Heat Transfer And Combustion in Multiphase Systems
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Lecture 21
Droplet vaporization models-I

Welcome to today's lecture! So, here what we are going to do is that as promised we would now take a look at the different droplet level models, right.

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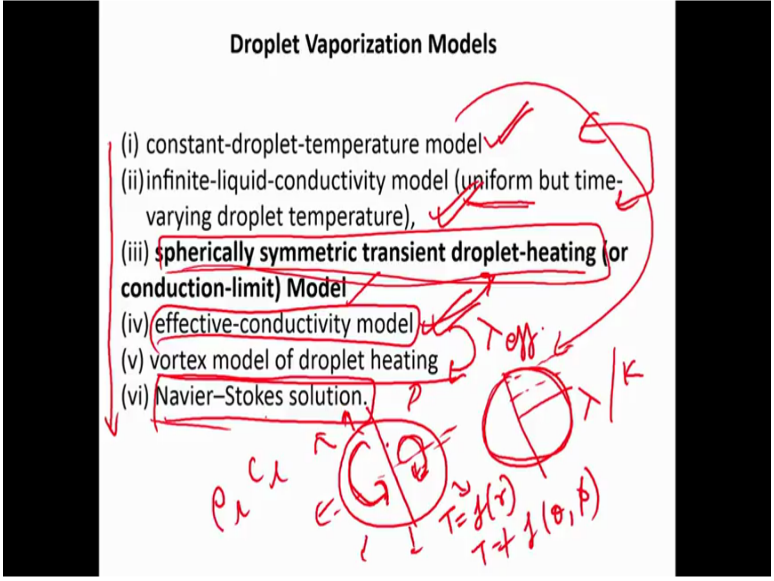

Droplet Vaporization Models

(i) constant-droplet-temperature model

(ii) infinite-liquid-conductivity model (uniform but time-varying droplet temperature),

(iii) **spherically symmetric transient droplet-heating (or conduction-limit) Model**

(iv) effective-conductivity model

(v) vortex model of droplet heating

(vi) Navier–Stokes solution.

Like the different droplet vaporization models and we are going to go one by one, okay. So, if you look at the first simplest model is a constant droplet temperature model, right. Now, this is the model that is going to yield you the d square law essentially, okay. There can be also infinite liquid conductivity model it means that the liquid conductivity, the liquid temperature.

So, if this is a droplet, okay. The temperature within the droplet is uniform spatially. That means the temperature will be like this. However, this temperature might actually vary with time, okay. It will go up with time. So, that is the infinite liquid conductivity model. In essence it means that a lambda or the K whatever you use to represent the liquid conductivity that is very high, okay.

So, this is akin to the lumped capacitance model that you have learnt in your heat transfer, right. So, because there is no variation of temperature within the droplet, okay. And constant droplet temperature model means the temperature is constant no matter what, okay. These two can be linked with each other and in fact they are linked with each other and we will see how in a in a little bit.

Then, there will be the spherically symmetric transcient droplet heating model. In this model, what happens is that the liquid temperature within the droplet, okay. It is not actually uniform, okay. So, that means there will be a temperature gradient of some sort, whatever the temperature gradient is, okay. So, that temperature gradient, so, there will be a variation of temperature within the droplet, okay.

So, within the droplet the temperature will be a function of r. But still it is spherically symmetric. That means, it is isotropic in nature, okay. So, there is no variation. Temperature is not a function of theta or Phi, got it. So, there is no as a azimuthal variation, okay. So, that is called a spherically symmetric transient droplet heating model.

There is something called effective conductivity model also which we will come across a little later. Then, there is something called the vortex model. The vortex model actually takes into account the fluid flow inside the droplet, okay. So, it actually is a convective model. So, it takes into account how the fluid, the liquid inside the droplet is actually churned, okay.

As a result of that you have to solve the momentum, the energy equation within the droplet. But what the vortex model is effectively does is that you can assume of flow filled within the droplet, some assumed flow field, okay. And we will see what that assumed flow field is. And you can get around solving the full momentum equation. You can just solve the energy equation, right. But it is a full slated convective solution.

