Heat Transfer And Combustion in Multiphase Systems
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Lecture 25
Droplet liquid phase transport

Welcome to today's Lecture. So, last time what we did was that we undertook a very detailed analysis of the gas phase and what we have done, what we are going to do now, we are going to look at the, some, improved formulation of the gas phase as well as look at the liquid phase recirculation and at the same time look at full fledged model of the gas phase flow field.
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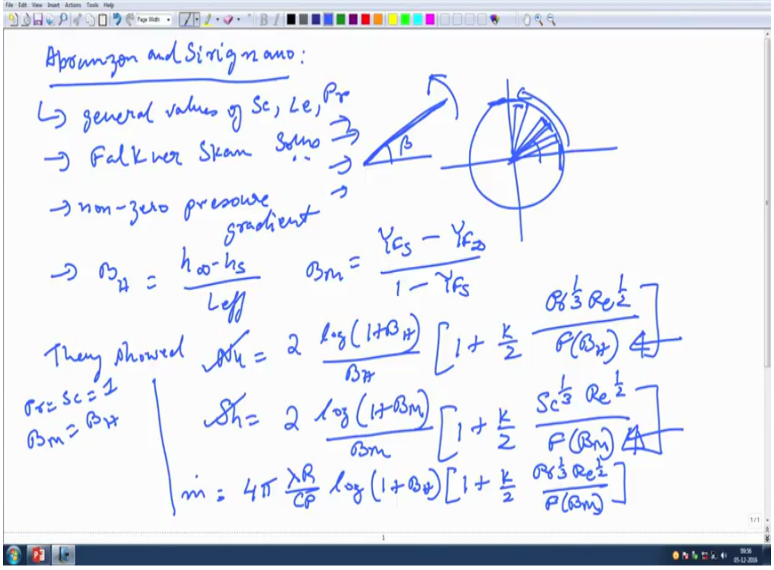

Abramzon and Sirignano:

↳ general values of Sc, Le, Pr

→ Falkner Skan Soln

→ non-zero pressure gradient

→ $B_H = \frac{h_\infty - h_s}{L_{eff}}$ $B_M = \frac{Y_{Fs} - Y_{F\infty}}{1 - Y_{Fs}}$

They showed $Nu = 2\frac{\log(1+B_H)}{B_H}\left[1+\frac{k}{2}\frac{Pr^{\frac{1}{3}}Re^{\frac{1}{2}}}{F(B_H)\cancel{4}}\right]$

$Pr = Sc = 1$
$B_M = B_H$

$Sh = 2\frac{\log(1+B_M)}{B_M}\left[1+\frac{k}{2}\frac{Sc^{\frac{1}{3}}Re^{\frac{1}{2}}}{F(B_M)\cancel{4}}\right]$

$\dot{m} = 4\pi\frac{\lambda R}{C_P}\log(1+B_H)\left[1+\frac{k}{2}\frac{Pr^{\frac{1}{3}}Re^{\frac{1}{2}}}{F(B_M)}\right]$

So, this work is primarily Abramzon and Sirignano, this is professor Bill Sirignano, previously of Princeton now of UC Irvine, okay. Now, what they did was that previously we showed that there are two classes of solution, right; one we took at the shoulder, okay where it was more like a flat plate and once we took near the equator where it was more like a stagnation flow.

And we say that the, these are the two extreme cases, right. And we did something in between. But Abramzon and Sirignano, they did higher work. They took that let us take general values of Schmidt number, Lewis number and Prandtl number. So, they did not restrict themselves to Lewis number =1 kind of a situation.

They also did undertake the Falkner Skan type of solution. Now, what is Falkner Skan type of solution? This is interesting. Falkner Skan type of solutions, if you know that these are basically wedges, right which makes any arbitrary angle beta with the flow direction. If you recall your few dynamics, okay.

Flat plate is a case of a Falkner Skan where this angle is basically equal to 0, right. So, you have this angle that changes from beta = 0, all the way up to beta = 180 degrees, so that is what the Falkner skan class of solutions are, right. So, what Abramzon and Sirignano did was, that if this is a droplet, right they took each section of the droplet to be locally like some wedges, right, okay.

They took it locally to be some wedges and they did the solution for the entire generic profile, got it. They did not restrict themselves to stagnation flow or a flat plate type of a flow, right. They did something which is equivalent, right which is like a Falkner Skan like a complete similarity solution taking locally, okay, the locally. And they also took non-zero pressure gradient, okay.

