Heat Transfer And Combustion in Multiphase Systems
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Lecture 22
Droplet vaporization models-II

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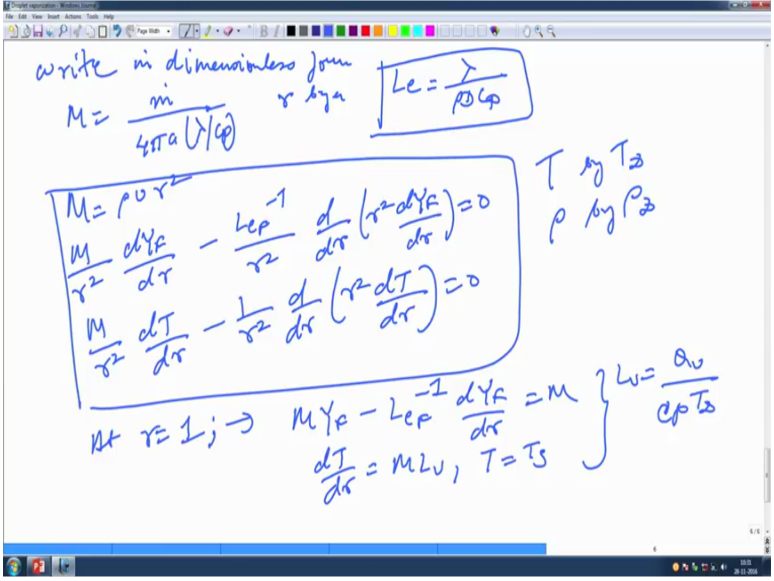

write in dimensionless form

$$M = \frac{\dot{m}}{4\pi a(\lambda/C_p)} \quad r \text{ by } a \quad \boxed{Le = \frac{\lambda}{\rho D C_p}}$$

$$M = \rho v r^2$$

$$\frac{M}{r^2}\frac{dY_F}{dr} - \frac{Le_F^{-1}}{r^2}\frac{d}{dr}\left(r^2\frac{dY_F}{dr}\right) = 0$$

$$\frac{M}{r^2}\frac{dT}{dr} - \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

T by T_∞

ρ by ρ_∞

At $r = 1; \rightarrow$ $M Y_F - Le_F^{-1}\frac{dY_F}{dr} = M$

$\frac{dT}{dr} = M L_v, \; T = T_s$ $\Big\}$ $L_v = \frac{Q_v}{c_p T_\infty}$

So, we can write all this equations in the dimensionless form. Let us form, okay. So, you can write like for example, M that is the mass can be written as m dot by a lambda by Cp. That is one term. This is the dimensionless mass flux you can normalize r by a, okay. So, I mean these are the typical non-dimensional parameters.

You define something called a Louis number which is basically nothing but the thermal diffusivity divided by the mass diffusivity, right. So, that is the Louis number. And this is very important, specially, in the case of combustion and other related things. If you do all the transformations, we can write the non-dimensional equations as o v r square, M by r square, dYF by dr, -L eF -1 r square, d = 0, M by r square dT by dr - 1 by r square, right okay.

So, these are basically the dimensionless versions of the whole thing. The temperature similarly can be normalized by T infinity and things like that, okay. These are not important but you can. So, it is basically T is normalized by T infinity Rho is normalized by Rho infinity, okay. So, that will take care of the things, okay.

So, at r = 1, that means non dimensional r = 1, okay. We could have written it as r bar also does not really matter. The same boundary conditions now will translate to something like this. This is where the Louis number comes in as you can see, right. And dT by dr = MLv T = Ts and where your Lv is basically Qv by Cp T infinity, okay.

CTp infinity is basically nothing but a h infinity, okay. So, these are some of the basic quantities that we have written, okay. Now, let us look at. So, this actually shows that what happens at the interface in terms of the normalized quantity.

