Heat Transfer And Combustion in Multiphase Systems
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Lecture 23
Droplet vaporization dynamics-I

Welcome to this lecture! In the last lecture what we covered was the spherically symmetric evaporation model, right. And before that, we did that, what led to the d square law. And we developed some of the relationships that when the spherically symmetric mortal is useful and things like that.

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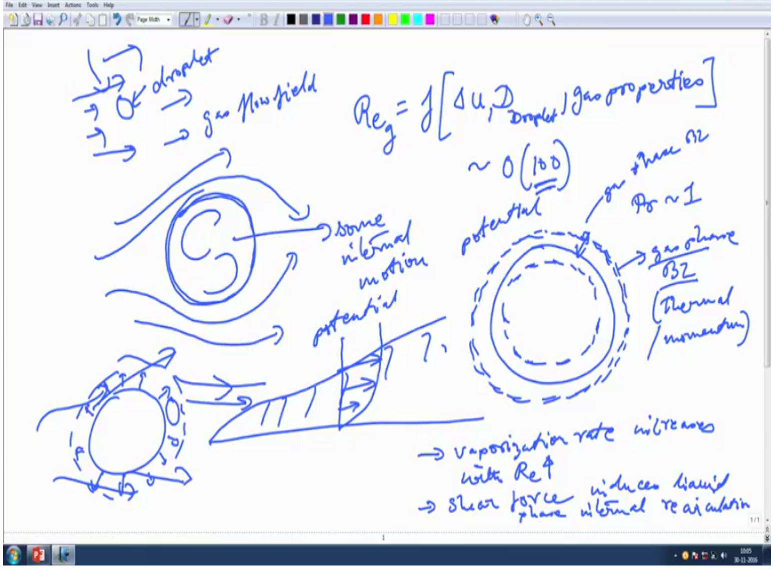

But as we know that when you, if you look at the screen, that in most of the cases, the droplets are injected in a very high speed kind of a gas flow field, right. So, this is gas flow field and this is the droplet. So, if you look at a zoomed in view of the droplet that means we are isolating the droplet, right.

What kind of a flow you would expect? There will be some flow going around it like this, right. It will be almost like flow over spherical objects like what you have done in your fluid mechanics. So, because of this flow field that is created, okay. It is apparent that there will be some motion that will be created inside the droplet.

Some kind of motion will be created, right. Some motion, some internal motion has to be created because this is not a solid sphere anymore, right. So, it is a droplet. So, there has to be some motion and that motion happens because there is a relative velocity between the gas and the liquid, right. The droplet's own velocity and the velocity of the gas flow field around it, is different.

So, as a result there is sheared that is created along the surface, right. As a result of that, there will be a flow that will be generated inside, right. The magnitude of the flow will be low but as we will find out in due course, the liquid phase Reynolds number is not small, okay. So, we will see to that that what that is exactly means. So, in most of these applications, therefore, this the droplet, therefore, is moving with some relative velocity.

So, if we define something like a Reynolds number, okay, for the gas phase, with respect to the gas phase, we will call it gas phase. But here I am writing it as Re g, just for the sake of clarity, it has to be a function of the relative velocity, right. It has to be a function of the droplet diameter, right, okay. And it has to be a function of the gas properties.

Gas properties means density, viscosity, etc., right, okay. In most of the cases, we will find that this Reynolds number is of the order of 100, okay, 100. So, it is laminar still, okay. But it is not 1, okay or below. So, whenever you have a Reynolds number in that particular region, what do you expect to see? This is the droplet. What do you expect to see, when you actually a have a Reynolds number like that?

You will have some kind of a boundary layer that will develop around the droplet, right. That is gas phase boundary layer, is not that so? So, the gas phase boundary layer will develop, right. So, one will be the momentum boundary layer; one will be the thermal boundary layer depending on what is the gas phase Prandtl number is, right.

Now, most likely, for example, if the gas is air the gas phase Prandtl number will be of the order 1, right. So, therefore, these two boundary layers will perhaps coincide with each other, right. So, you can have a gas phase boundary layer thermal or momentum or viscous right, okay. So, this Reynolds number value therefore creates that kind of a boundary layer effect, right. So, beyond that it is all potential, right.