Then, there is a Navier-Stokes solution that means you solve basically the entire thing, okay. You solve the liquid flow momentum equation, you solve the liquid flow energy equation and you solve the gas phase momentum energy in all those equations, right. So, it is in increasing order of complexity essentially, if you think about it, okay.

And depending on the situation that you are concerned with, like for example, in cases, where the surrounding temperature is not very high, okay you can get around using this and this, right okay. Or in cases where the liquid thermal conductivity is very high, okay, you can get around using this kind of a model, okay.

But, in cases where the droplet size is large, say, for example, size is large meaning; the thermal mass is large, okay. So, that means essentially your Rho l Cl is large right okay in those particular cases what will happen and your thermal conductivity is not that high in those cases you will find that the spherically symmetric transient droplet heating model, okay is the correct way to do.

That means you have to solve the energy equation basically solve the conduction equation within the liquid droplet, right okay. Effective conductivity model is a variation of this, except that, here the thermal conductivity lambda is actually revised to something called lambda effective, okay. This is done because if there is a flow within the droplet, okay.

What does the flow do when there is a convective flow within the droplet? What does the flow do? It basically promotes mixing, right. So, effectively you can think about like from a turbulent perspective, we were heard like there is something called effective viscosity and things like that.

Here also this is effective thermal conductivity that means, in effect, your droplets thermal conductivity as if is going up because of this flow that is generated within, okay. You want to have a little bit more fidelity you can use the vortex model in which you actually impose a flow solution within the droplet.

And then you solve basically the energy equation using the velocity term, using the convective derivative term. And then, you, of course, have the Navier-Stokes equation which is the full-fledged solution of the same. So, given this pollute, okay let us look at now the first of the problem that means the droplet vaporization using deriving basically the d square law okay.

So, I guess this actually tells you that what we are actually looking at and we will come back to this in a little while, okay.

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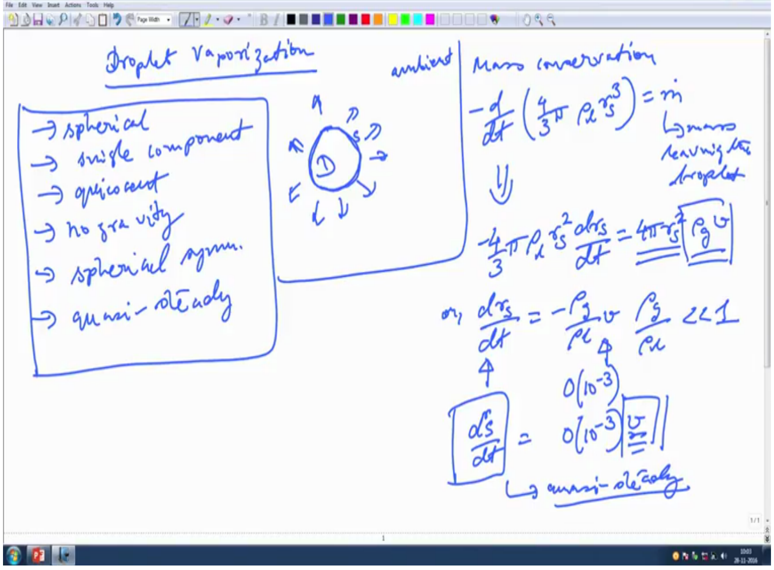

Droplet Vaporization

→ spherical
→ single component
→ quiescent
→ no gravity
→ spherical symm.
→ quasi-steady

Mass conservation

$$-\frac{d}{dt}\left(\frac{4}{3}\pi \rho_l r_s^3\right) = \dot{m}$$

↳ mass leaving the droplet

$$\Downarrow$$

$$-\frac{4}{3}\pi \rho_l r_s^2 \frac{dr_s}{dt} = \underline{\underline{4\pi r_s^2}} \boxed{\underline{\rho_g v}}$$

$$\text{or, } \frac{dr_s}{dt} = -\frac{\rho_g}{\rho_l} v \qquad \frac{\rho_g}{\rho_l} << 1$$

$$O(10^{-3})$$

$$\boxed{\frac{dr_s}{dt}} = O(10^{-3}) \boxed{\underline{\underline{v}}}$$

↳ quasi-steady

So, Droplet Vaporization, okay. So, as we know, if you recall the previous lecture, it is a spherical droplet vertical in shape; single component. Let us not assume multi-component droplets over here. That means if it is a water droplet is a water droplet; that there is no ethanol, there is nothing okay.