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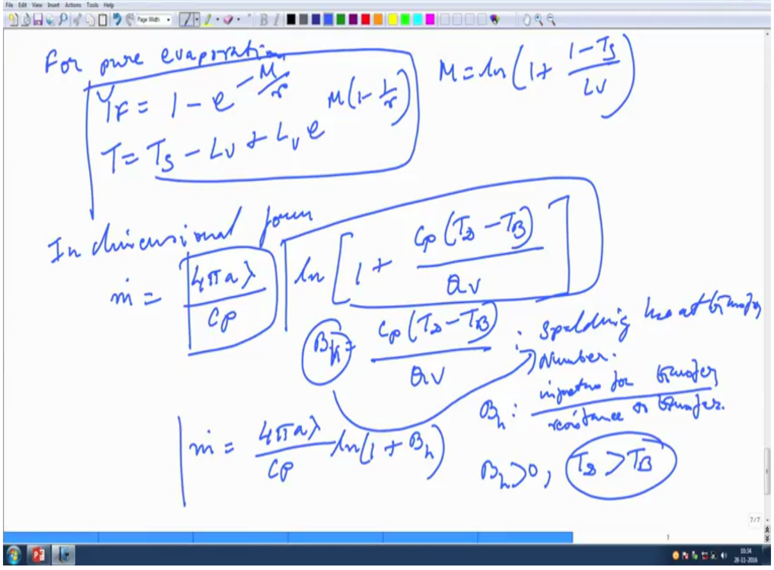

For pure evaporation

$Y_F = 1 - e^{-M/r}$

$T = T_s - L_v + L_v e^{M(1-\frac{1}{r})}$

$M = \ln\left(1 + \frac{1-T_s}{L_v}\right)$

In dimensional form

$$\dot{m} = \boxed{\frac{4\pi a\lambda}{C_p}} \ln\left[1 + \frac{C_p(T_\infty - T_B)}{Q_v}\right]$$

$B_h = \frac{C_p(T_\infty - T_B)}{Q_v}$: spalding heat transfer Number.

B_h : $\frac{\text{impetus for transfer}}{\text{resistance to transfer}}$

$$\dot{m} = \frac{4\pi a\lambda}{C_p} \ln(1 + B_h)$$

$B_h > 0$, $T_\infty > T_B$

Now for pure evaporation, if you solve for YF, it is basically 1 - m by r, okay. And T = Ts - Lv + Lv M = ln 1 + 1 – Ts by Lv, okay, so, this is the pure evaporation. In dimensional form you can just work out the maths because there is nothing more to do apart from algebra, okay. In the dimensional form, your m dot will be given by 4 Pi a lambda by Cp, okay.

So, here this particular factor, we define a factor called Bv or Bh, whatever you call it, CP -T infinity - TB by QV. This is called the spalding heat transfer number, got it. So, the mass flux is basically given by this into this, right. And this Bh is basically called the spalding heat transfer number, okay. So, m dot in that compact notation can be written as a lambda divided by Cp ln 1 + Bh, okay.

In some cases which will be written as Bv does not really matter. It is the same thing, okay. So, basically what does this represent? The Bh, the Bh basically represents the impetus for heat transfer, in this case divided by, the resistance to transfer, okay. So, it is the impetus versus the resistance to transfer, okay. In most of the cases Bh will be greater than 0 and T infinity will be greater than TB. That is the surface temperature, right.

That will be always be the case because it's a evaporation situation, right. Now, how do you go from the d square law from here? We have already established this expression, right. This is the; using the Spalding heat transfer number. Now, how to calculate that how the droplet will actually reduce in size? That is the most pertinent question that we have.

