
Liquid phase analysis

$Q_L = \dot{m}\left[\underbrace{C_{pF}(T_s - T_s)}_{BT} - L(T_s)\right]$

Intensive internal recirculation

$Re_L = \frac{U_s d \rho_L}{\mu_L}$; $Pe_L = Re_L Pr_L$

$Pe_L > 100, Pe_L \gg 1$

$\gg Re$

U_{max}: maximum velocity at droplet surface.

→ Hill's spherical vortex is actually a reasonable soln

Predict the temp. distribution

↳ k_l, λ_l is very high, T liquid is spatially uniform but varies temporally (infinite conductivity model)

↳ Conduction limit: HT inside the liquid phase happens only due to conduction. Surface T is uniform.

→ surface T is uniform.

↳ vortex model.

Now we once again revisit the liquid phase analysis now that we have done this the liquid phase analysis become whatever we did earlier right, let us put all the things in perspective now. Now the liquid phase analysis okay, what we have to do as I said earlier that we are interested to know what is the temperature distribution right and how it looks like with evaporation of the droplet.

So, you already know that QL is basically given by this right, correct, that is what it is right CPF this is the enthalpy, this is a specific sensible enthalpy part. That is a latent heat part right. So, this goes towards the inside the droplet. So, what we need to predict the temperature inside the droplet, so, our job is to basically predict the temperature distribution.

That is one of the important things, temperature distribution right to do that what people normally do is that we assume that there is an infinite conductivity model right that is what we did. Infinite conductivity model means that Kl of the liquid phase, Kl or lambda l whatever you call it is very, very high right.

So, that means the temperature is spatially uniform but temporarily vary, temperature the liquid is spatially uniform but varies temporally that was the infinite conductivity model right infinite conductivity model. We have already seen this right, similarly there could be a conduction limited model, conduction limit okay where heat transferred through the droplets only happens due to thermal conduction okay.

So, the conduction limit is that heat transfer inside the liquid phase happens only due to conduction right, solely due to conduction. And of course the surface temperature is uniform correct, is that clear. So, these were the two models that we already looked at earlier right the conductivity limited model and this right.

However now we have a situation as we said earlier there is intensive internal recirculation and what is the role for that. We are interested in this particular portion right. This is called as we said earlier by surface friction nothing else other than surface friction okay. When we already have defined the Reynolds number as Us into d of the droplet Rho L by Mu L and the Peclet number is given as well.

So, this is basically as we said is a U max or maximum velocity at droplet surface right. This is we already said that it is greater than equal to the Reynolds number, the normal gas phase Reynolds number right. So, here we already say it something about the vortex model and how it does a very good job in which the flow inside the droplet is represented by the Hill Spherical Vortex Scenario right.

So, here we said that Hill Spherical Vortex and we already said what are the equations that it solves; Hill Spherical Vortex is actually a reasonable solution, okay. And of course we continued that the surface temperature is uniform, we said uniform we did not say it is constant right.

We did not say that the surface temperature there is no spatial variation it is only at the surface but the temperature is uniform that means along the surface there is no temperature gradient okay which would have brought the Marangoni and other kind of stresses into the picture. And in reality that can happen actually because you have a wet behind the droplet okay and all those things.

But for the time being for in this particular consideration for the vortex model we are assuming and for many other models also we are assuming that this surface temperature is actually uniform across the across all levels okay. And this Peclet number is also high it is actually some times greater than 10, greater than 100 in most of the times it is actually greater than 10 at least okay so high Peclet number situation okay.