But, if it is the fuel droplets, it is the pure fuel droplet. The ambient is basically quiescent, okay. That means there is no fuel, there is no gravity which you know already, it is true, okay. Then, there is a spherical symmetry, right. And there is a state of quasi steadiness which we already established why, when we can consider the flow field to be quasi steady, right.

And I am not including the combustion parameters here, okay. That we will cover a little later. So, these are basically the assumptions; the basic assumptions, okay. Now, let us look at each and everything in details now, okay. And we already have a diagram that how a droplet actual evaporates. So, this is the droplet evaporating like this, correct, okay.

So, this is the ambience, ambient okay. And this is the surface of the droplet this is the surface, okay. Now, based on this let us see how we can actually solve this, okay. So, let us consider the mass conservation first, okay. So, the mass conservation is basically how the droplet is basically losing its mass, as simple as that okay.

It is four third Pi Rho l rs cube m dot, correct. This is basically the deduction in mass of the droplet, okay which is happening over time and this is basically the mass that is leaving the droplet, very simple okay. Now, if we now want to do this you want to write this further we can take density to be constant, the liquid density to be constant. So, it comes out of the other bracket r s square drs by dt, got it?

And that is the right hand side, okay. If we write it in terms of the gas phase flux okay rs square Rho g into some velocity, correct, okay. Is not that correct, okay? So, this is basically the mass flux. This is the area, okay and this is the flux with which the gas is actually leaving just to have to match because whatever the mass the liquid is actually the deficit, that is happening in the liquid has to be carried out by the vapour in essence okay.

So, or if we do this now like that it will become Rho g by Rho l into v, right. And as we know that Rho g by Rho l is much, much less than 1. This is true for all the cases, right. So, this actually implies that this rate which we already established in our last class is much, much lower than this, right.

It is Rho g by Rho l, right. So, it is a order of 10 to the power of -3, correct. So, this would mean that drs by dt is basically ordered 10 to the power of -3 into v, right. So, essentially it means that v is much, much faster compared to drs by dt, right. This is what we established last class also, okay. Essentially means that a regression of the droplet diameter is very, very slow compared to the diffusive flux, right okay.

In that particular case, we can say that this is basically this justifies our quasi steady assumption, okay. We already knew this is another proof that and we have shown over here, got it. So, now let us write after that we have established this okay we do not really need the liquid phase anymore okay.

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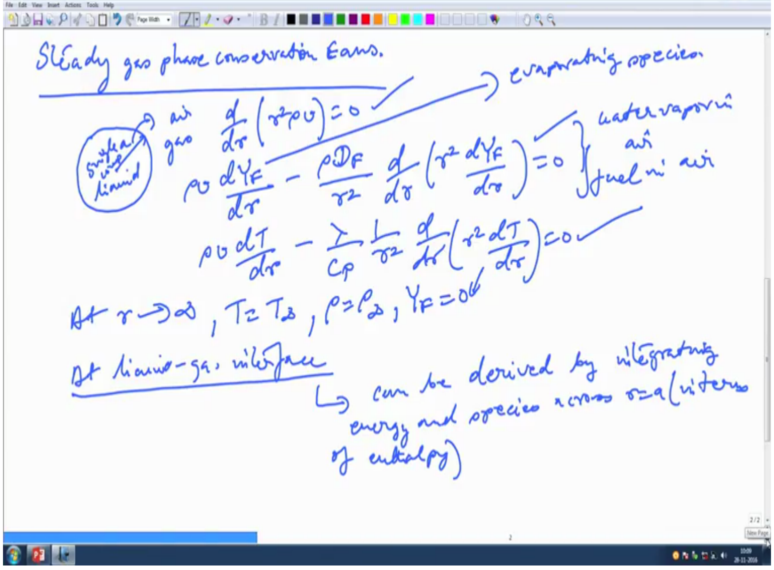