That is what normally we are accustomed to seeing, right. That it is potential flow outside only in the boundary layer you have all these effects, right. If you recall your fluid dynamics course, there you already have done. Say, for example, if there is a flat plate boundary layer, right. What happens? This is how the boundary layer evolves, right. So, here it is potential, right. And here the all the effects are confined to this particular region, right.

So, here you have the velocity varying like this and things like that, correct. If you recall your fluid dynamics course, right so, here also something very similar thing actually happens.hmm. So, the boundary layer there is a gas phase boundary layer that exists, okay. And it is quite natural also to see two other things. One is the liquid vaporization rate, right.

The vaporization rate, rate increases with Re, okay. Now, if you have a clear idea why this happens, let us try to step back and explain that, that why should the vaporization rate increase with Reynolds number, right? With Reynolds number going up, if you take a cup of tea, how do you actually cool it, okay? Sometimes we will blow air, right.

We try to blow air over the surface of the tea, right to cool it, okay. That is big but if you keep a cup of tea open to this ambient, it will cool down, right. It cool down after some time but in order to cool it faster, what do we do? We blow air, right and what does that actually do it basically blows away the accumulated vapour that is around the droplet surface.

And that is how it actually promotes. So, it is basically a conviction effect. It is not a diffusion effect because if you recall earlier what we did was that, how does the diffusion transfer happens? This, vapour gets from the droplet surface and they form a layer, right of high concentration vapour around the droplet.

But and after that there is a diffusion gradient which takes away this vapour, okay slowly with the do, the diffusion flux, to the far field, right. And that is how it relieves the vapour concentration here, so that more liquid can escape, right. Now, if you have a convective flow that is happening, it will blow away this vapour, okay much more conveniently, right.

As a result of that your vaporisation rate should actually increase. So, that is one of the key important points. So, it increases. The other thing is that the Shear force, this we may already mentioned, Shear force induces liquid phase, phase internal recirculation, got it, okay. So, these are the two prime things.

And there exists a gas phase boundary layer; does there exist, a liquid phase boundary layer also? You bet there also exists, a boundary layer in a liquid phase, right. The structure of that boundary layer, these are very hard things to compute. But we will see what we can do about that, right, okay? So, these are the two important things that we have mentioned over here. Then, let us look at the next one.

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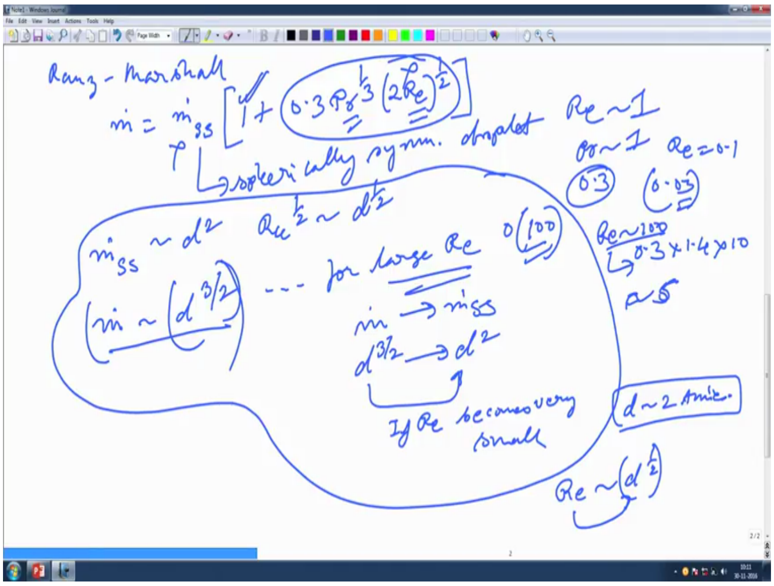

That, there have been many people who have given that in a steady state or in a diffusive environment, there is a certain evaporation rate, right. Now, can we actually prescribe some evaporation rate for this convective environment, right. So, Ranz Marshall and you may have heard the name is called Ranz Marshall Correlation, okay.

So Ranz Marshall gave a correlation in which m dot is given by m dot ss, ss is basically nothing but the steady state 1 plus 0 point 3 Prandtl number to the power of one third to Reynolds number to the power of half. This is for this, corresponds to a spherically symmetric droplet, right. So, you know the spherically symmetric droplet. You know how are the analysis is? If you know, what is the Reynolds number value and what is the Prandtl number, right.