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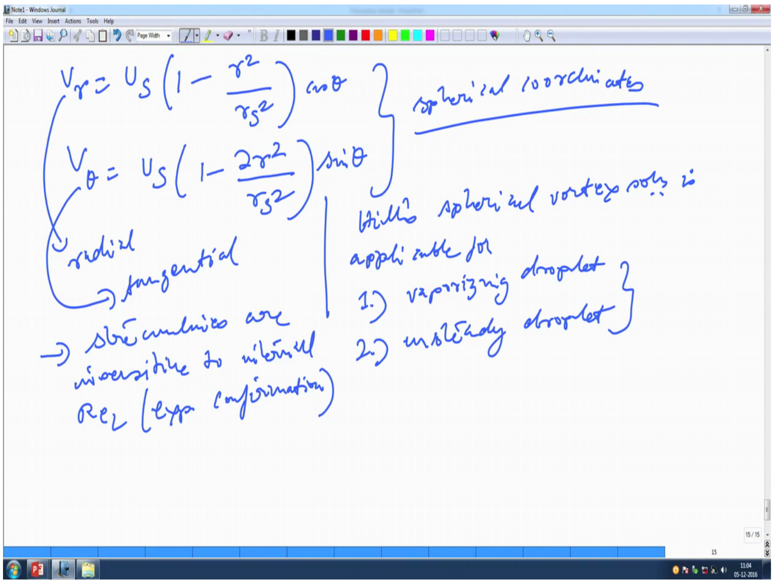

$$V_r = U_s\left(1 - \frac{r^2}{r_s^2}\right)\cos\theta$$

$$V_\theta = U_s\left(1 - \frac{2r^2}{r_s^2}\right)\sin\theta$$

spherical coordinates

radial

tangential

→ streamlines are insensitive to internal Re_L (exp. confirmation)

Hill's spherical vortex soln is applicable for

1.) vaporizing droplet

2.) unsteady droplet

Now let us look at from the Hill Spherical Vortex what we have if you cast it in terms of velocity this is a radial component of the velocity okay. And this is the corresponding tangential component of the velocity okay. So, these are velocity is given in the spherical coordinate, if you are unsure about the spherical coordinates I suggest that you pick up a fluid mechanics book and see okay that how it happens in a spherical coordinate.

So, this is basically the radial component of the velocity this is basically the tangential component of the velocity. So, these are the two components okay. We also assume that the Hill Spherical; this velocities are by the way variable okay. They are not constant okay but we assume that the Hills Spherical Vortex applies to the unsteady situation also.

And during the vaporization of the droplet the simple reason is that we already established that the vorticity transport happens in a much shorter time okay and that is the reason why this we can actually go buy it okay. So, even when the; so, formerly writing it Hills Spherical Vortex solutions is applicable for number one vaporizing droplet right okay.

Vaporizing droplet and unsteady and we already established why that should be the case okay. And we also see that the streamlines are insensitive this is an important observation insensitive to internal Reynolds number that is ReL. So, the streamlines are basically insensitive there is an experimental confirmation also on this.

People who have measured the velocity inside the droplet you can see that streamlines are basically insensitive to the liquid phase Reynolds number okay. Now to look at some of this thing okay and in order to explain one final time, so we already established let us go to the next page.

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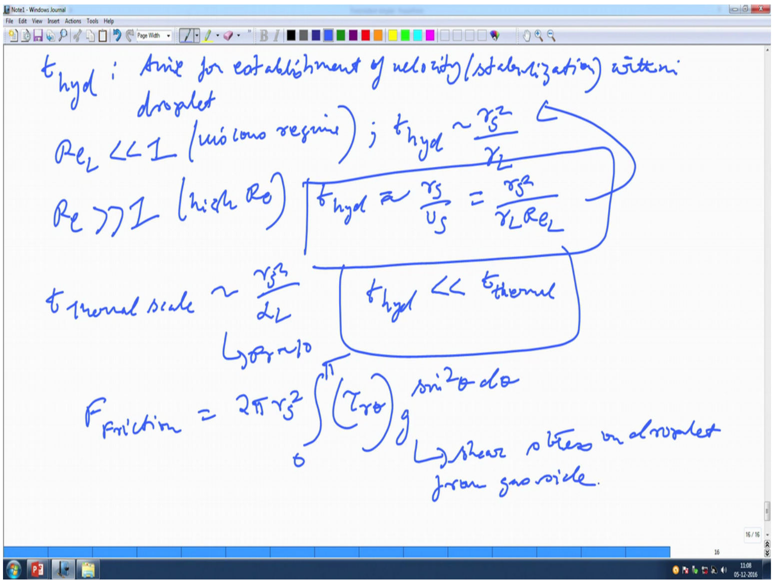