Steady gas phase conservation Eqns.

$$\frac{d}{dr}\left(r^2 \rho v\right) = 0$$

$$\rho v \frac{dY_F}{dr} - \frac{\rho D_F}{r^2} \frac{d}{dr}\left(r^2 \frac{dY_F}{dr}\right) = 0$$

water vapor in air / fuel in air

$$\rho v \frac{dT}{dr} - \frac{\lambda}{C_p} \frac{1}{r^2} \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

At $r \to \infty$, $T = T_\infty$, $\rho = \rho_\infty$, $Y_F = 0$

At liquid-gas interface

↳ can be derived by integrating energy and species across $r = a$ (in terms of enthalpy)

So, we will now try to look at the corresponding steady gas phase conservation equation, okay, so, gas phase conservation equations, okay. Now, let us assume that this is the droplet once again following the notation of Martian Matlon we are using this. This is the liquid. The surface is denoted as a, this is the gas. It can be any notations by the way it can be rs, it can be a, it can be whatever you can think of.

But the basic thing does not really change, okay. So, this is a steady gas phase conservation equation. We already are using the word steady because we have already proved that it is a quasi steady gas flow field, right. So, we do not need to carry the temporal derivative anymore, okay. And this is the gas phase. So, in the gas phase what can we write in the gas phase?

And I am not using Rho g anymore, okay. So, not using Rho g, so, when we use the liquid we will use Rho l. We will use the subscript l, okay. So, that is what we have that is a gas phase conservation equation, right; the mass conservation equation, okay. So, the next equation that we are supposed to write is basically the species, right okay.

Though it is a single component in the liquid phase right. But as the vapour comes to the vapour phase, right it is a multi-component system, correct; because there is fuel and then there is air, okay. There is whatever the species that is evaporating. It is evaporating into the ambient which is air, right. So, you have to write the species balance equation for the vapour that is coming into the ambient, right.

So, under this assumption what we do is: And we are writing the evaporating species as dYF. The notations are a little bit different because we are writing it in the spherical system. So, this is like for example water vapour in air or fuel in air, if you are considering the combustion of droplets, right and YF is basically the evaporating species, correct okay.

This should be clear. So, so, that is the first equation. That is the species. Now, we can write their, the temperature form also, okay. So, that is basically the energy equation, right. So, this is the tempura all of these are written in the gas phase. Remember, we are writing nothing in the liquid phase. And if I have understood correctly as I said in the last class the liquid is always single component, right okay.

So, the vapour that is coming out is going into air. So, it becomes a multi-component system. The gas phase is multi component; at least binary, okay. It is air and tough fuel, okay. So, we are considering it to be like that. And these are the three equations, mass conservation, species and the corresponding energy, right and they are we have written it in a very succinct way.

Now at r goes to infinity; that means far away from the droplet, right. What will be the temperature? The temperature will be some kind of an ambient temperature at T infinity, okay whatever the temperature may be. The Rho will become equal to Rho infinity. That means the density and YF will become equal to 0, correct, okay. In case of fuel in air, there is no real fuel present in air normally, right.

So, YF will have the value of zero, right. Even in the case of water vapour, the component amount of water vapour that is present in air is so, so very less, okay. That we can almost consider to be equal to 0 as a very far field, right. So, that means it is almost like relative humidity is equal to 0 there, right okay.

So, normally that will be that if, if you are considering a system in which the evaporating species is already present in ambient in substantial quantities you have to use the appropriate values over there, okay; whatever is appropriate value, got it. That is whatever it is a typical value, say, for example, in if it would have been like saying example some other gas, right.

