Heat Transfer And Combustion in Multiphase Systems
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Lecture 24
Droplet vaporization dynamics-II

So, in the last lecture, we, we showed that the Liquid phase Reynolds number is, it can almost be as large as a Gas phase Reynolds number, right. So, the way of stating this is that, what will be the nature of the liquid phase boundary layer. Remember that region 3, the region 3 that we mentioned within the droplet.

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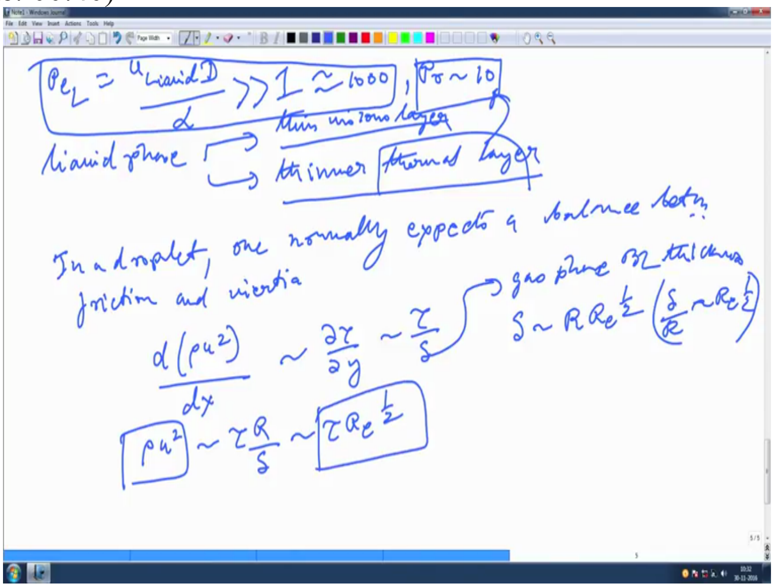

Another number that is a practical relevance is the piclet line number, piclet number is what u liquid into d by alpha. This d can be the droplet diameter can be the radius of the droplet also this Peclet number is normally much, much greater than 1, okay. In fact, this is of the order of thousand actually, the liquid Peclet number, okay.

And the Prandtl number of the normal liquids are of the order 10. Usually they are not allowed at 1. So, in the liquid phase, what you expect? In a liquid phase, you expect to see a thin viscous layer, right and you can get thinner thermal layer. So, thin viscous layer and thinner thermal layer.

Now, the Peclet number as I said is of the order of 1000, okay. So, the liquid flow velocity plays a major role and the Prandtl number is of the order of 10, okay. So, thinner thermal layer comes because of this Prandtl number, okay and the thin viscous layer comes because the Peclet number value is high, okay.

So that is the liquid phase description, right. So, it will also have a boundary layer that is the bottom line. It will also have a boundary layer. In fact, in most cases, the thermal boundary layer is much, much thinner, okay. Now, let us now return to the two Corollary or Postulate that we mentioned. One was that there is no torque, right; the other is that the friction force is of the order of 1 over the Reynolds number, right.

So, these were the two postulates, okay. So, in a droplet normally you expect a balance to happen between inertial force and friction forces, right. In a droplet, one normally expect, expects a balance between friction and inertia, right. So, from an order of magnitude perspective, this is your inertia. This is your friction force, which can be written like this, right. Delta is nothing but the gas phase boundary layer thickness, right.

And if you have done your fluid mechanics course, you will find that Delta is always scales as R and Reynolds number to the power of half, right. This is from the boundary layer scaling. If you recall, this comes from delta by R scales as Reynolds number to the power of half. If you have done your flat plate boundary layer, you will see that that is the expression that we normally use, right okay.

So, using that what you will see here is Rho u square scales as Tau R by delta. That is Tau Reynolds number to the power of half, right okay. That is what you are going to get. So, basically it is a combination of inertia and the corresponding, the balance between inertia and friction, right. Now, whenever as I said that this asymmetry normally happens due to some fluctuation, right.