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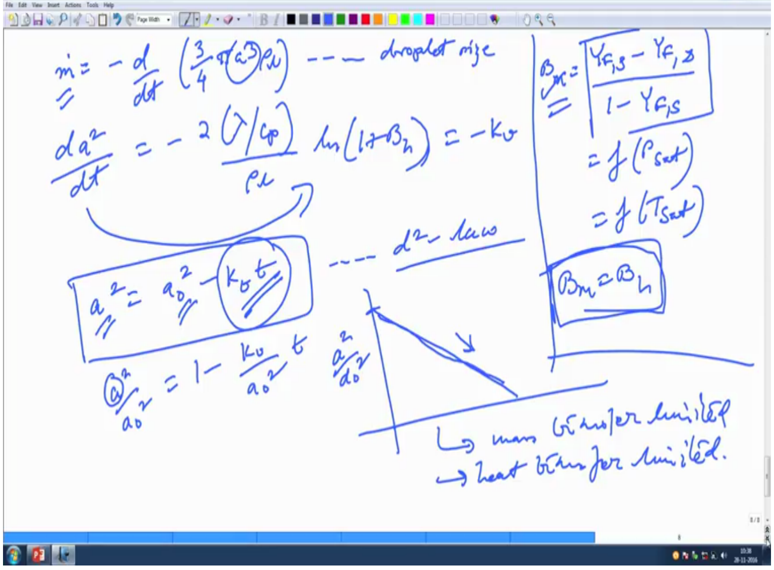

$\dot{m} = -\frac{d}{dt}\left(\frac{3}{4}\pi \boxed{a^3} \rho_l\right)$ --- droplet size

$\frac{da^2}{dt} = -2\frac{(\lambda/C_p)}{\rho_l}\ln(1+B_h) = -K_v$

$\boxed{a^2 = a_0^2 - K_v t}$ --- d^2- law

$\frac{a^2}{a_0^2} = 1 - \frac{K_v}{a_0^2}t$

$B_M = \boxed{\frac{Y_{F,s} - Y_{F,\infty}}{1 - Y_{F,s}}}$

$= f(P_{sat})$

$= f(T_{sat})$

$\boxed{B_M = B_h}$

Now, m dot is also given by three-fourths a cube Rho l, which is basically, basically given on the droplet size, right. Is not that so? It is given on the droplet size. Now, if you take d a square by dt okay, that will be given by 2 lambda by Cp by Rho l into ln 1 + Bh, right, because this m dot is the same as the m dot that we established here, right. So, what can be done is that you actually differentiate this but retain the a square term right, okay.

And then, you show that this is equivalent to that. Now, this is actually equal to - Kv, okay, whatever. That is just a constant that we have done over here. So, basically if we integrate this, this will become a square equal to a naught square minus Kv t, right. This is nothing but the d square law, got it, okay because it is a square given as the initial diameter reduces at a rate, okay which is basically linear, right.

So, you can also normalize this a square by a naught square to 1 - Kv a naught square into t, right. So, this will show a dk characteristic like this, right. So, it will start with when a naught is the same as a one, right, okay. So, it will start. So, the time axis will start from there, right. And you can see it is a gradual reduction of the slope which is expected, right because a decreases with time.

So, this is basically the d square law that we have derived out of this entire exercise, right. So, this square, now, there is another thing to this d square law we have determined it based on the heat transfer spalding heat transfer number. There is also spalding mass transfer number which is nothing but at the surface this is a far-field 1 minus the surface, okay.

So, this as you know, is a function of the P saturation pressure, right which in turn is a function of that P saturation pressure, right because P saturation and T saturation are related to each other, right. So, the spalding mass transfer number is given in terms of this. Ideally B m should be equal to Bh, okay.

You can derive this d square law from either of the two situations, okay. And this should be the, this should be the equivalent to each other, okay. So, this is mass transform if they are not equivalent the lower of the two basically actually governs the process, okay. So, it is either called mass transfer limited or heat transfer limited, okay.

So, there are basically two things, okay. Mass transfer limited or heat transfer limited, okay. So, these are the two cases that you have over here, okay. So, now that we have done all these situations, okay.