And air already has say, 5% of that gas. Then, this value will be 5% equivalent value will be 5% over there, right. So, at the liquid gas interface okay. So, the liquid gas interface is very, very important, okay. So, the liquid gas interfaces because we are dealing with basically all these interfaces, okay.

So, how it can be derived? It can be derived by integrating the energy and the species equation across r = a in terms of enthalpy, got it. How it is derived? Let us write it. It can be derived by integrating, okay. Energy and species across r = a. That means across two sides of the interface and we usually do it in terms of enthalpy, got it, clear on this particular part that at the liquid gas interface and we will see how we are exactly going to do this, okay.

Now, that is my interface. Interface is the most important thing, right because that's what we have been doing for so long, okay.

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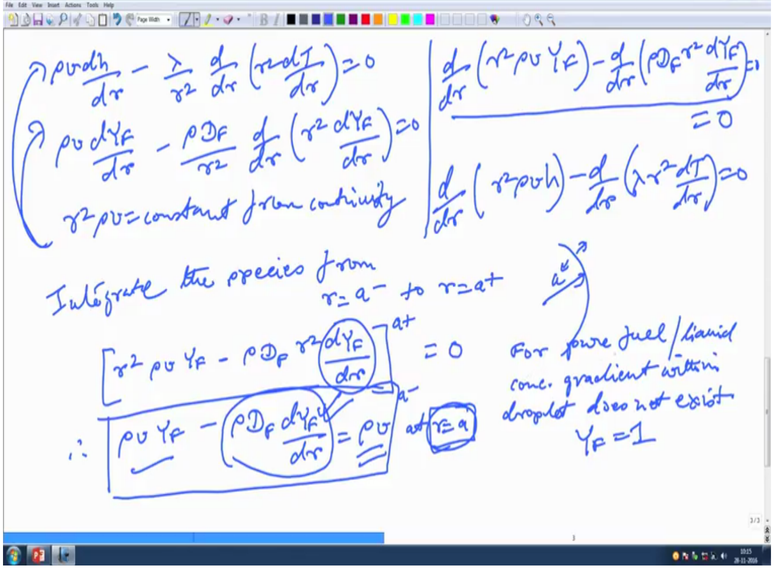

$$\rho v \frac{dh}{dr} - \frac{\lambda}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

$$\rho v \frac{dY_F}{dr} - \frac{\rho D_F}{r^2}\frac{d}{dr}\left(r^2\frac{dY_F}{dr}\right) = 0$$

$r^2 \rho v =$ constant from continuity

$$\frac{d}{dr}\left(r^2 \rho v Y_F\right) - \frac{d}{dr}\left(\rho D_F r^2 \frac{dY_F}{dr}\right) = 0$$

$$\frac{d}{dr}\left(r^2 \rho v h\right) - \frac{d}{dr}\left(\lambda r^2 \frac{dT}{dr}\right) = 0$$

Integrate the species from $r = a^-$ to $r = a^+$

$$\left[r^2 \rho v Y_F - \rho D_F r^2 \frac{dY_F}{dr}\right]_{a^-}^{a^+} = 0$$

$$\therefore \boxed{\rho v Y_F - \rho D_F \frac{dY_F}{dr} = \rho v} \text{ at } \boxed{r = a}$$

For pure fuel/liquid conc. gradient within droplet does not exist

$Y_F = 1$

In terms of the enthalpy, the equations are written like this, okay. And r square Rho V is equal to constant that you know from continuity, okay. So, including this over here, okay; that means we write it in the conservative form, right. We can always write every equation in the conservative form.

So, that will give you, sorry, that is equal to zero. So, if you cannot see it right in bigger over here similarly there is nothing fancy. I have just included the continuity equation inside the energy and the momentum equation. Let us call the conservative way of writing the equation, okay, got it, okay. So, this is like at a compact.

How you can write it compactly, okay? So, now, what we do is that we integrate the species and that is what we said, right. That we will integrate the species; species from r = a - to r = a

+. That means, on either side of the interface, right; minus plus minus means is within the liquid plus it is just in the gas phase, right.

