Heat Transfer And Combustion in Multiphase Systems
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Lecture 26
Comprehensive droplet vaporization model and correlations-I

So, the last lecture we looked at the approximate liquid phase analysis except in a heuristic way. Because you see that complications about these kinds of problems are that you can never do the full math okay. Because the full math itself are basically research problem okay. So, we are just giving you ideas that how this problem are actually approached okay. And that is how we are actually solving them okay.

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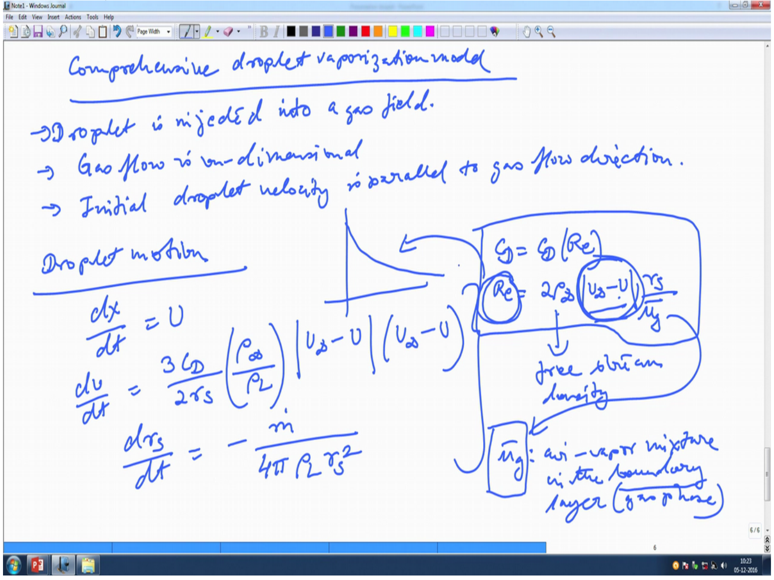

Comprehensive droplet vaporization model

→ Droplet is injected into a gas field.
→ Gas flow is one-dimensional
→ Initial droplet velocity is parallel to gas flow direction.

Droplet motion

$$\frac{dx}{dt} = U$$

$$\frac{dU}{dt} = \frac{3C_D}{2r_s}\left(\frac{\rho_\infty}{\rho_L}\right)\left|U_\infty - U\right|\left(U_\infty - U\right)$$

$$\frac{dr_s}{dt} = -\frac{\dot{m}}{4\pi\rho_L r_s^2}$$

$C_D = C_D(Re)$

$Re = 2\rho_\infty\left|U_\infty - U\right|\frac{r_s}{\bar{\mu}_g}$

free stream density

$\bar{\mu}_g$: air-vapor mixture in the boundary layer (gas phase)

So, here we are going to put forward a comprehensive droplet vaporization model and that will make things a lot clearer okay. A comprehensive droplet vaporization model now what happens in that comprehensive droplet vaporization model is that we are going to look at all the phases now and we try to connect them actually together okay.

So, that will give you the clear cut idea. So, here what happens is that you are assuming a scenario in which our droplet is injected into gas field gas phase flow field right okay. That is where the droplet is actually injected okay. So, there are several assumptions that goes with it that the gas flow is one dimensional.

So, that means we are not really concerned, so much about the gas flow. The initial droplet velocity is parallel to the gas flow direction got it. So, it is like a droplet that is injected into a gas flow field the droplet is velocities parallel to the gas flow. So, we are going to analyze all the things that we have learned so far and put them in one under one umbrella.

So, that he can understand them right that you can actually solve them also like that. So, the droplet motion let us take the droplet motion first, this is the bulk motion of the droplet dx by dt = U du by dt = 3CD by 2rs Rho infinity by Rho l this we already know, I am just writing it. So, that you get the complete picture right.

If you recall this is what we did in a drag right that is a change of velocity is basically given by that drag force right whereas the change in position of the droplet is basically given by the velocity right that is very simple. And drs by dt that is the rate of change of droplet diameter is basically the vaporization rate, like that right.