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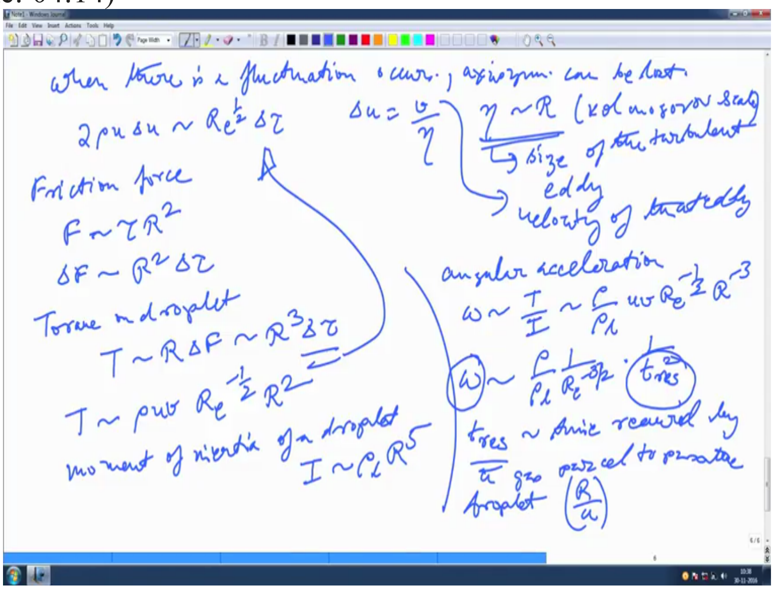

So, let us go to the next page and write. When there is a fluctuation of some kind, okay this axis symmetry can be lost. And the difference in velocity and stress can be found out. The axis symmetry is usually lost, can be lost, okay. So, what you find is that if you just differentiate the previous form that we had, you will find Rho u into delta u scales as Reynolds number to the power of half into delta Tau, right, okay.

So, this is just a differential right. Now, delta u, what is delta u? Delta u can be written as v by Nita and we will see what those v and Nita are. Nita is basically the same as the droplet diameter. So, this essentially means that the fluctuation, why should the fluctuation in velocity happen? This can happen because of turbulent Eddy's, right of different sizes.

Now, the turbulent Eddy of the pertinent size in this particular application of the size of the droplet diameter, right because that is what the droplet can see, right. So, it is a scale thing. It is a scale issue. So, this is basically nothing but the size of the Eddy size of the turbulent Eddy, okay. And v is the corresponding velocity of that ad, that Eddy, so to say, okay. So, that is the fluctuating Eddy, okay.

This is normally this, this assumes that this Eddy lies in the Kolmogorov scale. We are not going to go into deep into Kolmogorov scale as of now, okay. So, this is the scale, if this is the smallest scale in the turbulence spectrum, okay. So, in the Kolmogorov scale, okay this eddy size is the same as a droplet size and the velocity of that Eddy is given by that v, okay. And that is the expression that we have.

Now, the friction force, what can we write about friction force? Friction force scales as Tau into R square, right, is not that so? It is a torque into the corresponding R square. So, the change in the friction force is basically given by R square into delta Tau. Once again it is a simple differential, right. So, the torque on the droplet, what will be the torque value on the droplet?

The torque value on the droplet is basically R into this delta F, right the change in friction. That is basically given as R cube into delta Tau just by substituting the expression for friction, right. Now, already this delta Tau expression is linked here, as we know, right. So, this Tau is therefore given as Rho u square or Rho uv, whatever you call it. Reynolds number to the power of half into R square, okay, got it, okay.

So, now the moment of inertia is the torque, right, moment of inertia of a droplet. Droplet is given as I is Rho l into R to the power of 5, okay. So, the angular acceleration that happens is basically given as T by I which is also given as Rho by Rho l. This you can write it if you want as u,v that will be the correct way of writing it, okay.

So, this is Rho by Rho l into u into Re, Reynolds number to the power of minus half R, got it. So, that is the angular acceleration, okay. So, the angular acceleration can also be therefore written as Rho by Rho l divided by Reynolds number to the power of -3 by 2 into 1 by some T residence times square.

Now this T residence time is nothing but the time required by a gas parcel to pass the droplet, okay and this residence time as you can rightly guess it can be given as R by u, R being the relevant non-dimensional, R being the relevant diameter, right. So, with u that is the traversing time that thing actually takes, okay.