t_{hyd} : time for establishment of velocity (stabilization) within droplet

$Re_L << 1$ (viscous regime) ; $t_{hyd} \sim \frac{r_s^2}{\gamma_L}$

$Re >> 1$ (high Re) $\boxed{t_{hyd} \approx \frac{r_s}{U_s} = \frac{r_s^2}{\gamma_L Re_L}}$

$t_{thermal\ scale} \sim \frac{r_s^2}{\alpha_L}$ ↳ $Pr \sim 10$

$\boxed{t_{hyd} << t_{thermal}}$

$F_{friction} = 2\pi r_s^2 \int_0^\pi (\tau_{r\theta})_g \sin^2\theta \, d\theta$

↳ shear stress on droplet from gas side.

To have the screen, so, the hydrodynamic characteristic time of establishment of velocity, so this is the time for establishment of velocity. Within the, or stabilization or velocity stabilization within the droplet, now that is given by ah when the Reynolds number of the liquid phase is very, very low, much, much less than one basically viscous the entire droplet is basically viscous okay.

Let us take you; take a droplet of honey okay or a glycerol droplet which viscosity is very high okay and that will give rise that the velocity inside the droplet will be very low. So, in that particular case your t hydrodynamic is basically scales as rs square divided by gamma L right which is the kinematic viscosity.

Now in the high Reynolds number that means there are also some are much, much greater than 1 high Re. The same t hydrodynamic therefore now will be, now will scale rs by Us, Us is nothing but the surface velocity right. So, in other words, this will be equal to rs square divided by gamma L into Reynolds number L, right.

So, what we can see is that if the Prandtl number now; so, in this particular case okay one can readily see that this hydrodynamic time scale is much, much shorter than that, no doubt right this hydrodynamic time scale should be much, much shorter than that, okay. Similarly if you talk about some kind of a thermal scale okay thermal scale which is responsible for some kind of a say the temperature differential okay.

That is still scale as rs square by alpha L right and into the usual case is t hydrodynamic is actually much, much shorter than this t thermal okay. So, this basically justifies that why the Hill Spherical Vortex is always a good solution okay. And why it is actually a quasi steady kind of a thing okay. And this happens also because your Prandtl number is always of the order of 1 of the order of 10 over here okay.

Now that key feature that evolves out of this, if you look at the previous slide. So, you can see that there is in order to compute these two velocities we need this, this Us which is the surface velocity right, we need that. So, how to actually calculate that is the question that is a long standing question.

So, it should definitely come from the friction because that velocity is induced by friction right. So, let us calculate the friction force so when we calculate the friction force this is rs square is the r theta g sine square theta d theta so this is a shear stress on droplet from gas side coming from the gas side right that is why I wrote g over there.

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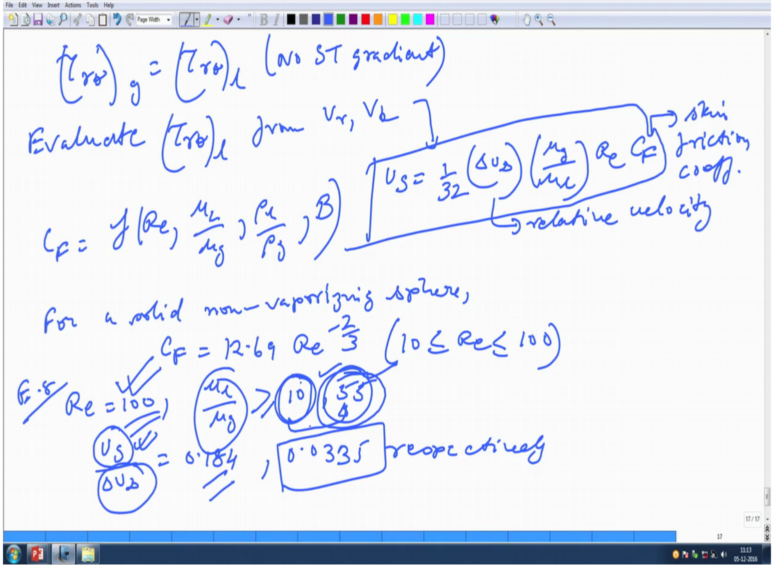