And what did you learn earlier that CD the drag coefficient is basically a function of Reynolds number right, the external Reynolds number where that Reynolds number once again is given by U infinity - U rs by okay. Now this is basically what this is a free stream density it is not the mixture density right.

This is a free stream density whereas this guy over here okay is basically the Mu g is basically the air vapour mixture and it is the average viscosity in the boundary layer. So, this boundary layer is basically the gas phase boundary layer right. So, it is the average viscosity in that boundary layer got it, here, okay.

So, it is the average density in that boundary layer. So, this is CD, this is, that now one thing that you can readily see over here that this Reynolds number is supposed to come down right, the gas phase Reynolds number why because this depends on the relative velocity of the two of the droplet and the free stream flow right.

So, when a droplet is injected into a flow field it is natural that the droplet velocity will increase okay and ultimately at some point of time it will become very close to the gas phase of the velocity right that is what happens. So, when that happens, this Reynolds number value is supposed to die down very quickly right.

So, you can expect a profile in which this Reynolds number will go down right that is supposed to happen. So, that is what you get from this from this analysis okay. So, the care is taken that you are computing the viscosity in the boundary layer whereas you are taking the density in the free stream okay.

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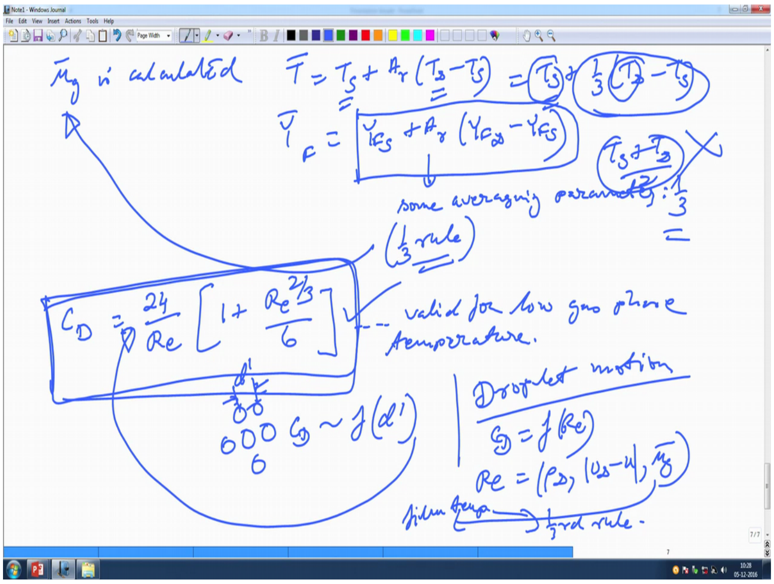

$\bar{\mu}_g$ is calculated

$\bar{T} = T_s + A_r (T_\infty - T_s) = T_s + \frac{1}{3}(T_\infty - T_s)$

$\bar{Y}_F = Y_{F_s} + A_r (Y_{F_\infty} - Y_{F_s})$

some averaging parameter : $\frac{1}{3}$

$\cancel{\frac{T_s + T_\infty}{2}}$

($\frac{1}{3}$ rule)

$C_D = \frac{24}{Re}\left[1 + \frac{Re^{2/3}}{6}\right]$ --- valid for low gas phase temperature.

d'

$C_D \sim f(d')$

Droplet motion

$C_D = f(Re)$

$Re = (\rho_D, |u_D - u|, \bar{\mu}_g)$

film tempe $\rightarrow$ $\frac{1}{3}$rd rule.

Now this Mu g now you can ask that how is Mu g then calculated in the boundary layer right that is the next question. So, where we are advanced and each and every question that you might have. So, this Mu g is calculated, so you need two things you need the temperature right because you need the temperature because viscosity is a function of temperature. And you also need the composition.

Because we do using the composition and the temperature you are going to find out what is the mean or the average viscosity will be in the boundary layer. So, that is calculated as T bar that is where it is calculated. This is the surface temperature to some factor and this is T infinity - Ts.