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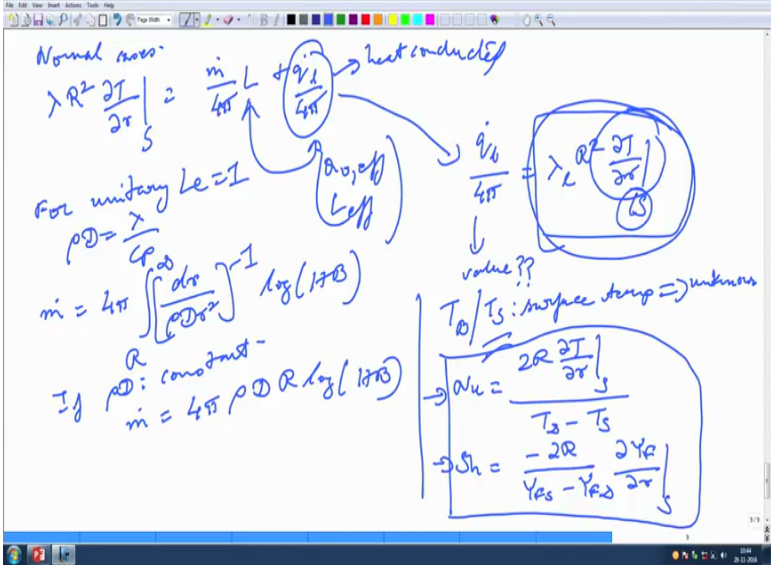

Normal cases:

$$\lambda R^2 \frac{\partial T}{\partial r}\bigg|_s = \frac{\dot{m}}{4\pi} L + \frac{\dot{q}_l}{4\pi}$$

→ heat conducted

$\alpha_{v,eff}$, L_{eff}

$$\frac{\dot{q}_l}{4\pi} = \lambda_L R^2 \frac{\partial T}{\partial r}\bigg|_{LS}$$

value??

For unitary Le = 1

$$\rho D = \frac{\lambda}{C_p}$$

$$\dot{m} = 4\pi \left[\int_R^\infty \frac{dr}{\rho D r^2}\right]^{-1} \log(1+B)$$

If ρD : constant

$$\dot{m} = 4\pi \rho D R \log(1+B)$$

T_B / T_S : surface temp ⇒ unknown

$$Nu = \frac{2R \frac{\partial T}{\partial r}\Big|_s}{T_\infty - T_s}$$

$$Sh = \frac{-2R}{Y_{Fs} - Y_{F\infty}} \frac{\partial Y_F}{\partial r}\bigg|_s$$

Now, let us look at, the first model is done that is the d square law part is done, okay. Now, let us look at the more the next phase of the models, right. So, because as I say d square law is limited we already established from the presentation what the d square law entails, right. Now, under certain cases, of course, in normal cases, okay what will happen is that your droplet, you cannot actually neglect like this.

You have to take into account the gradient inside the droplet, okay. Now, normal case is what we wrote was simple, right. Normal cases, just now, we wrote it. So, lambda a square or r square whatever you call it, right. Sirignano know uses capital R. So, you can stick to if you are reading Sirignano's book. Anyway does not really matter.

So, one part is basically your m dot by 4 pi into the corresponding Qv. Sirignano writes it as L. That is a latent heat it does not really matter either way. And we have already seen that a q l by 4 pi. This is basically the heat conducted; okay which in Matalon's notation, we combine these two, okay.

And say it that this is Qv effective. Sirignano writes it as L effective essentially means the same thing, okay. These two are basically the same, okay. So, you really do not need to bother about the notation. But whatever location you use, you use it consistently, okay. Now, we already know that this particular term and we did something to indicate that this is basically nothing but lambda l into r square into dt by dr coming from the liquid side.

The liquid side of the interface we can write it as LS, okay. Or you can write it as a minus, okay. Anyway once again does not really matter, okay. Now, normally, if you cannot neglect this, last time what we did was that, we say that this is equivalent to dt by the time, the time dependence of temperature.