So, the Tau residence time is this. So, the angular acceleration that is created is basically equal to 1 over the Tau residence time, okay.

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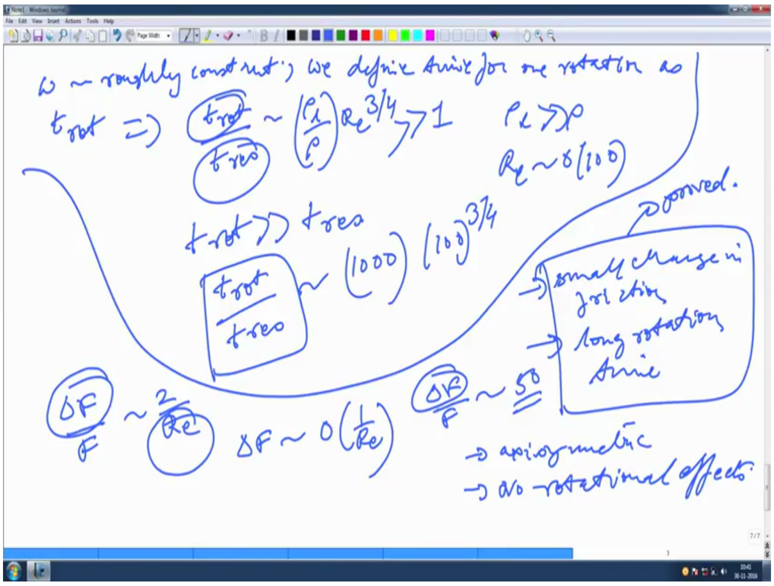

Now, this Omega, the angular acceleration is roughly constant, right. So, we define time for one rotation as t rot, okay. This implies t rot by t res scales as Rho l by Rho Reynolds number to the power of three fourth, right. Normally this will be greater than 1 why because Rho l is much, much greater than Rho, right. And Reynolds number we have said just now, that, it is of the order of 100, right for this kind of a situation.

So, naturally the rotation time and the residence time, this is much, much greater than 1, right. So, this would imply that the t rotation is much, much greater than the t residence for most of the cases because this is of the order of thousands. We can estimate this in fact at t rot by t res is almost equal 1000 and if the Reynolds number is 100. So, it is of the hundred to the power of three fourth, right.

So, it will be like several orders higher, okay. We would expect this to be several order higher than this right, okay so, at least 3 to 4 orders higher, right. So the t rotation time is much, much longer than the t residence time which justifies our assumption that we say it later. If the droplet rotates, it rotates so slowly, right that many gas parcels actually passes over it, okay.

So, there is no asymmetry as such, okay because this is a very slow process, okay. On the other hand, we also defined that let us look at the nature of the friction force. So, the friction force delta F by F basically scales as 2 by Re, right. So, this also shows that the delta F is basically of the order of 1 over Re. So, if Re is very large, the change in friction force is very small. Percentage change is very small, because Re is of the order of 100, right.

So, delta F by F should be like on the order of 50. That is the change in friction force is roughly 50 times lower than the value of friction. So, there is what we can say, there is a small change in friction, right and long rotation time. So, you have proved through our scaling argument that that is true. So, the rotation or asymmetric circulation is not significant for droplets with large Reynolds number values, okay.

So, whatever we, we portrayed in that, PPT, where we showed that what are the regimes within the droplet, that is still valid. We can take it to be axis symmetric nice and axis symmetric, okay. So, that would be the first thing, okay and there is no rotational effect. So, we can safely do the problem as if the flow is axisymmetric and there are no rotational effects, got it, okay.