So, the temperature that is normal because external temperature is T infinity surface temperature is Ts, so the boundary layer temperature will be somewhere in between right. So, that is what the assumption is. So, this A we will come a little later, what that is, similarly the composition with YFs on the surface.

It is Ar YF infinity - YFs right, so, here also this A is basically some averaging parameter okay. And usually the value is one third this is in some literature it is called the one-third rule okay. So, it is basically what it means is Ts + one-third T infinity – Ts. So, this is like a wetting okay it is not like just a mean of the tube.

It is not like Ts + T infinity by 2, it is not like that right because the temperature is predominantly Ts and there is a little bit of a contribution coming from the difference right. So, this is an similar thing happens with composition okay it is not Ts + T infinity by 2 that is wrong okay.

So, this has come through experiments and empirical correlation that this is called the one-third rule. This is how this viscosity is actually calculated. So, you can see it is biased more towards Ts than towards T infinity you can see that right because it is a two-third, one-third kind of a rule right.

It is a one-third rule that is what actually we call it in the literature right okay. So, that is how the Mu is calculated, the Mu g is calculated. The CD looking at now the variation of CD because CD we said is a function of Reynolds number. So, that is given by 24 by Re okay, Re to the power of to third by 6.

This is however valid for low gas phase temperature not for high temperature systems like for example plasma or flames okay where the droplet can be even in a flame right. It can be in a very hot flow okay. So, it depends on the Reynolds number in this particular way. Now many of these things are empirical many of them are devised through experimental studies.

Again that is how these numbers are coming to the picture. Also you should note that this is valid only for a droplet if we have a spray configuration for example right. In a spray what happens that the droplets are close to each other correct. In that way it is CD okay will also be a function of this inter droplet spacing.

Let us call that some d prime this will be dependent on that inter droplet spacing also right in some way, we do not know. How that is? Why it is very hard to measure that CD for a cluster of droplet because you do not know how they are going to behave right. So, this is a empirical formulation valid for a single droplet.

This term also has to be incorporated into this somehow okay. Because of the shielding effect that the droplet actually creates okay. So, the CD, how the CD will depend on that that is also an open ended question, got it. So, this actually takes care of the droplet motion part whatever we did this is droplet motion.

The key takeaway things are that there is something called CD which is a function of the Reynolds number. The Reynolds number itself okay is a function of Rho infinity, the relative velocity which is U infinity - U and the corresponding Mu g bar, this Mu g bar requires evaluation of the film temperature.

Film temperature is the temperature of the gas phase boundary layer and there we have used the one-third rule right. Using the one-third rule we have found out what is the Mu g going to be, got it, okay. So, all these come under our unique umbrella now over here okay.
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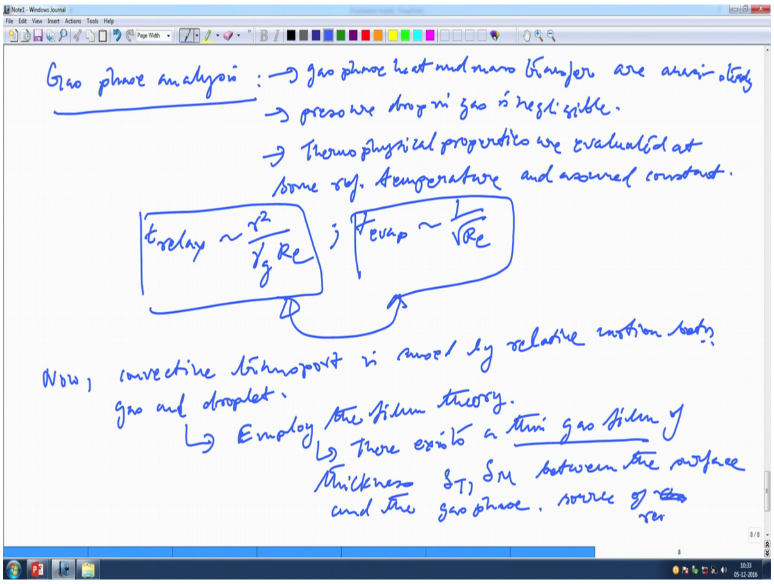

Gas phase analysis : → gas phase heat and mass transfer are quasi-steady
→ pressure drop in gas is negligible.
→ Thermophysical properties are evaluated at some ref. temperature and assumed constant.