And of course, the standard definitions of the heat and mass transfer number the spalding heat and mass transfer number, okay. So, so, you understand. So, they basically took this to be a Falkner Skan type of solution, a wedge, basically took it to be like a wedge, okay. Locally each part of the droplet is like a wedge, okay. So, they took the generic solution for the wedge and that is how they actually solved it.

They showed that Nusselt number and we are not going to the math because it is extremely complicated. What are this F and this? We will come a little later. So, that is a Nusselt number then you get the corresponding Sherwood number for the mass transfer. As you can see both of their forms a kind of very equivalent except Prandtl number is replaced by the corresponding Schmidt number and this is a function of BM, okay.

So, what these factors are, okay? That will come a little later, okay. And we will see what those factors can be, okay. These are more like correction terms that you can see over here, okay. So, using the class of solutions they found what is the Nusselt number and what is the Sherwood number. And the important part is, of course, the evaporation rate which is m dot, right. So, that is given as 4 pi lambda R CP log 1 plus Bh 2 1 + k by 2 Prandtl one-third Reynolds half F BM, okay, got it.

Now, there is one other assumption when Prandtl number is the same as Schmidt number, is equal to 1 your heat and mass transfer numbers becomes equal to each other, right. That means BM it becomes equal to BH, okay.

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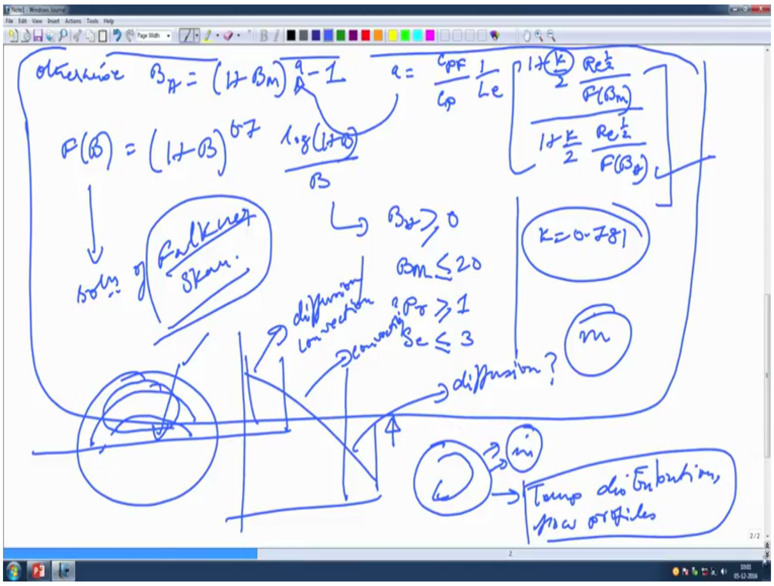

Otherwise, if that is not the case, then, normally your BH is given as 1 + BM to the power of a - 1, okay. That is otherwise. That means when that condition is not valid; when Prandtl and Schmidt number are not equal to 1, okay. Now, this a okay, CPF by CP, CPF is nothing but the mixture specific heat or the specific heat of the gas mixture including the fuel vapour, okay.

1 over the Lewis number 1 + k by 2 Reynolds number to the power of half F BM is the same F factor that we have, 1 + k by 2 Reynolds number to the power of half F Bh. That is the total expression that we have, okay. That is that factor a, which goes and sits here, okay. Now, the correlation of the numerical results for Falkner Skan actually shows that your FB is actually equal to because FB is an unknown that we stated over here.

It is about .7 log 1 + B by B, okay. Now, this class is valid usually for BH greater than equal to 0 BM less than equal to 20 Prandtl number greater than equal to 1 and Schmidt number less than equal to 3, okay. So, that is the thing. And this k that you see over here it k is about . 781. So, this comes from the solution of Falkner Skan.

So, this is a really ingenious way that Sirignano and Abramzon actually used to solve it. They took the droplet into a piecewise kind of locally to behave like a Falkner Skan type of a solution. Locally they are like wedges right, okay. And then they use to the similarity profile, the similarity solutions that we did in the last lecture, right.

The similarity solution, okay, f, f triple prime + f into f double prime is equal to 0, right. That similarity kind of a solution, of course, in Falkner Skan, is a little bit different. They use that to find out expressions for the Nusselt number, Sherwood number and mass flux, right because that is what you need, right for this class of solutions.