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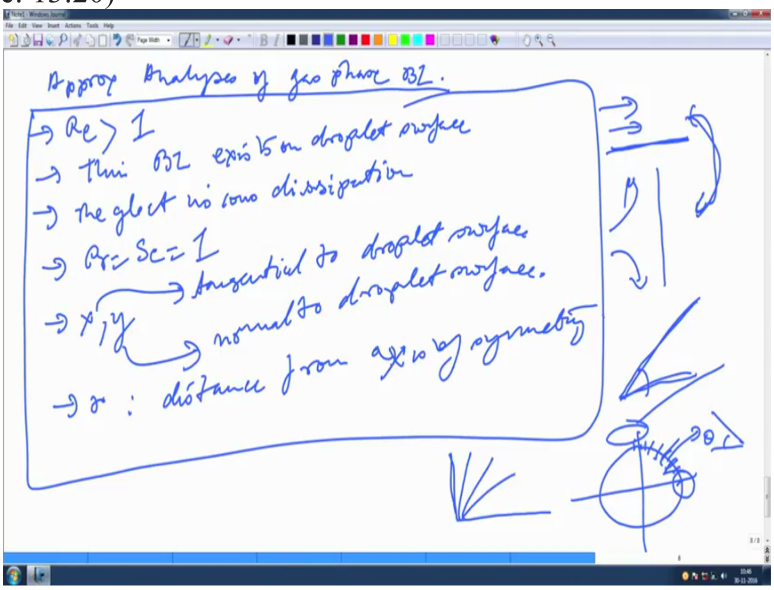

So, now that we have done it now let us look at the first level of analysis that means we would now start to look at approximate analysis of the gas phase boundary layer, okay, Gas phase boundary layer. So, we are first going to analyze the gas phase boundary layer because that is step number one. We have to also come and do the liquid phase and analysis because this is a full convective model that we are looking at, right.

So, the first assumption is basically that your Reynolds number of the droplet is greater than one which is obvious because that is why that is when the convection becomes an important part of the whole thing. And we are assuming that thin boundary layer; thin boundary layer exists on droplet surface. This is already established but we are putting this as an assumption.

We are going to neglect viscous dissipation and we are going to assume Prandtl number equal to Schmidt number is equal to 1, okay, xy in our expression means, x means a tangent tangential to the droplet surface to droplet surface, okay and y means normal to droplet surface, got it, okay. And r designates the distance from axis of symmetry, okay so $r = R$ basically means the droplet surface, okay.

So, these are that for the approximate analysis of the gas phase boundary layer. These are the few assumptions not assumptions per se, okay. Or they are very valid, okay. So, based on this, we can start working on the problem. And here what we will do is that we will take abundant use of the existing literature that is available on gas phase boundary layers say, a flat plate.

Say, for example, a droplet can be now analyzed in like a flat plate boundary layer or like a stagnation flow. Stagnation flow is like this, as you know, right. It is also called the Hyman's flow. And there is the flat plate boundary layer. Combining these two and also if you know the Falkner Skan type of solutions that means.

Any angle if you have a surface inclined at any particular angle, there is a class of solutions that are available which are called the Falkner Skan, okay. So, that looks like a balsier solution but it is applicable for any level of inclination. Last year solution is only available for a flat plate, right with zero pressure gradient right.

In the case of Hyman's flow it is different and this Falkner's Skan basically includes these two, as two special cases, okay. So, Falkner Skan is applicable for any wedge, right whatever the magnitude of that wedge is right. So, when the wedge becomes 90 degree it becomes basically what we call a Hyman's or a stagnation flow.

When it becomes 0 it becomes like a flat plate boundary layer, right. So, our droplet basically what we are trying to say over here it can be actually resolved, you know in pieces and locally it can be made to approach a situation like this. One way it can be like a flat plate; one way it can be like a stagnation point and in between the curvature changes, right.

So, it is almost like these wedges if you take this section out you will find that is like a wedge, right. Like a wedge of a certain angle, got it. So, we can use locally, Falkner Skan class of solutions to solve the boundary layer, right because it is a droplet with, with pieces, right, in pieces, in locally, right.

It behaves like a Falkner scan class of solutions, okay. And there are extremes like this and this where it behaves like a normal flat plate boundary layer or like a stagnation flow solution, okay.