$(\tau_{r\theta})_g = (\tau_{r\theta})_l$ (No ST gradient)

Evaluate $(\tau_{r\theta})_l$ from V_r, V_θ

$$U_s = \frac{1}{32}(\Delta U_\infty)\left(\frac{\mu_g}{\mu_l}\right) Re\, C_F$$

→ skin friction coeff.

→ relative velocity

$$C_F = f\left(Re, \frac{\mu_l}{\mu_g}, \frac{\rho_l}{\rho_g}, B\right)$$

For a solid non-vaporizing sphere,

$$C_F = 12.69\, Re^{-\frac{2}{3}} \quad (10 \leq Re \leq 100)$$

E.g. $Re = 100$, $\frac{\mu_l}{\mu_g} > 10, 55.4$

$\frac{U_s}{\Delta U_\infty} = 0.184$, 0.0335 respectively

And we already said that this r theta g is the same as r theta l right why that happens because there is no surface tension gradient we already said that right uniform temperature across the surface right. So, what we do immediately after this is that now that we know evaluate r theta l from Vr and V theta that is easy right.

Because you know what Tr theta is right, so it is basically the cross derivatives well once we know that from here and I am not showing the steps again the steps of messy. So, this is Us is 1 by 32 delta u infinity which is basically nothing but the relative velocity Mu g by Mu l Re into CF. Now this is basically a relative velocity the velocity okay between the gas phase and the liquid phase.

This is basically a skin friction coefficient comes from the friction right, so this is like a skin friction coefficient that we have. Now this screen friction coefficient in turn is a function of the Reynolds number Mu l Mu g of course Rho l Rho g and it also depends on the spalding heat and mass transfer numbers.

This is like simple common sense intuitive sense should actually tell you that if this is the velocity profile that we get at the surface then it must be dependent on these parameters okay. Now for a solid non vaporizing again; we move to the same type of analysis for a solid non vaporizing sphere I think what we have CF is basically given as 12.69 Reynolds number to the power of minus two-third.

This is valid between Reynolds number all the way up to 100, 10 to 100 right. So, for example if your Reynolds number is 100 okay Mu l by Mu g is greater than say 10 or 55 okay this Us by delta U infinity that means the relative velocity will be given as 0.184 and . 0335 respectively.

It is just to give you an example it understood that the Reynolds number external Reynolds number is of the order 100 is Mu l by Mu g is of the order of 10 or 55 you can see when it is 10 that means the ratio of the density of the of the viscosity between the liquid and the gas is only 10 times okay.

The gas is obviously 10 times less viscous than the liquid you have that your Us that is the velocity that is created at the surface of the droplet inside the liquid phase right is actually about 20 percent of the velocity of the relative velocity right. So, if the relative velocity is say 5 meter per second this will be more like 20 percent of 5 meter per second, got it, okay.

On the other hand as we increase the viscosity ratio to 55 that means the liquid phase perhaps has become a more viscous fluid say for example it is become glycerol, glycerol and water mixture something like that right okay. You will find that the velocity of Us for the same velocity in the external phase is actually a much lower number it is only about 3.5 times, 3.5 percent right.

So, you can see as you make that inside of the droplet more viscous less will be the velocity that will be created by this surface friction right. If then if the viscosity difference is very small okay like of the order of 10 right significant amount of velocity is actually created inside the liquid droplet.