So, using the gas phase results and using the model that there is a spherically symmetric kind of a droplet evaporation because that was our starting point, right. And after that we included the convection effect, within the, within the liquid. So, for this convection effect, okay using the gas phase part of the analysis, this is what Sirignano and others actually showed.

That this is what is actually valid, got it, okay. Now, obviously we have done this we are, we know what is the heat and mass transfer. But there is a certain catch now, okay. People may be interested to know what is happening, within the gas, within the liquid phase itself, right. Why that is important, that is important in many things, okay.

Say for example, you have a liquid, okay which you may not be just interested in that m dot, okay, m dot is a global parameter. You may also want to know that how the heat and mass transfer is happening within this droplet, right. That means what is the temperature distribution, right? What is the temperature distribution, correct and what is the flow profiles look like, okay?

How does it vary during the droplet lifetime? That means a droplet, as we know; say it evaporates in a fashion like this, right. In this portion, is it diffusion or is it convection dominated, right? If the middle section is that convection only and in the later section, is it diffusion? So, these are kind of questions, right that I am posing over here, okay which you need to know, right.

You just cannot, you know, what is the information about m dot right the global information okay. That of course, does not tell you that how, for example the isothermal contours look like, the isotherms look like we did the droplet. That means if I take a droplet like this and let us take half of the droplet because it is axisymmetric in a way, right across that access, okay.

Now, how would the contour actually look like? Whether it will be like nice round like this? Signifying that there is a pure diffusion dominated thing? That means the isotherms are basically circles, concentric circles. Is it going to be like this or is it going to be something else, okay? We do not know or is it going to resemble the flow profile, right, okay.

So, that kind of a thing you can only analyze unless after you know that what is happening in the liquid phase, correct.

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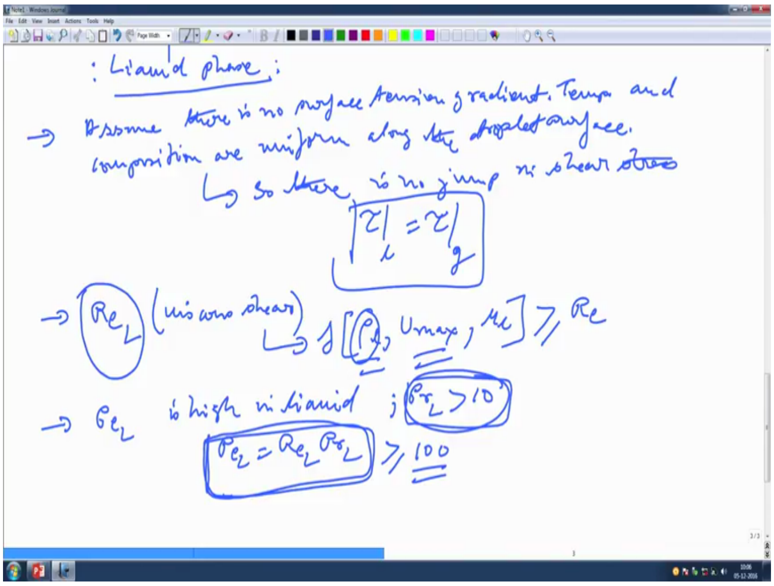

: Liquid phase :

→ Assume there is no surface tension gradient. Temp and composition are uniform along the droplet surface.

↳ So there is no jump in shear stress

$$\boxed{\tau|_l = \tau|_g}$$

→ $\boxed{Re_L}$ (viscous shear) ↳ $f[\rho_l, U_{max}, \mu_l] \geq Re$

→ Pe_L is high in liquid ; $\boxed{Pr_L > 10}$

$$\boxed{Pe_L = Re_L Pr_L} \geq 100$$

So, the liquid phase analysis, we will go, we will do in a certain way that first and foremost we will say that the liquid phase, we will just introduce a problem here. And then we will take it up when we do the full fledge droplet modelling, okay because that way it will be more clear. So, in the liquid phase what we have is that normally there would be a jump in the shear stress, right across the interface.

Why that thing happens? Because there is a gradient in surface tension, usually that was our original generalized form that we wrote when we actually devised the mathematical equations, right. So, here, of course, assume that there is no surface tension gradient. There is no surface tension gradient.

Now, that is a valid assumption, if there is no temperature or composition variation across the surface which is what we have assumed in the first place, okay. So, temperature and composition, composition are uniform, uniform along the droplet surface, is it a good assumption? We will see, got it.