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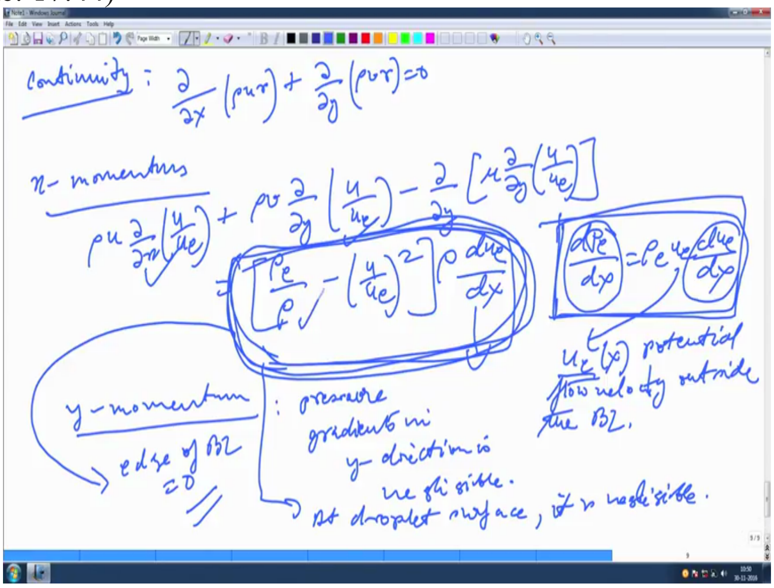

So, let us write the equations which will be useful. So, continuity, Good part is that we do not have to solve all of them but the math we are not going to work out the math like that. But we are just going to pose the problem and we have already explained how the problem can be solved. So, it requires that Falkner Skan type of solution.

So, that is the continuity equation X momentum equation. And you will find a lot of similarity with your flat plate boundary layer. We will see, say, what u e is, just hold on for a second, till we establish this equation. Where do you recall that? This is the, this comes from the Euler's equation in outside the boundary layer, right.

So, ue is basically nothing but the potential flow, flow velocity outside the boundary layer. This we already knew, right from your, if you recall your classes where, we actually did this Falkner’s scan type of solutions, or whatever courses that you may have taken, this is the solution.

This is basically comes from the Euler's equation which basically relates the pressure gradient with the velocity, right. There is Euler's equation. If you look at the X momentum equation carefully, you will find that there are variations of u, with respect to x and y. There is an additional term that we are carrying over here which solely comes; because there is a variation of the free stream velocity or basically the variation in the velocity ue.

That is the velocity outside the boundary layer with respect to x. There is a spatial variation, right. Once it is a spatial variation, this term has to be included because its origination this is also true, in the case of your Falkner Skan and other things, okay. So, that is very important, okay and what about the y momentum equation?

Now, the y momentum equation, the pressure gradients in the Y Direction is, negligible. Gradients in y direction is negligible, right. So, this particular term, on the other hand, this term that we have mentioned, okay this has got a peculiar feature. For example, at the edge of the boundary layer this term becomes equal to 0 right because of the edge of the boundary layer it becomes equal to 0. It is also negligible at the droplet surface.

At droplet surface it is negligible, okay. Why it is negligible at the droplet surface because the transverse variation of the dynamic pressure is small at the droplet surface. So, this term also have got a particular feature that feature being that it is negligible at the droplet surface and at the edge of the boundary layer it becomes equal to zero, okay.

That is because these two terms becomes equal to zero to each other. So, they become one right. So, they become equal to zero, okay. This term clearly drops out, okay.
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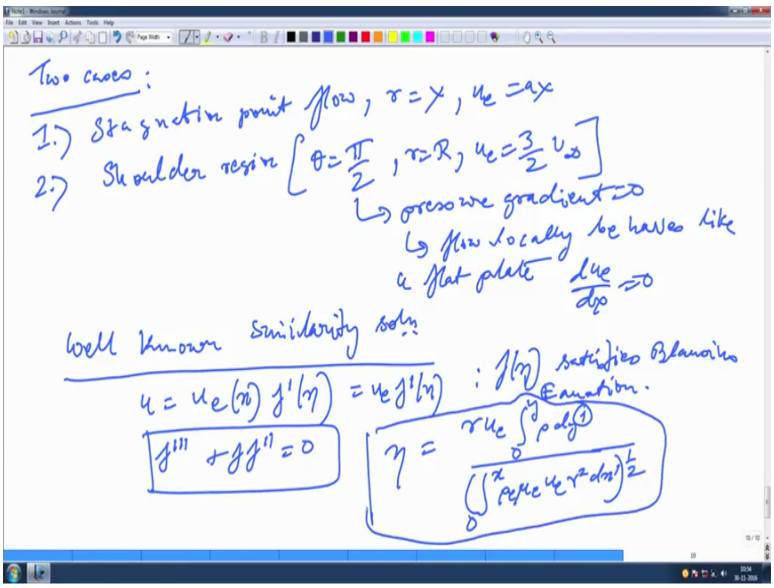