These are all gas phase, remember. We can actually predict what will be the mass flux, the revised mass flux, okay? So, as a rule of thumb, can you see certain interesting features coming out of this particular correlation? The first interesting feature is that we already prescribed through the d square law and things like that and related things, okay.

That m dot ss was already kind of a functional d square, right, okay it was a function of d square, right. Now, Reynolds number to the power of half, if we write it, that is a function of what that is d to the power of half, right scales. So, your m dot basically scales as d to the power of 3 by 2, right because it is a multiplicand of this with that, isn't that so? Right.

So, if you multiply the two, so, this will be a d to the power of 3 by 2 for large Re. However, if the Reynolds number value is very small, right this term becomes negligible, okay. Say, if the Reynolds number is of the order one, okay and Prandtl number is already of the order one then, what will this term be?

This will be of the order of .3 correct, okay. So, if the Reynolds number is very less than 1 that means less than 1 means we are slowly going to the pure diffusion case, right, that if the Reynolds number value is say .1, right. So, this factor now will become .03, right. So, it is only three percent compared to this, right.

So, there is only a three percent increment. Similarly, the Reynolds number decreases faster this m dot actually approaches m dot ss, right. So, in other words the if we plot the evaporation flux, okay it goes from d to the power of 3 by 2 to something like d to the power of 2, right, okay. This is, this happens if your, Re become very small. Got the idea, got the clear-cut idea of what this could be, right.

So, this is for large Reynolds number it is d to the power of 3 by 2 large that means Reynolds number of the order of 100 because when it is of the order of 100 this is 10. So, .3 into 1.4, I think right into 10. So, it will be something like 3 into 1.4, right. 3 into 1.4 around 5scales around 5, right. So, it is so this is 1 this is 5, okay.

So, you just we do get a lot of increase right in the m dot. So, m dot increases by almost five times compared to the steady state situation, right when Reynolds number is of the order of 100, got it, okay. Clear on this part. So, once now that we have made this clear, okay in many practical situations, the Reynolds number remains high for most of the droplet lifetime.

Can you guess why? There is another trick to the whole thing, okay, right. So, Reynolds number, as we know, is a function of the droplet diameter raised to the power of half, right. So, if the droplet mass reduces by 8 times, say for example, okay, 8 time the droplet mass-produces, right.

So, the actual diameter reduces by only 2 times approximately, right. So, the Reynolds number change that you are going to have is going to be miniscule. Even if your droplet shrinks a lot, right because one is volume, okay; the other one is basically your Reynolds number is basically square root of this. So, it does not change by much.

So, the Reynolds number if it is of the order of 100, it remains around that order throughout the droplet lifetime, okay, for almost part of the droplet like that, got it okay.

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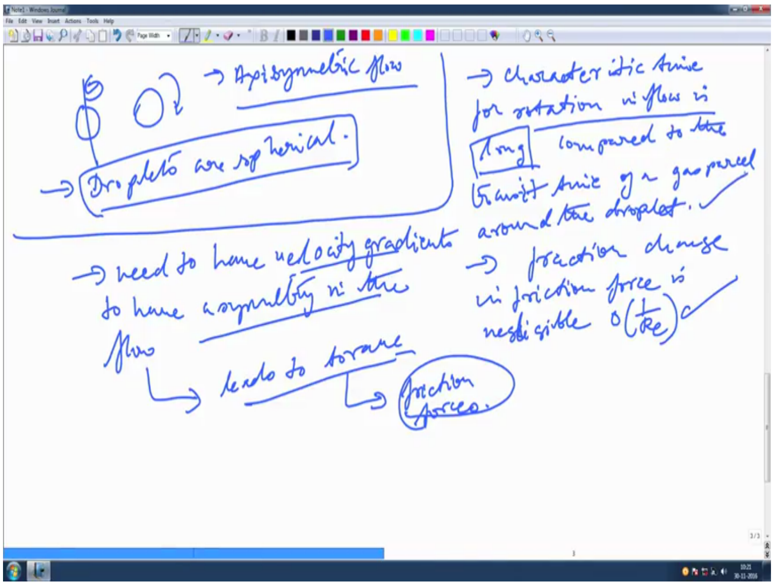

→ Axisymmetric flow

→ Droplets are spherical.