$\boxed{t_{relax} \sim \frac{r^2}{\nu_g Re}}$; $\boxed{t_{evap} \sim \frac{1}{\sqrt{Re}}}$

Now, convective transport is caused by relative motion betn gas and droplet.
↳ Employ the film theory.
↳ There exists a thin gas film of thickness δ_T, δ_M between the surface and the gas phase. source of ver

Now, so, let us look at now what we are going to do about that gas phase analysis. Now the gas phase analysis okay you should consult the previous lectures that we did okay. And if you have forgotten some part of it let me just recollect that gas phase; heat and mass transfer are quasi steady.

We have already explained many times why it should be quasi steady we will do that one more time so the pressure drop in gas is negligible okay and thermo physical properties are evaluated at some reference temperature and is considered constant.

So, if you have forgotten why it is quasi steady let me say that that relaxation time of the gas phase right scale as r square this is the viscosity into the Reynolds number where as the evaporation time usually scaled as root over of Reynolds number if you recall the two arguments.

So, for high values of Reynolds number what we had was the relaxation time was much shorter compared to the evaporation time right. So, we could assume that the droplet evaporates much slowly. By droplet evaporating much slowly we mean that the diameter regression is very slow okay.

The gas phase quickly relaxes okay before that droplet actually vaporized. So, it is basically a quasi steady solution this we considered much earlier and we showed why is that is the case. This is just to remind you that this is what actually happens. Now we also say it now the convective transport this is important.

Convective transport is basically caused by friction or in other words the relative motion. Friction is due to relative motion, the relative motion between gas and droplet okay. So, what we do here in this particular case is employ the film theory okay. So, what is a sound theory need okay. As the name suggests, it is basically nothing but a thin boundary layer.

Let me put it that way it is like a nothing but a film of the vapour mixed with the gap that exists in the vicinity of the droplet very close to the interface right. So, this assumes that the resistance to heat and mass transfer between a surface and the gas may be modelled as gas films of constant thickness.

So, this is basically what is it, this is basically a boundary layer concept right. So, there it what we mean is that there exists a thin gas film right of thickness dt and dm one is the mass, one is for the temperature but is it thermal and one is the corresponding mass basis okay.

Between a gas this is; that there exists a thin film of thickness varies between the surface right and the gas phase there is the bulk gas phase. It is exactly the same as a boundary layer concept okay and this is what provides the resistance okay to the heat and mass transfer that happens between the gas phase and the liquid phase right.

So, this is what is actually provides the resistance okay. So, we are going to model this, this is the source of resistance basically, got it, okay. So, that part should be clear.

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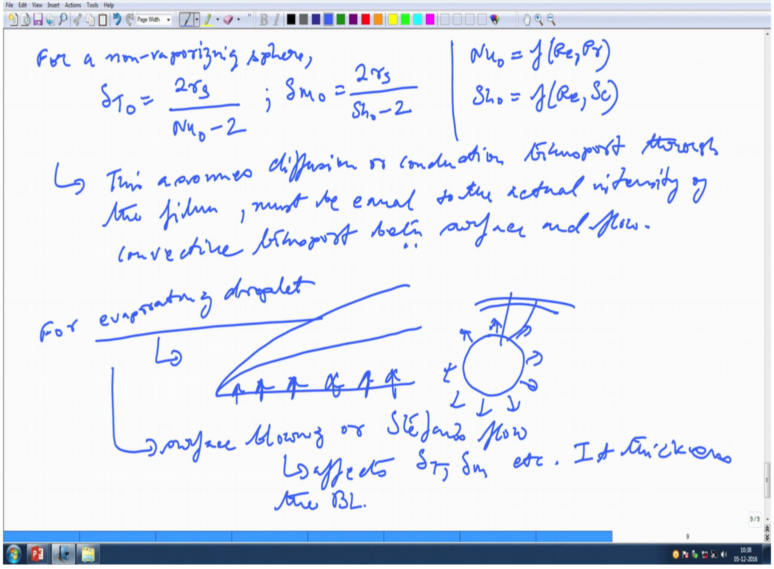