So, based on that so that is the part, so there are two special cases, of course, as you can see that it is tough to solve it, okay, holistically. So, we will take 2 cases needs to be taken. First one is basically the stagnation point flow when r = x and ue = ax you already know the stagnation flow. If not just do a very quick recap of Panton or S M White's fluid mechanics book for you can actually read the part on the viscous flow.

And there will be flows like stagnation point flow. Stagnation point flow is very common. So, one is the stagnation point flow and you can guess which part is a stagnation point flow. The other one is the shoulder region of the droplet, right. There theta if you consider theta from the droplet equator, that is at 90 degrees, right.

So, theta at 90 degrees when r =R ue is basically 3 by 2 u infinity, okay. So, that is the shoulder region where the pressure gradient in the shoulder region, the pressure gradient is basically equal to 0, okay. That is exactly like a flat plate. So, the flow locally behaves like a flat plate. And in the flat plate as you know udu dx is basically equal to 0, right. That is a flat plate boundary layer, right.

In because, there is no pressure gradient there is zero pressure gradient inside a flat plate boundary layer, right. This you know from your Blasier solution. In fact that is what the Blasier solution is all about understood. So, whatever it is, the well-known similarity solution and this we will just use the Blasier's form and you will see that how powerful this thing hard.

The idea is how we can actually take a solution from a very different perspective, you know, and apply it in a very different perspective also and still get results which are physically very meaningful, insightful results. And that is the beauty of doing fluid mechanics, okay. So, the similarity solution becomes ue x and this is f prime nita, we, where you know that it is basically ue into f prime nita.

Where you know that f nita satisfies Blasier's equation, okay. It satisfies the Blasier's equation and so when you write it this is basically f triple prime plus f into f double prime is equal to 0. Do you recall the form of this equation, okay? That is the Blasier's flat plate solution, right, okay. And nita obviously, we have to define what is nita?

That is ru e do not worry about that sign that I have mentioned here. That is basically the dummy variable, okay. So, that is how Nita is actually defined, right. If you do not even get, you just have to recap your flat plate. Just go and recap that, okay. If you are not getting the feel of this similarity solution, okay.

But this is basically a similarity solution and you know what similarity transformations are. We are not going to spend time on that. If you have, if you have learnt your fluid mechanics you already know that. What similarity transformation exactly means, okay.

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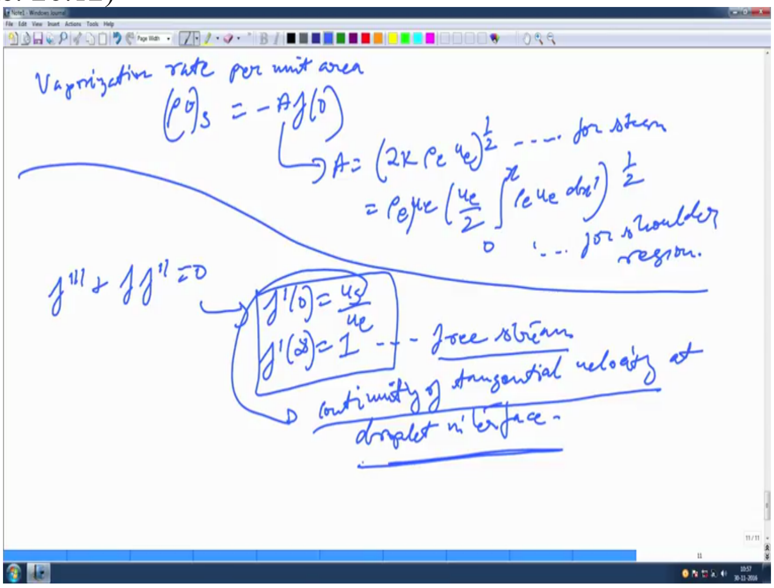