And then, we say that we can be neglected, if it is in a wet bulb limit, this can be neglected. And all those things, we said, but, we are taking a case in which all these things cannot be neglected. Let us assume the more generic case like that, okay. So, so, in that particular case, if you look at, this particular expression once again over here, okay.

So, this shows that is in the liquid side of the interface. We normally do not know what is this conducted heat flux value is going to be? What is its value, okay? It is nominally unknown, okay. The droplet surface temperature that means either TB or TS, okay. Once again is a notation thing which is basically nothing but the surface temperature, okay. That is also unknown, okay.

So, someone needs to solve the heat diffusion equation within the droplet to know about all these answers, right. Otherwise your surface temperature is unknown; your heat flux is unknown; your gradient is also unknown. So, there are many unknowns which you need to solve, okay.

Now, before we go this, we can establish a few things over here, okay that will help us a few quantities, okay. Some of the quantities we already knew. Now, for unitary Louis number that is Le = 1, right. Louis number means you are basically thermal diffusivity and your mass diffusivity at the same, they are the same value, right basically Rho d = lambda by Cp, okay.

Now, m dot if you recall the expression for m dot, okay log of 1 + B, right. That is what we wrote in the last one, right. Now, if your Rho d is constant which in normal cases it is, right. We already established this. So, I am just reiterating the same. This we already established, right. In just in the two slides back in the last lecture, we established this particular thing, right.

And we are established to the two mass transfer quantities, right, okay. Now, based on this, we can also define the Nusselt number which is basically 2 r dt by dr at s divided by T infinity - TS. Similarly, you can define something like called the Sherwood number, okay which is basically the mass transfer equivalent YFs - YFs infinity, dYF by dr at s.

So, the Nusselt number is basically Nusselt number and Sherwood number is taken like that, got it. These are definition. This is no mathematics, okay. We have just defined the, what is the Sherwood number is going to be like. This will be useful quantity but as we will see later, okay.

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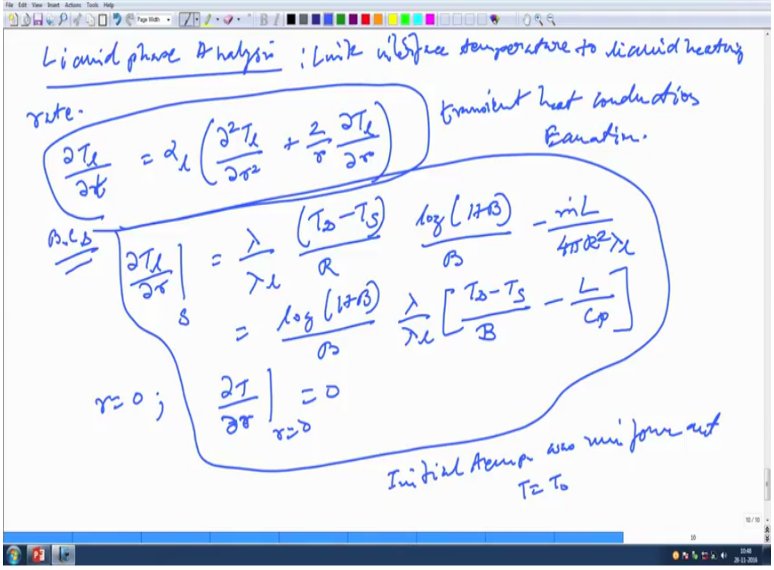

Liquid phase Analysis : Link interface temperature to liquid heating rate.

$$\frac{\partial T_l}{\partial t} = \alpha_l\left(\frac{\partial^2 T_l}{\partial r^2} + \frac{2}{r}\frac{\partial T_l}{\partial r}\right)$$

transient heat conduction Equation.