This is quite obvious that is because if the viscosity which actually determines what kind of a velocity you will generate inside the liquid phase for a given outer phase flow field right. So, we have kept the outer phase flow field that is the Reynolds number to be the same at 100 in both the cases except we have played with the viscosity.

As you make the viscous viscosity ratio higher and higher you get lower and lower surface velocities in the liquid phase. So, that is an interesting observation that we have over here okay.

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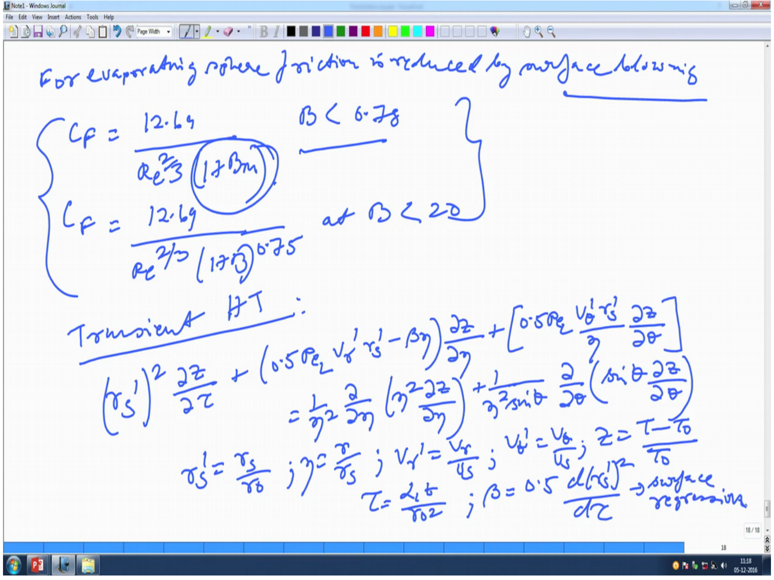

For evaporating sphere friction is reduced by <u>surface blowing</u>

$$\left\{\begin{aligned} C_f &= \frac{12.69}{Re^{2/3}\boxed{(1+B_m)}} \qquad \underline{B < 0.78} \\ C_f &= \frac{12.69}{Re^{2/3}(1+B)^{0.75}} \quad \text{at } B < 20 \end{aligned}\right\}$$

<u>Transient HT</u>:

$$(r_s')^2 \frac{\partial z}{\partial \tau} + \left(0.5 Pe_L v_r' r_s' - \beta\eta\right)\frac{\partial z}{\partial \eta} + \left[0.5 Pe_L \frac{v_\theta' r_s'}{\eta}\frac{\partial z}{\partial \theta}\right] = \frac{1}{\eta^2}\frac{\partial}{\partial \eta}\left(\eta^2 \frac{\partial z}{\partial \eta}\right) + \frac{1}{\eta^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial z}{\partial \theta}\right)$$

$$r_s' = \frac{r_s}{r_0}\;;\; \eta = \frac{r}{r_s}\;;\; v_r' = \frac{v_r}{u_s}\;;\; v_\theta' = \frac{v_\theta}{u_s}\;;\; z = \frac{T - T_0}{T_0}$$

$$\tau = \frac{\alpha_l t}{r_0^2}\;;\; \beta = 0.5\frac{d(r_s')^2}{d\tau} \rightarrow \text{surface regression}$$

So, moving on to the next one okay, so, for evaporating sphere, so this is non evaporating or evaporating sphere okay friction is reduced by surface blowing the friction is reduced for your CF will be 12.69 into Reynolds number to be the power minus two-third. Sorry wrong expression.

So, when you actually have the Reynolds number; you have the skin friction coefficient this 12.6 line still remain this Reynolds number two-third also remains. You have 1 + BM that comes over here this is valid for B less than 0.78 whereas the same expression will be 12 .69 divided by Reynolds number to the power of two third 1 + B raised to the power of .75 at B less than 20, got it, okay.