So, the vaporization rate, rate per unit area, area is given as Rho vs as - A f 0, where this A is 2k Rho e ue to the power of half for stagnation flow. And this is equal to Rho e mu e sorry ue by 2 okay for the shoulder region, got it, okay. That is the vaporization rate. Now, the boundary conditions of course if you look at the nature of this equation, f double prime f f double prime =0.

How many boundary conditions it needs? It requires three, right because the triple derivative that we are talking about. So, the boundary condition, two boundary conditions are readily available, f prime 0 = us by ue. That is at the surface f prime infinity is basically equal to 1. This is basically the free stream. These are the, are the applied at the two boundary layer interfaces, right.

And this basically comes from the continuity of tangential velocity at the droplet surface; of tangential velocity at droplet interface, got it, okay. So, these two boundary conditions as you can readily see one is applicable for the free stream when the flow where basically u approaches the free stream, the other one, when it is the tangential velocity were at the droplet interface.

In most of the Blasier solution, you will find that that will be equal to the 0 because there will be a no slip, right. But here that no slip is not valid, it is no slip, but it is not equal to 0, okay because there has to be a continuity of that tangenial velocity across the droplet interface. And that is very important, okay.

Now, we need basically a third boundary condition, right. Third boundary condition is needed. So, in order to get that, let us write the other, other boundary conditions for energy.
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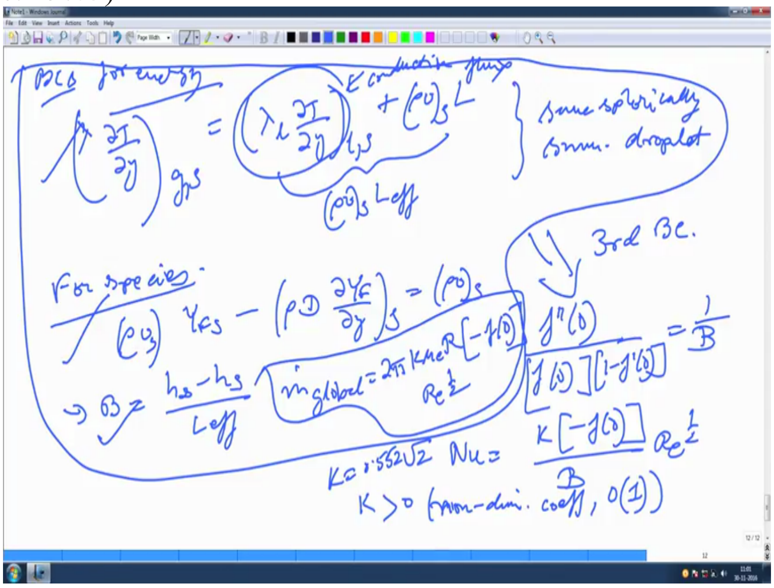

Say, for example, I have not written the energy equation because we have already written it earlier for the gas phase, okay. So, once again, if you recall what the energy equation was, it is g at the gas interface, gas side of the interface. Then this is basically on the liquid side of the interface, right plus there was this Rho vs which was basically the mass flux into the latent heat. Do you recall?

That was the latent heat, right and this sometimes we wrote it as Rho vs into L effective is no't that so, right. And then we had a lot of arguments regarding the nature of this, this is basically that conduction heat flux, right, is not that so. It is the q dot that we mentioned earlier right, okay. So, this is basically the same as radically symmetric droplet. This is the same as radically symmetric droplet, right.

For species, similar thing once again, we have written it already for the species earlier. We are not writing it once again. Rho vs YFs - Rho D YF by dy, right. Sorry, this is not okay and then we have the definition of B is equal to h infinity - hs by l effective right okay. So, using all these things and we are not going to show how because it is very complicated. And you also do not have to do it, okay.