B.C.s

$$\left.\frac{\partial T_l}{\partial r}\right|_s = \frac{\lambda}{\lambda_l}\frac{(T_\infty - T_s)}{R}\frac{\log(1+B)}{B} - \frac{\dot{m}L}{4\pi R^2\lambda_l}$$

$$= \frac{\log(1+B)}{B}\frac{\lambda}{\lambda_l}\left[\frac{T_\infty - T_s}{B} - \frac{L}{C_p}\right]$$

$r = 0$; $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$

Initial temp was uniform at $T = T_0$

Now, if we do now only the liquid phase under liquid phase analysis because that is something that we never did. And that is what we promised, right. The liquid phase analysis, this is basically to link the interface temperature to the liquid heating rate, okay. How to link interface temperature to liquid heating rate? Okay

So, okay, so, we have to somehow link the two. So, the boundary conditions and you can show this, got it. That is also equal to log 1 + B by B, L is basically a latent heat, got it. And the symmetry condition prevails at r = 0 that means your dt by dr at r = 0 is 0, right, you got it. So, that is the basic set of equations, okay. And the initial temperature of the droplet, you can say, initial temperature was uniform.

And it was T = T naught, okay. So, the time derivative is considered in the liquid phase but in the sorry the time derivative is considered in the liquid phase but not in the gas phase. And that is because the gas phase, as we already know, we have already established that the gas phase does not require, right the gas phase does not require any such thing because it is fast, okay.

I am sorry. And so it, so that is a question, okay. So, that is a, this is basically a transient heat conduction equation, okay. So that we had done with that so that is the symmetric condition, okay. So, how about estimating? So, so you need to solve this equation, okay. You need to basically solve this equation to get an idea of what will be the interface temperature and how it is linked with the liquid heating rate.

Now, let us look at, okay the question that I am going to pose over here is that, we know what is the ratio of the, important question to know is that what is the ratio of the liquid heating time to the overall lifetime of the droplet, because if the liquid heating time is very small, then, we do not need to solve all this liquid phase equation, right.

We are just concerned with whatever is the wet bulb the temperature, right because the liquid is not getting heated anymore, it is not getting evaporated. So, whatever heat that is coming in is going towards evaporation, right.

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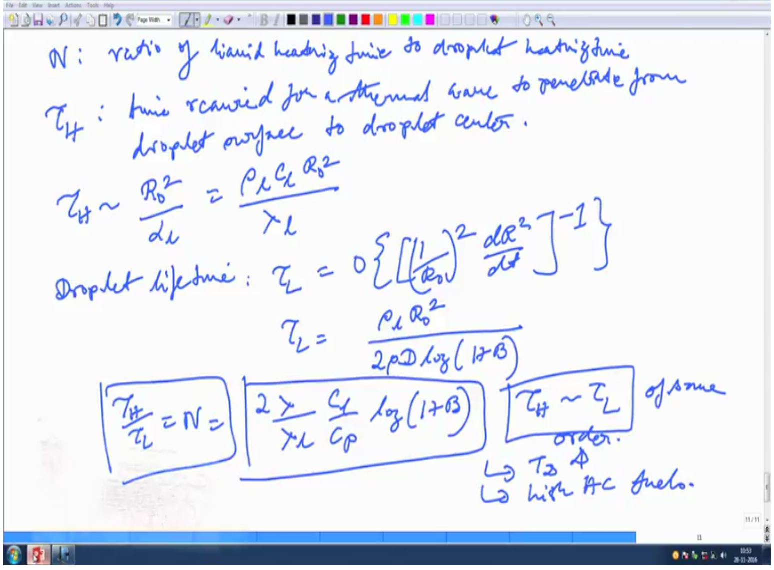

N : ratio of liquid heating time to droplet heating time

τ_H : time required for a thermal wave to penetrate from droplet surface to droplet center.

$$\tau_H \sim \frac{R_0^2}{\alpha_l} = \frac{\rho_l c_l R_0^2}{\lambda_l}$$

Droplet lifetime : $\tau_L = O\left\{\left[\left(\frac{1}{R_0}\right)^2 \frac{dR^2}{dt}\right]^{-1}\right\}$

$$\tau_L = \frac{\rho_l R_0^2}{2\rho D \log(1+B)}$$

$$\boxed{\frac{\tau_H}{\tau_L} = N =} \boxed{2\frac{\lambda}{\lambda_l}\frac{c_l}{c_p}\log(1+B)} \quad \boxed{\tau_H \sim \tau_L} \text{ of same order.}$$

↳ T_B ↑

↳ high HC fuels.