We got it so, this particular part is quite obvious now the skin friction coefficient is given, it is actually reduced because of the surface blowing because of the thickening okay of the boundary layer right. So, this will in turn reduce your Us that means the surface velocity that you are going to generate inside the liquid phase.

Now we are in a perfect now that we have analyzed everything you know Us and everything now the transient heat transfer now is transient heat transfer now can be solved. How it can be solved and this is a long equation with all non dimensionalized. So, you can open it up and you can see it yourself also.

This beta is not the same as the wedge will show you what that is, sine theta complete spherical coordinates got it, long equation nothing to be afraid of. It is a spherical coordinate we have written basically the advective, convective advective equation okay within the liquid phase. So, your rs prime is basically equal to rs by r naught, r naught being the initial diameter.

Nita is the normalized r access Vr prime is basically Vr by Us, V theta prime is basically V theta by Us, Z is basically T - T naught by T naught, Tau is basically alpha lt by r naught square okay and beta is basically equal to not the wedge angle I know that there are some notational issues always d Tau okay.

So this is basically proportional to the surface regression, got it okay. So, this is the complete equation written in a non dimensional fashion for you okay.

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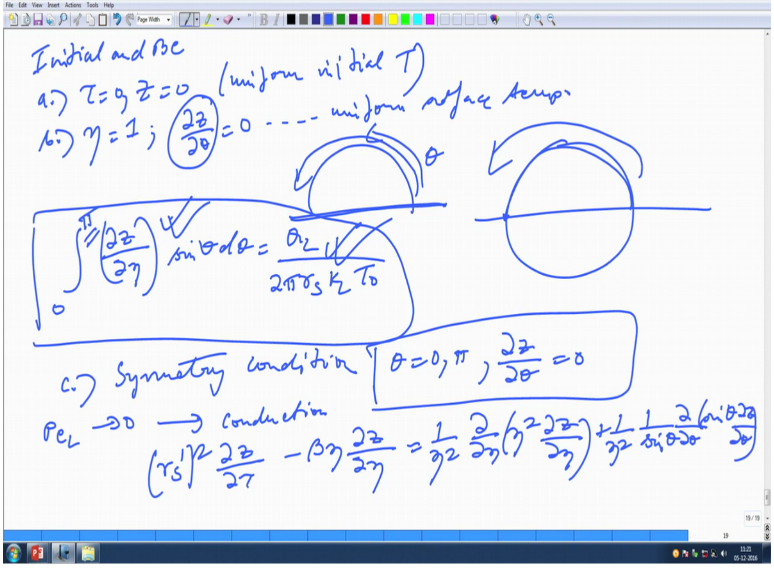

Initial and BC

a.) $\tau = 0, Z = 0$ (uniform initial T)

b.) $\eta = 1; \boxed{\frac{\partial Z}{\partial \theta}} = 0$ ---- uniform surface temp.

θ

$$\int_0^{\pi} \left(\frac{\partial Z}{\partial \eta}\right) \sin\theta \, d\theta = \frac{Q_L}{2\pi r_s k_L T_0}$$

c.) Symmetry condition $\theta = 0, \pi, \frac{\partial Z}{\partial \theta} = 0$

$Pe_L \to 0 \longrightarrow$ conduction

$$(r_s')^2 \frac{\partial Z}{\partial \tau} - \beta \eta \frac{\partial Z}{\partial \eta} = \frac{1}{\eta^2} \frac{\partial}{\partial \eta}\left(\eta^2 \frac{\partial Z}{\partial \eta}\right) + \frac{1}{\eta^2} \frac{1}{\sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial Z}{\partial \theta}\right)$$

So, this has this is subjected to some initial and boundary conditions a. Tau = 0 t = 0 this is the uniform initial temperature right as the initial temperature, b. at Nita = 1 that means at the surface of the droplet dz by d theta = 0 okay this is also again uniform surface temperature at any point. If we said right the surface temperature is always uniform regardless right, okay.