→ need to have velocity gradients to have asymmetry in the flow

↳ leads to torque ↳ friction forces.

→ characteristic time for rotation in flow is long compared to the transit time of a gas parcel around the droplet.

→ fraction change in friction force is negligible $O\left(\frac{1}{Re}\right)$

Now, there are several other things to this whole tale, okay. The Reynolds, so, the droplet can also when the droplet is injected into a flow field like this, right. So, the droplet can actually rotate. It is free to rotate, right; it is free to roll, right, okay. And this is rotation about that axis is free to roll, okay and free to spin, okay; whatever you can call it, okay. All these things the droplets are perfectly capable of actually doing, right.

But most of the analysis is you will find that people assume that there is an Axisymmetric flow, right. How feasible is that? We will look at it, okay in a little bit that whether it is feasible or not. And another leap of imagination that people normally take is that the droplets are spherical. Now, this is true in some cases, not always right because droplet deformation and atomization can happen, right.

And it is a very strong function of the flow field as well, right. So, it is not like that, the droplets will always remain spherical but that is the rule of the Assumption of the droplet.

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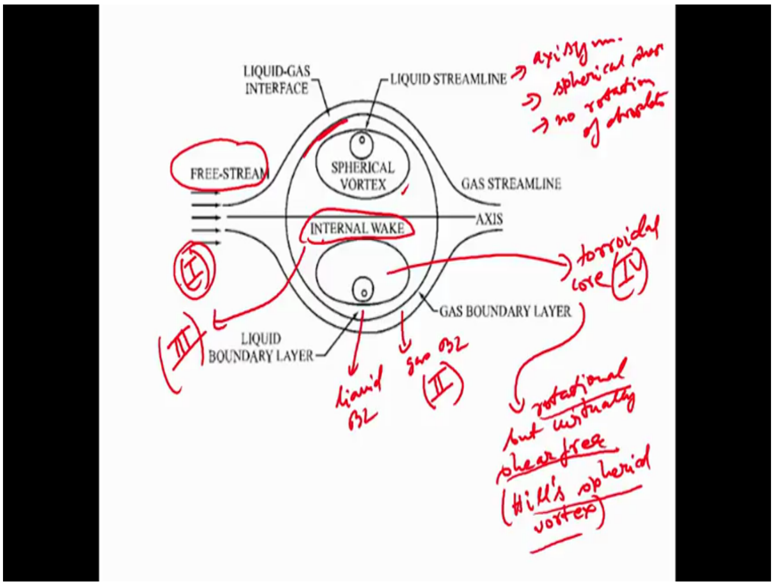

Now, based on this, let us look at the PPT of the flow field that we are going to see in the droplet, okay. So, based on those assumptions that what we say it axisymmetric flow field, then, the droplet actually does not retain this spherical shape, okay. And the droplets do not really turn in the flow, no rotation as such, droplets, okay, got it.

So based on these assumptions, we can see some of the interesting regions that what we believe is going to happen in the flow field. So, this is outer slope is the free stream that you can see, okay. So, this is basically the gas liquid interface. This is what is happening in the interior of the droplets a liquid stream line.

This is basically the gas phase boundary layer that we talked about right, okay. And this is the liquid boundary layer it is on the other side. This also, we say it, right there exist, right, okay. Now, based on, if this is what the situation looks like, let us see what are the regions in this particular flow field, right? So, there is of course an invicid free stream region which is marked as one, right.

This is in invicid free stream, right. So, there is no effect of any viscous thing. This is pretty much what happens if you analyze any flow over an obstacle, right. For field is always like a free stream. Then, there is a gas phase boundary layer. This is marked as region two, okay and the mere wake region, okay. So, you can see that this is the gas phase boundary layer is marked as two over here. Then, there is a liquid viscous layer and a liquid internal wake.

So, this is the internal wake in the liquid and this is the corresponding viscous layer. So, this is termed as three, got it, okay. So, the liquid boundary layer and the liquid internal wake and then there is the, what we call, the torroidal core, okay the torroidal core okay. Now, in this torroidal core, it is rotational.