So, there is no jump in shear stress, got it. There is no jumping shear stress across the interface, okay. That is what we need, okay. And so the shear stress is basically continuous or whatever sheared that you write, coming from the liquid phase will be same as a shear that you have from the gas will, okay across the interface.

That is going to be valid like regardless. Now, also another thing that we mentioned earlier and I want to reiterate. Now, I will continue to reiterate that the Reynolds number in the liquid phase that is created, it is mainly due to viscous shear, right. It is due to the friction between the droplet and the flow, right.

And as we know that this is a function of Rho L, your Umax, that is a maximum velocity of the, of the liquid phase and mu l which is basically the liquid phase viscosity. This should be much, much greater; sorry it should not be much, much greater always. Let us see. So, this will be like greater than equal to the Reynolds number of the gas phase.

This we already established because your Rho l is about thousand times higher though your U max is quite a bit low, okay. Here, U max is one order lower, but your Rho l is several times higher, right. So, that makes it that this Reynolds number that is actually introduced into the liquid phase is not a low quantity.

And the third thing that we said is the Peclet number is high. What happens when Peclet number is high? When Peclet number is high, Peclet number is high because Peclet number is what? the Peclet number is Reynolds number of the liquid phase into Prandtl number of the liquid phase, right. So, Prandtl number of any liquid is usually high, right.

It is usually greater than 10 in many of the cases, okay for what water is about 6, right. So that Prandtl number is usually a high enough quantity. So, the Peclet number is usually high. Sometimes it is greater than equal to 100, okay in many of the cases that we are going to consider over here.

Now, when Peclet number is that high that implies okay that convection is much more dominant, right than the diffusion essentially that is what the Peclet number significant is, right. So, the heat and mass transport is different from the vorticity or the momentum transport.

Or in other words, so that is what we are going to see in the later part of the course that momentum transport or vorticity transport happens very quickly, right, okay. So, that is what we are going to see in the next part of the author of the lecture.
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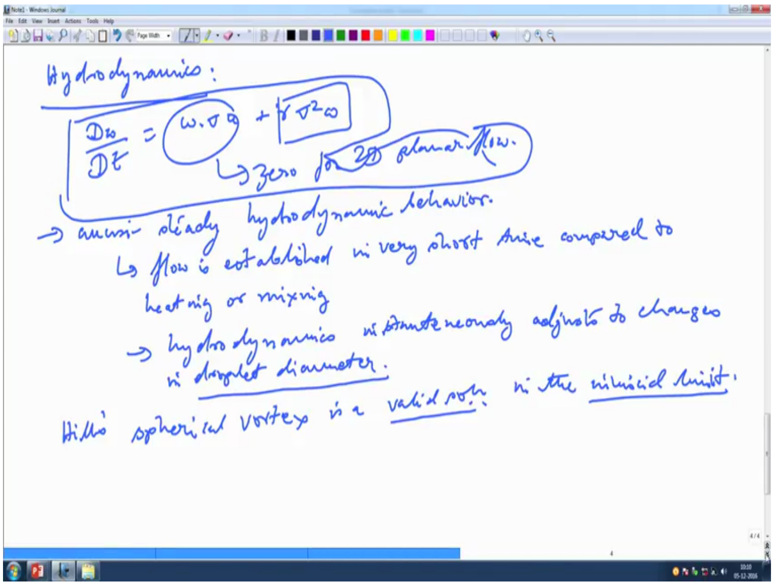

Hydrodynamics:

$$\boxed{\frac{D\omega}{Dt} = \boxed{\omega\cdot\nabla u} + \boxed{\nu\nabla^2\omega}}$$

↳ zero for 2D planar flow.

→ quasi-steady hydrodynamic behavior.

↳ flow is established in very short time compared to heating or mixing

→ hydrodynamics instantaneously adjusts to changes in droplet diameter.

Hill's spherical vortex is a valid soln. in the inviscid limit.

Now, in order to solve this liquid phase that usually the hydrodynamics is solved by writing the vorticity transport equation. That is the convective derivative. That is the diffusional term, right. This normally will be equal to 0 for 2D planar flow. That is because your vorticity is pointing in a plane, okay away. It is out of plane component, right, okay; whereas your U, okay U is basically the in-plane component, okay.