So, but in, in we in order to estimate that ratio if we say that ratio is, right capital N, which is nothing but the ratio of, of liquid heating time okay to droplet it in time, right. So, there are two ratios, right. The droplet heating time and the liquidity time and there can be a very significant way of finding this, okay.

So, let us assume, the liquid heating time is governed by Tau H, right. So, what is Tau H? Tau H is defined as a time required for the thermal wave which is originating and the droplet interface to move to the core, right. It is the time required for a thermal wave, okay to penetrate, from droplet surface, okay, to droplet center, okay. So, Tau H is therefore given as R naught square by, okay.

That is basically the diffusion scale, right. That time that it needs if it proceeds by diffusion that is the time scale you already know about all these things, right. For the time that is required for the thermal wave basically to come from the surface all the way up to the center. So, that is some kind of a liquid heating, heat up time, so to say. That is where the droplet center actually feels that there is something going on at the surface, okay.

Is equivalent to saying like this? You are hitting that wall, okay far away from me. I am sitting over here. I am not able to feel that there is something action is going on at the surface right. I will only feel it when the temperature gradient will come and hit me, right. So, when I will feel that there is a rise in temperature, right so that we can call about that time lag, right.

It is basically called that effect actually penetrates and comes all the way up to me. So, this is something like a, so, from a droplet perspective. That is something like the droplet heat up time, okay. Now, the droplet lifetime on the other hand lifetime is given by Tau L. What is the order of this? The order is 1 minus R naught square, okay, dr square by dt - 1. Tau L is basically given as Rho L R naught square divided by 2, Rho D log 1 + B, right, okay.

So, you know if we take the ratio Tau H by Tau L which is given as n is basically given by 2 lambda Cl by Cp log on 1 + B, okay. So, that is something that can be computed, right. So, that is the computation. In most of the cases you will find, that Tau L and Tau H are of same order, okay. This is particularly valid if the droplet has if the large portion of the droplet life time.

And it especially happens, when your ambient temperature is high particularly happens when your T infinity is quite high, okay. And you have higher hydrocarbon fuels, okay. Higher hydrocarbons will always show this kind of characteristics where the droplet heat up time and the temperature are almost the same.

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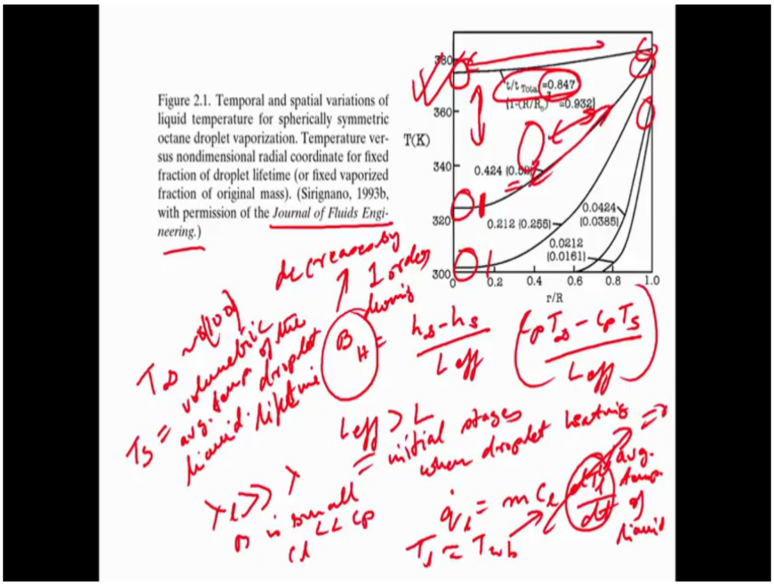