For a non-vaporizing sphere,

$$\delta_{T_0} = \frac{2r_s}{Nu_0 - 2} \; ; \; \delta_{M_0} = \frac{2r_s}{Sh_0 - 2}$$

$Nu_0 = f(Re, Pr)$

$Sh_0 = f(Re, Sc)$

↳ This assumes diffusion or conduction transport through the film, must be equal to the actual intensity of convective transport betn surface and flow.

For evaporating droplet

↳

↳ surface blowing or Stefan's flow

↳ affects δ_T, δ_M etc. It thickens the BL.

So, now if you look at for a non vaporizing sphere what happens is that delta T naught is given by 2rs divided Nusselt numbers 0 - 2 and I will come to that. The corresponding mo okay you remember that Nusselt number the function of Reynolds number and Prandtl number.

This you know from your heat transfer this is a function of Reynolds number and Schmidt number right. So, for a non vaporizing sphere that is the case. So, this essentially assumed that is the conduction model right non vaporizing sphere is a conduction model okay. So, this assumes that there is conduction or diffusion through the film must be equal to the actual intensity of the convective transport right.

So, in other words we have to write it this assumes diffusion or conduction transport through the film must be equal to the actual intensity of the convective transport of convective transport between surface and flow correct okay. So, evaporating droplet from expression, same expressions are used basically. So, this is for non-evaporating right.

So, for evaporating drops it, so, that was non-evaporating, so, you could get that very easily done for non-evaporating we use the same thing. Now we incorporate the effect of surface blowing okay. So, what happens in the evaporating droplet is that you vaporize the liquid right. So, there is some kind of a surface blowing so this we almost equivalent to the problem that you have a flow over a flat plate.

There is some kind of a surface blowing happening through the plate what happens with that the boundary layer gets actually extended right. So, there is a thickening of the laminar boundary layer so to say right. So, imagine the same thing is like a droplet there are vapours are being vented out right.

Because of evaporation rate right, so if there is a boundary layer that existed on the liquid surface right. On the liquid surface there is some kind of a boundary layer right. That boundary layer usually is extended or it becomes thicker okay. So, this is called the surface blowing surface blowing or Stefan's flow.

So, it affects, Stefan flow basically affect delta T delta m etcetera okay. So, in other words it thickens the boundary layer right. It thickens the boundary layer that is what it does right.
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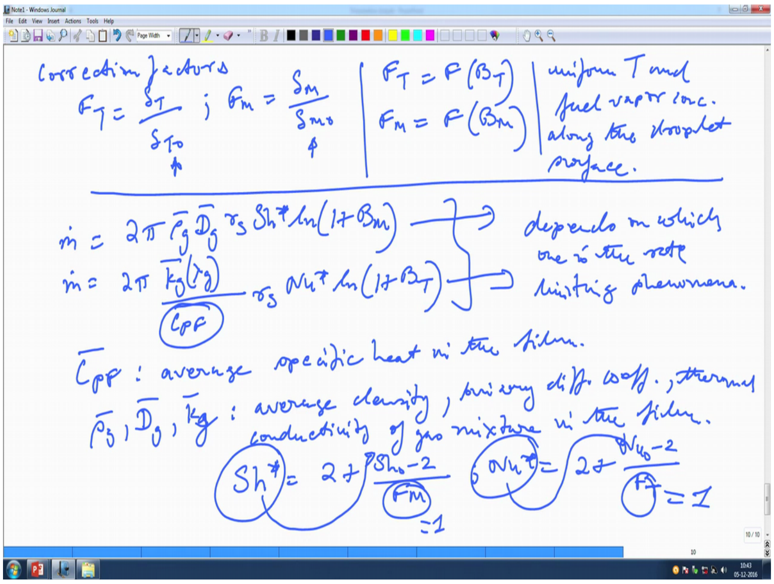

Correction factors

$F_T = \frac{\delta_T}{\delta_{T0}}$; $F_M = \frac{\delta_M}{\delta_{M0}}$

$F_T = F(B_T)$

$F_M = F(B_M)$

uniform T and fuel vapor conc. along the droplet surface.