Using all these cases we can now device the third boundary condition that is needed and the third boundary condition is basically a little bit complicated to look at. Do not bother about the math because that is not important over here, okay. It will take a lot of time for you if we have to work out this, okay.

That is given as 1 over B right, so that is the third boundary condition, okay which combines the energy, species and everything. And uses the definition of B and combines weekend we can have this particular form, right. And let us define the Nusselt number also so the Nusselt number is given as this by B Reynolds number to the power of half. Now, this k is not the thermal conductivity, k is basically a greater than 0, non-dimensional coefficient.

Non-dimensional coefficient, the order is unity of the order one, okay and it is determined when the heat flux is averaged across the droplet surface. This is averaged across the whole droplet surface, right. And this k value normally nominally is given as something like point 5 5 2 into root over 2 okay about 0.7 81 or something like that, it comes, okay.
Similarly, one can write m dot global that is the global vaporization rate, 2 Phi k mu e r f 0 into Reynolds number to the power of half, got it, okay. So, that is the expression.
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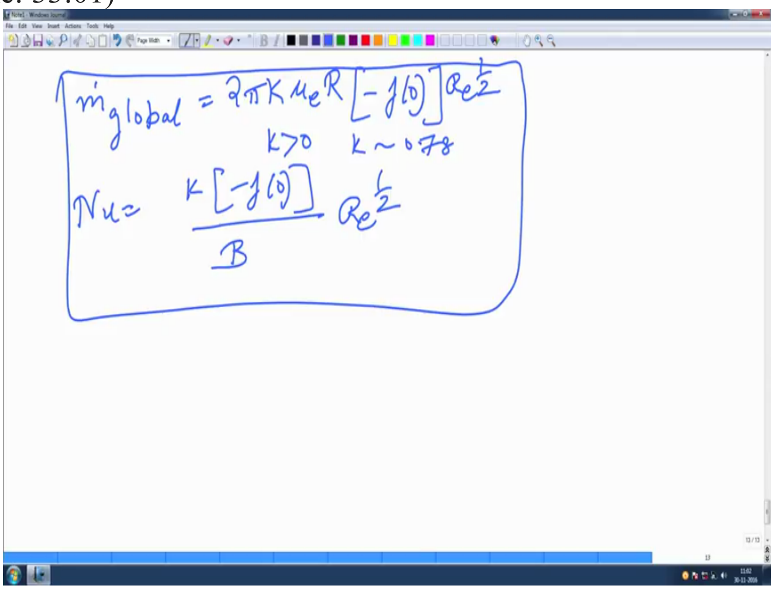

So, let me write it in a fresh way so that you do not have to global and basically 2 phi k. This is not the same k as thermal conductivity, Reynolds number to the power of half, okay. When this k is greater than zero, normally k is about .78 something like that, okay and the Nusselt number so that you can use it if given has - f0 by B to Reynolds number to the power of half.

These are the two main actors that even industry people are going to use this again and again, right, okay. So, this is how we have solved it, just doing a bit of a recap. We divided the droplet into several pieces, right and we consider the stagnation flow and the flat plate boundary layer. So, you use the similarity transformation to basically link the velocity and other things.

And there we establish three boundary conditions, the last boundary conditions comes from the energy and the species combining it. And then, we have not shown the math that how exactly the steps are. We have devised that there are two quantities of importance one is the nusselt number; which is for the heat transfer coefficient and the other one is basically the global evaporation rate.

Both are solved in terms of this similarity variable of the Blasier solution, right, okay. So, with this, we have shown that how one of the model would actually work, okay. But there is another type of model which was done by Sirignano. No, this does not involve the Falkner Skan by the way.

So, what Sirignano and Abramson actually did was that they actually incorporated the Falkner Skan class of solutions and devise a more rigorous approach to do this, okay, do this was not rigorous. This is like ad hoc it just took like the shoulder region and that the two extremists is essentially, right; shoulder region and the stagnation region, okay.

And so, you can assume that all the other solutions will lie somewhere in between that was the whole argument, right. So, but, Sirignano and Abramson did basically the Falkner Skan class of solutions using non zero pressure gradient, okay. And in the next lecture or in the next class, we will find out what that exactly is okay, thanks.