So, the dot product between the two is going to give you a zero quantity, okay. But the viscous term obviously will be retained. So, that is the strictly the vorticity transport, okay. Now, here what we say is that we assume a quasi steady hydrodynamic behaviour, okay. So, that means, this essentially means that the flow is established, established in very short time, very short time compared to heating or mixing, okay.

And the hydrodynamics and we will see why this happens a little later. And hydrodynamics instantaneously adjusts, okay adjust to changes in droplet diameter, got it. So, the flow is established in very short time compared to heating or mixing. That is once again a quasi-steady kind of thing. That means whatever happens the flow adjusts the fastest, right and the hydrodynamics instantly adjust to the change in droplet diameter.

That means if the diameter, droplet diameter is actually shrinking, right. So, the hydrodynamics if it had just instantaneously that means we can also take it to be quasi steady in terms of the droplet itself correct, okay. Now, usually for this of the equation that you have over here that you have over there, right, okay.

The Hill's Spherical Vortex which we mentioned very briefly earlier, the Hill's Spherical Vortex is a valid solution, okay in the inviscid limit. This is valid in the inviscid limit which is true that is because your liquid boundary layer is very thin, okay. It is restricted mainly towards the droplet surface, right. And we already showed if you recall your diagram.

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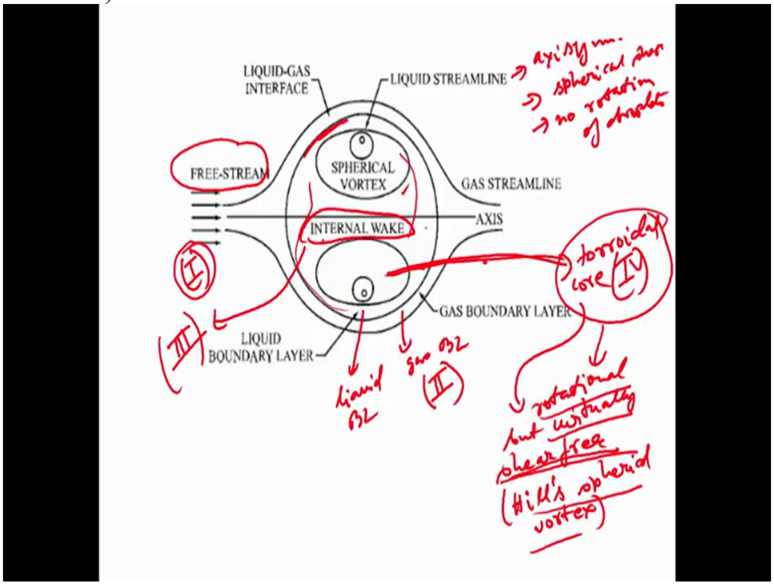

Let us pull that up if you look at, if you recall this particular slide, okay you will find that the liquid boundary layer is restricted to a very small region correct okay. Let us spend one or two minutes from this so, if you recall this particular expression and it is useful, okay. You will find that most of these things, okay.

This, this core that you see over here, okay this toroidal core, right that is basically rotational; but virtually shear free, okay. So, it is basically like an inviscid core, all right where the boundary layer is restricted only in these regions, got it, okay. So that is what the Hill Spherical Vortex is all about.

So, keeping that in mind, so the vortex is a valid solution in the inviscid limit okay. So, now a solution of this kind is usually written as:

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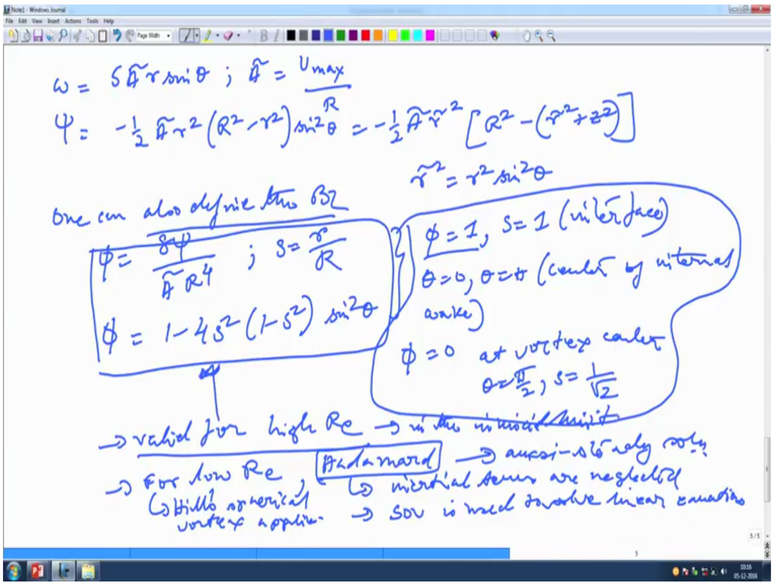