$\dot{m} = 2\pi \bar{\rho}_g \bar{D}_g r_s Sh^* \ln(1+B_M)$

$\dot{m} = 2\pi \frac{\bar{k}_g(r_g)}{\boxed{\bar{C}_{pF}}} r_s Nu^* \ln(1+B_T)$

→ depends on which one is the rate limiting phenomena.

$\bar{C}_{pF}$: average specific heat in the film.

$\bar{\rho}_g, \bar{D}_g, \bar{k}_g$: average density, binary diff. coeff., thermal conductivity of gas mixture in the film.

$Sh^* = 2 + \frac{Sh_0 - 2}{F_M}$, $F_M = 1$; $Nu^* = 2 + \frac{Nu_0 - 2}{F_T}$, $F_T = 1$

So, in other words to take into account that thickening we introduce correction factors. So, the correction factors are FT is basically given by dT by dT0 and so it basically just packed some ratio so this is m divided dm0 right. So, these were basically for the non vaporizing case right.

Now if we assume that the distribution of temperature and fuel vapour and uniform across the droplet surface okay. We are can safely comment that FT is basically a function of the Spalding heat transfer number and Fm is basically a function of Bm that is mass transfer number. This happens because we have uniform temperature and concentration and fuel vapour concentration.

But concentration along the droplet surface got it, so along the droplet surface we have this uniform vapour concentration right okay, so, we can write it like that now. Based on all these things m dot which is a vaporization rate of the liquid can have actually two forms okay instantaneous vaporization rate.

One is given by this Rho g Dg rs Sherwood number + Bm, the other one equivalent kg or it can be written as alpha d lambda g also CPF rs a Nu star ln 1 + BT right. One is come from the heat transfer argument; one comes from the mass transfer argument. We already know that right, because of these two evaporation rate. These are the two functions okay.

Now one might ask which one to use between the two now that is a complicated question. What do you think? Which one should be used? Normally because normally Bm is not the same as BT right. We just established that it is only equal to BT when your Prandtl number and Schmidt number are same right but it is normally not equal correct.

So, when can we actually use it, we can use it, which one to use it okay that depends on which one is a rate-limiting step right. So, actually the smaller of the two is basically what actually governed okay that means you can transfer a lot of heat okay. So, if it is only heat basis you can evaporate it probably at a faster rate.

But it is the mass limitation that means still this accumulated vapour around the droplet has to be transported out for the next level of evaporation to take place right. So, it becomes at which one is the rate-limiting step over here and that is important okay. So, it depends on the, depends on which one is the rate-limiting phenomena heat or mass.

So, the smaller of the two is basically what is called the rate limiting. So CPF bar that we have written over here right so, that requires this particular guy that you see over here is basically nothing but the average specific heat in the film, film means that boundary layer right whereas this Rho g bar, D ge bar, Ks bar, Kg bar rather kg bar is basically the average density binary diffusion coefficient okay.

And thermal conductivity sorry thermal conductivity of gas mixture in the film okay got it. That is how you evaluate it; the evaluation part is not that difficult okay. So, we have also introduced two numbers like Sherwood number star that is basically 2 + a Sherwood number for the non vaporizing case divided this Fm let me just now said okay.

Similarly there is a Nusselt number counterpart to it that is nothing but 2 + Nusselt number 0-2 divided FT okay. So, you can imagine that when there are no surface blowing effect this becomes the same as the Nusselt number 0 right. This also becomes the same as this, so when there is no evaporation means Fm = 1 FT also will become equal to 1.