The flow is rotating but it is virtually Shear free, got it, okay. So, that is called, represented by, something called Hills Spherical Vortex, right. So, that is basically your region four, got it. Let us recap, okay that the free stream region is denoted as one, right; no viscous effect, the gas phase boundary layer is denoted as two; not this at all, this is two, got it, okay.

So, this is the gas phase boundary, the liquid boundary layer and the internal wake region there, right, this whole region that you see. This is basically called region 3, okay and the rotating torroidal core, that you see, what is denoted as Spherical vortex, right. It is rotational but it is virtually shear free, right.

And it is represented by the well-known Hill's Spherical Vortex solution. So, that is the four region, okay that you can mark when the droplet is actually showing all this kind of motion when it is injected in a gas phase flow field, got it, okay. So, in order to solve this particular problem, you need to kind of taking back on all these parameters, right.

You need to solve the gas phase boundary layer. You need to solve the liquid flow field and the liquid boundary layer. You also need to know what is happening elsewhere, right in the heart stream. Of course, the actual task is a little bit more difficult, okay and it is not so very easy to do, okay. But we will see what, what happens, okay.

So, this actually let us end the show here. So, I think you got the idea that what this thing exactly means, right. Go back to our journal entry. So, now coming back, so, let us now try to investigate a few of these estimates, right. Like, for example, we say that there is no rotation of the droplet and there is access symmetry of the flow field.

So, therefore we have stated basically two postulates, right. That this can happen if the flow is axisymmetric and the droplet does not show any rotation, okay. Then, there can be two corollary is essentially it can be based. It can only happen if this is true the characteristic time for rotation in flow, right is long and I highlight the word long, compared to the transit time of a gas parcel, right. Parcel around the droplet, you got it.

What does this essentially means? That if you see that the droplet is rotating, right so, it takes a certain amount of time for the droplet to rotate itself, right. During that time if a pocket of gas actually passes over, right that liquid sphere and if that time scale is very short compared to the rotational timescale, okay then, we can say that the effect of rotation is negligible, okay, so to say okay.

So, that's why I say the characteristic time for rotation inflows is long compared to the transit time of a gas parcel, right. And the second corollary is that the fractional change in friction force. And why this is important? Change in friction force is negligible and it should be of the order of 1 over the Reynolds number, right. Order, order means that, it is of the similar order.

When we say ordered we do not mean the exact number. We mean the scale essentially. So, where does the friction comes into the picture, okay? See a droplet can only rotate if there is a torque, right. If there is an axisymmetric flow field over the droplet, then, there cannot be any torque, as such that can be produced, correct.

So, there has to be a net moment due to some variation of the flow, some asymmetry, some pulsation, something, right. There needs to be something of that sort, okay. So, there is a need to have, have velocity gradients to have, to have asymmetry in the flow, okay. So that asymmetry leads to torque, got it, okay. And this torque is further dependent on the friction forces.

And friction forces we know that depends on the velocity gradient, right from the normal viscous shear, okay. So, friction force; so, velocity gradients needs to estimate torque which is due to the origin of this friction forces over here, right. So, we have to show that these two things are correct.

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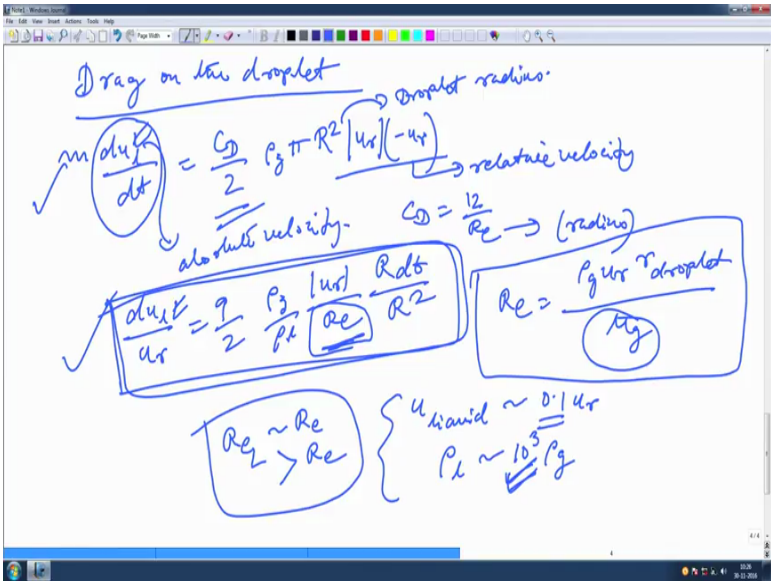
Drag on the droplet

$m\frac{du_1}{dt} = \frac{C_D}{2}\rho_g \pi R^2 |u_r|(-u_r)$

→ Droplet radius.