$\omega = 5\tilde{A} r \sin\theta$; $\tilde{A} = \frac{U_{max}}{R}$

$\psi = -\frac{1}{2}\tilde{A} r^2 (R^2 - r^2)\sin^2\theta = -\frac{1}{2}\tilde{A}\tilde{r}^2\left[R^2-(\tilde{r}^2+z^2)\right]$

$\tilde{r}^2 = r^2\sin^2\theta$

One can also define the BL

$\phi = \frac{8\psi}{\tilde{A}R^4}$; $s = \frac{r}{R}$

$\phi = 1-4s^2(1-s^2)\sin^2\theta$

$\phi = 1$, $s = 1$ (interface)

$\theta = 0$, $\theta = \pi$ (center of internal wake)

$\phi = 0$ at vortex center

$\theta = \frac{\pi}{2}$, $s = \frac{1}{\sqrt{2}}$

→ valid for high Re → in the inviscid limit

→ For low Re, Hadamard → quasi-steady soln

↳ Hill's spherical vortex applies

↳ inertial terms are neglected

→ SOV is used to solve linear equations

Let us go to the next page before we write it, okay. So, Omega is basically given as 5 A tilde r sine theta. We will come across what that exactly means here, A tilde essentially means U max by R, okay and you can also write the same thing, okay. So, that, that is what the Omega is, okay. And you can also define like a stream function which will be given as minus half A tilde r squared capital R square - r square.

Capital R square being the droplet radius because this is equal to minus half A prime square R squared - r tilde squared + z square. Now this r tilde square and basically r square sine square theta, okay got it. So, that is the stream function and the corresponding vorticity. One can also define, so define the boundary layer as Phi = 8 Psi A tilde r to the power of 4, s = r by R. So, this is just a coordinate transform.

And this is also written as 4 pi s square 1 minus s square sine square theta. That is also a way of writing it. So, this is subjected to the condition that Phi = 1 when S = 1. That is at the interface. S =1 is basically the interface, okay. And when theta = 0 and theta = PI that means the center of the internal wake, okay. That is also valid, the same thing.

And Phi = 0 at vortex center which is basically theta = PI by 2 = 1 over root over 2, okay. Now you have to, do not have to understand the math altogether. What it essentially means that we have used the Hill's Spherical Vortex and we will do a little bit of more discussion on this. Hill's Spherical Vortex is a reasonable assumption, okay in the inviscid limit.

It satisfies also the interface conditions and we can also define the boundary layer using this expression over here, right. And using a combination of these two, we can define that what is going on inside the droplet. This solution is valid for high Reynolds number remaining behaviour, okay; the viscous terms are obviously neglected.

So, that the inviscid solution for the internal liquid flow, essentially, so, this is valid we can say for high Re, essentially. For low Re this is, there is another solution that is given by Hadamard, okay which is also a quasi steady solution given by Hadamard, okay. The, there also the inertial terms in the liquid and the gas is basically neglected because it is a low Reynolds number solution.

Snd the linear equations are solved by separation of variable and the Hills spherical vortex even applies there got it. So, in the Hadamard solution what did they assume is that the inertial terms neglected; terms are neglected, okay. Then, separation of variables which is so b, if sob is used, used to solve the equations.

The linear equations basically it is linearized because convection term is the only non, non-medium term to solve the linear equations, okay. And Hill spherical vortex is still valid. Hill Spherical vortex applies, okay though the external flow solution is very different, okay in those cases, okay.

So, for high Reynolds number case, you have this solution. For low Reynolds number case. What we have not covered it Hadamard gave a solution which is essentially uses the same Hill Spherical Vortex but neglecting the inertial terms, okay. So, that is what the main framework of this incident is that we are going to solve by using the Hill Spherical Vortex solution, right.

And that is normally valid for high Reynolds number in the inviscid limit, got it, okay. So, now, let us in the next lecture what we are going to do is that we are going to look at the full fledged droplet vaporization model where things will become a little bit more clear. And we will use some of these Hills spherical vortex and other notional concepts and find out what is going to happen in those cases, okay, okay, so in the next class.