So, naturally what happens when this becomes 1 these 2 and that 2 get cancelled. You are left with the Nusselt number 0 right. So, that means it is the same as a non vaporizing sphere which is quite obvious right okay. When there is surface glowing you have a different value of Sherwood number and Nusselt number because of that surface glowing effect, because of the thickening of the laminar boundary layer and things like that right okay.

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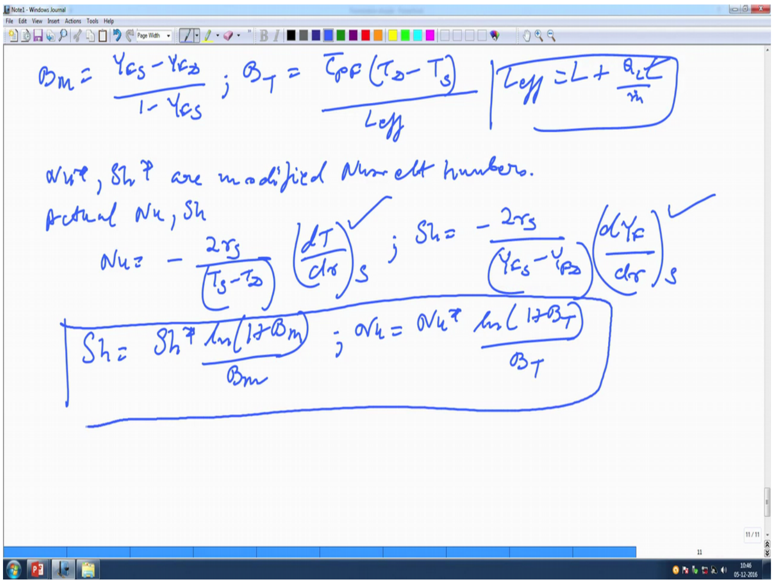

$$B_m = \frac{Y_{Fs} - Y_{F\infty}}{1 - Y_{Fs}} \; ; \; B_T = \frac{\bar{C}_{pF}(T_\infty - T_s)}{L_{eff}} \quad \boxed{L_{eff} = L + \frac{\dot{Q}_L}{\dot{m}}}$$

Nu^*, Sh^* are modified Nusselt numbers.

Actual Nu, Sh

$$Nu = -\frac{2r_s}{(T_s - T_\infty)}\left(\frac{dT}{dr}\right)_s \checkmark \; ; \; Sh = -\frac{2r_s}{(Y_{Fs} - Y_{F\infty})}\left(\frac{dY_F}{dr}\right)_s \checkmark$$

$$\boxed{Sh = Sh^* \frac{\ln(1+B_m)}{B_m} \; ; \; Nu = Nu^* \frac{\ln(1+B_T)}{B_T}}$$

It is also imperative to write once again to remind you guys that what is the Bm. So, Bm was YFs - YF infinity - YFs and the corresponding BT was CPF bar this is once again I have written it as it could have written it as Hs also that was not a problem and this is L effective. Where L effective if you recall was the latent heat plus it can be any QL by m dot also remember this QL was the amount of heat that was going inside the liquid phase right okay.

And l was basically the latent heat if you recall your earlier classes that is what we did in those classes as well, got it. So, now that this star are basically called the modified Nusselt numbers right star, Sh star are modified Nusselt numbers. So, actual Nusselt numbers are these are modified right actual Nusselt numbers are actual Nu and Sh are this is given from the first definition right.

If you look at the definition of Nusselt number this is what the definition is, at the surface right that is what Nusselt number is right. And Sh you given as 2rs divided by YFs - Y dYF by dr s right, got it. These are the actual definitions of the Nusselt number that you can see over here.

So, these muscles number these are the two definitions of the natural numbers that we have. This is the generic right the definition okay. So, these two Nusselt number and Sherwood numbers are linked with each other. The modified, this Nusselt number is given as Nusselt number star ln 1 + BT divided by BT okay, right.

So, these are the two definitions of way if you include, so, this includes the heat and mass transfer numbers over here okay. So in the next lecture we are going to go through, now and look at the liquid phase okay as well as the gas phase once again revisit it and try to cast everything actually together right. So, this lecture ends here.