Figure 2.1. Temporal and spatial variations of liquid temperature for spherically symmetric octane droplet vaporization. Temperature versus nondimensional radial coordinate for fixed fraction of droplet lifetime (or fixed vaporized fraction of original mass). (Sirignano, 1993b, with permission of the *Journal of Fluids Engineering.*)

Let us look at two graphs to illustrate this particular thing taken from journal of fluids engineering, okay the temperature versus non dimensional radial coordinate, right. And you can see even at 90 percent or 85 percent of the droplet lifetime, okay. Still there is a slope. Let us see that it is just about reaches there, okay.

It has not reached here. This portion is largely shielded is not that so? Largely shielded, right from the influence of what is happening there, there, right. So, you can see even at this is taken at 52 percent of the droplet lifetime even at 85 percent of the droplet lifetime still you retain a little bit, okay where this has not really sensed, right.

So, this actually shows that you know the droplet heating time, okay is actually may be a substantial portion not always but it can be a substantial portion of the overall lifetime of the of the drop, okay. And that is a very important concern over here, right because our analysis and all those things depend on this, okay.

Now, if you look at the definition of Bh. what it was? It was h infinity - hs by L effective, correct. Something likes that, well. Well, you, you are familiar with Cp - C infinity - Cp into Ts divided by L effective, equivalent definitions, okay. It does not really matter, okay. Now, here you can see that, okay.

Initially your L effective, okay must be greater than L, right because it is the initial heat up time. This is this happens during the initial stages, okay when droplet heating is going on and evaporation is marginal, okay droplet it. Because evaporation, as you know, is a function of the P sat, right. Because asked with temperature P sat goes up. So, your evaporation rate also starts to climb, okay.

So, this Bh but initially this is large, but then subsequently what happens is, it starts to decrease okay. Now, in that particular case, your Bh your value of the Bh actually changes over the droplet lifetime, okay. In fact Bh you will find it decreases by 1 order. 1 order during droplet lifetime, got it, okay. It happens. It decreases actually by one order during the droplet lifetime, okay.

So, if you recall that, what we did, if you recall this? That the type II model, model was basically where we consider that is the infinite liquid conductivity and type III model is this current one, that we are actually doing, right. It is a spherically symmetric assumption, right. Now, type III model, actually type II and type III model are actually becomes the same, okay.

If you assume that the lambda l is much, much greater than lambda, right, that means, the liquid phase thermal conductivity is very, very high and B is small okay. And Cl okay it is much, much less than Cp, okay. So, ambient temperature for a few 100 degrees that means T infinity is of the order of 100 degrees, okay.

This actually leads to the surface temperature Ts is the same as the volumetric mean temperature, the volumetric average temperature of the liquid that happens, okay. It is always equal to the volumetric temperature of the liquid in, essence; it essentially means that the instantaneous droplet mass. And then, so, the ql that we conjectured earlier is basically therefore, if you recall, this expression, right when that happens, okay.

So, the instantaneous droplet mass and this is basically what we call the average, temperature of the liquid, the liquid front, okay. At one point of time, as we said earlier, that Tl becomes equal to the T wet bulb, right to the wet bulb temperature. When that particular thing happens, okay this term basically boils down to 0, got it, okay.

So, we have said that a character; so, we have established over here, okay that when do we actually require to solve for the temperature field. We need to follow it for these cases, right. Where there is a strong temperature gradient as you can see. It is not negligible and it persists for quite a bit of a droplet lifetime. In that case you have to solve this and you have to link it to the outer gas flow field via the interface condition, right, okay.

So, that is very important. So, the next class, what we are going to do is that we are going to now consider what happens, when there is convection inside. The convection part is very important, okay. When there is convection inside, what happens to the analysis, okay. So, see you next class.