→ relative velocity

absolute velocity.

$C_D = \frac{12}{Re} \rightarrow$ (radius)

$\frac{du_1}{u_r} = \frac{9}{2}\frac{\rho_g}{\rho_l}\frac{|u_r|}{Re}\frac{R\,dt}{R^2}$

$Re = \frac{\rho_g u_r r_{droplet}}{\mu_g}$

$Re_l \sim Re$

$> Re$

$u_{liquid} \sim 0.1\, u_r$

$\rho_l \sim 10^3 \rho_g$

But before we show that, let us look at the drag that is developed on the droplet because we have talked about friction. Let us do the drag on the droplet, okay. Now, the drag on the droplet is simple we can be written as reading, writing it fast, where ur is basically the relative velocity.

This is basically the absolute velocity. CD = 12 by Re where Re is based on radius, rather than diameter, okay. And R is basically the droplet radius. It is a very simple equation, right. This is how you would write if you inject not just a fluid blobs; but something like that, like a solid particle in a flow field, right.

This is how you would like write that the rate of change in velocity how the main fluid blob is dependent on the drag force, where the drag force is not dependent on the actual absolute velocity depends on the relative velocity as you can see. As the liquid velocity increases, right slowly, this relative velocity will become negligible, right.

That is what happens that is how a droplet is actually entrained into the flow field, okay. So, if you do the, okay. As I said earlier, that this particular form is more or less valid because the Reynolds number do not really change significantly, even if there is a large reduction in the mass, okay. It reduces by less than one order roughly normally, okay.

So, we expect that there will be always a laminar boundary layer that will be established on the droplet for the most part of the process, right. So, the Reynolds number here is basically written as Row g, ur, r droplet divided by Mug, right, okay so that you can cross validate that what that number will be, okay.

Now, so, the drag coefficient is given by the drag of the liquid or the rate of change of velocity of the liquid is actually a function of the Reynolds number, right, okay. And this is the equation that actually governs it; this equation and this equation. So, you can solve it and you can find out how the velocity of the droplet actually evolves.

That does not require any solution of the internal flow field, right, okay because you can just find out what will be the velocity of the droplet, how the velocity of the droplet will evolve, okay regardless of its evaporation. That is because as we established even with evaporation that Re does not reduce by that much.

And the equilibration of that of the velocity happens, very fast, okay. Very fast the droplet getting trained into the flow filter, okay so, there is a velocity deficit actually reduces pretty fast, okay. So, this is the expression for the Reynolds number that we are going to use for the gas phase. Now, the velocity of the liquid phase due to internal recirculation is one order of magnitude lower than a relative gas velocity.

So, if we look at the velocity within the liquid phase that is velocity in the liquid not absolute velocity. Do not confuse this with that. These are not the same ul and u liquid are not the same, right. So, this velocity is roughly .1 of the velocity of the gas phase that means ur, so to say, right, roughly about one order lower, okay one order of magnitude lower.

But as you can rightly guess Rho l is however 10 to the power of three times Rho G that is the typical values like 10 to the power of 2 to 10 to the power of three. So, therefore if one compute something like a liquid phase Reynolds number Re l, okay that Reynolds number, okay will be large will be of the same order.

So, it will be either the same as Re in some cases which can be even greater than Re, right because you have an order three change over here, right. Though the velocity actually reduces by one order this actually increases by three orders of course the, the viscosity also needs to be readjusted that is also high, okay.

But you, you combine all these things normally the liquid phase Reynolds number and the gas phase Reynolds number will be very compatible, okay. Sometimes the liquid phase Reynolds number can be actually a little higher than a gas phase Reynolds number, okay. So, with that we stop on this lecture.

We will pick up in the next lecture, okay where we will show that how this Reynolds number where does it come into the picture and how we can actually establish that the droplet actually does not rotate by that much, okay, okay.