So, there is no theta variation, theta radiation means there is no angular variation if you take this as your droplet so this is your theta so in this theta there is no variation right which is make sense correct okay. And dz by d Nita sine theta d theta = Q L divided by 2 pi rs kL T naught, correct, okay. So, this is nothing but the total heat flux.

This has been integrated across the entire surface and we this is a symmetry line we have done it only up to PI right because this is half the droplet this is a full droplet is a symmetrical so from here to here is basically your Pi right. So, you have integrated, so this means that it has been integrated across this entire surface right to give you the total heat flux right that should be quite obvious okay.

Then there is the symmetry condition right symmetry condition is at theta = 0 and Pi d theta by dz = 0 right, okay, got it. So, now that we have done all these things, now you should recall that when Peclet number approaches zero that means the flow velocity within the droplet actually goes to zero.

This reduces to a conduction problem approaches a conduction problem okay. And so the equation basically gets changed, so if you write the same equation right, okay because I was just taking off the Piclet number term okay.

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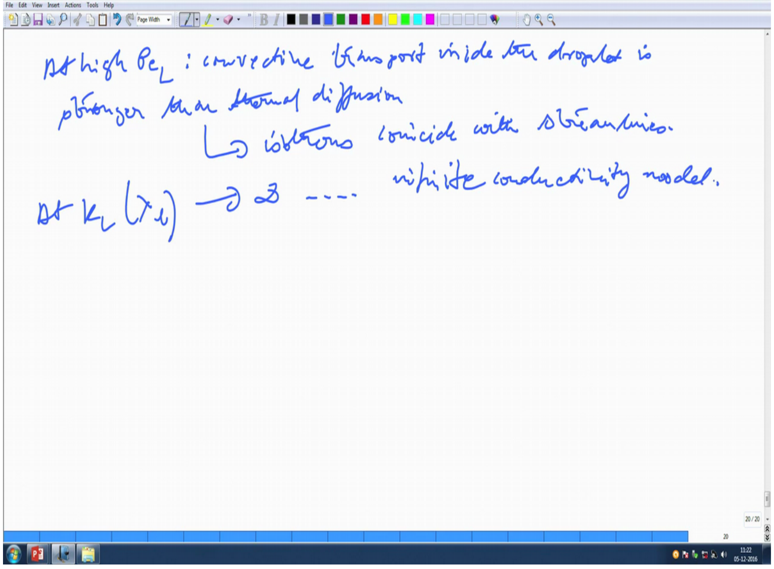
At high Pe_L : convective transport inside the droplet is stronger than thermal diffusion

$\hookrightarrow$ isotherms coincide with streamlines.

At K_L (λ_L) $\rightarrow \infty$ .... infinite conductivity model.

So, so at high Peclet number, this is for low Piclet number, for high Peclet number okay the convective transport inside droplet, the droplet okay if stronger than thermal diffusion obviously that is the case right and so the isotherm basically coincide with streamlines okay.

And of course there is other limit KL or you can write it as lambda L as that approaches infinity this is called basically the infinite conductivity model, got it, okay. So, this particular equation there is no way that you can solve it. So, you have to solve this numerically. So, in the next lecture what we are going to do is that we are going to see that how what are the numerical outcomes of these processes okay.

So, numerically these are solved cannot be solved in any other way right but we have shown that from the main governing equation what you can expect, you can expect at your stream lines and your isotherm should kind of match each other okay that is the case when you have a high Peclet number.

For a low Peclet number this would be more like a diffusion based kind of a problem right. So, these two things if you can analyze it and we do in a droplet lifetime also this can actually change because Peclet number is not constant across the droplet lifetime because your Reynolds number is varying outside. So, you are Reynolds number inside the droplet will also vary right.

As a result of that your Piclet number is also going to be vary right. So, all these things dynamics combined we need to show see some results basically to understand what are the physical interpretations. So these are basically solved in a numerical fashion, so numerically these problems are basically solved because there is no other way of solving it.

And we will look at some of the numerical results in the next class. We are also going to look at some alternative approaches are also there okay. So, those alternative approaches we will see in the next class okay, thank you.