On either side of the interface, this is your a, right. So, this is just around the interface on both sides, okay. Now, the concentration gradient, Now, once we actually integrate this. Let us write the form of the integration because all the integration all are with respect to r, right. So, it is easy to integrate, right. There is no problem. Now, you can understand why we got everything inside so that we can integrate it out easily, right okay.

So what it will be is after integration what we get is, right? That is what it is right, it is a definite integral so we are integrating it from a- to a+, and that is equal to 0, right. Now what can we say? Now that we have established this, what about the concentration gradient within the droplet, right? Since it is a pure fuel, there is no question of any concentration gradient.

Recall the lecture, that we did when we had a, when we say that when there is a pure component, okay. There is no concentration gradient because there is only one component, right. How can there be concentration gradient within the droplet, right. So, for pure fuel or whatever is a liquid that you are concerned about, okay.

The concentration gradient within the droplet, gradient within the droplet does not exist. There is no concentration gradient, right okay. Or within the droplet you have basically YF = 1 has to be the case always, correct okay. So, therefore, Rho V YF - Rho DF dYF by dr must be equal to Rho V at r = a, okay, getting the point.

So, this should be the equation that one should be able to write that this is the expression rather. At r = a, that means at the interface right r = a. That is the interface basically. So that is the expression that we have written over there okay. So it is basically this minus this is equal to that.

And this, remember, that one is in the liquid phase, one is in the vapour phase. So, do not confuse yourself. In the liquid phase that means at a minus this particular thing does not exist, right. So, you only have that, right. For it is a square Rho VYF, correct? When you are actually dealing within the vapour phase, of course, this will stay. It has to stay, right. So, into it will stay in that particular form, right.

So, combining those two things we actually have got this particular relationship. So, I think this is understandable, okay.
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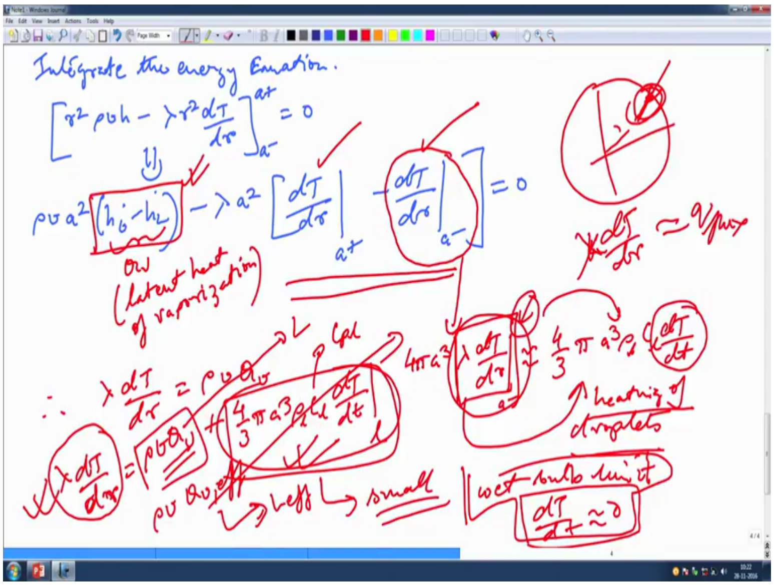

Now, let us integrate the energy equation. That is important. So, the energy equation is integrated in that similar way what you have is a Rho vh lambda r square, okay. That is it right. It is again the integration of the similar form, right. Now, if we now substitute, start substituting the values at a - and a +, okay. What are we going to get from this?

Just follow the term. There are two lambda a square, got it, okay. This is basically nothing but V into a into the corresponding enthalpy of the vapour and the liquid, correct. Now, that vapour and the liquid enthalpy, if we consider, what is this term actually called? This is basically called QV or the latent heat of vaporization is not that so? Is the latent heat of vaporization, right.

It is a latent heat of vaporization because it is the difference in enthalpy of the liquid and the vapour right, okay. On the other side you see two expressions. Now, one is basically the temperature gradient in a vapour phase; one is the temperature gradient in the liquid phase, right. Now, here is where the plot actually thickens a little bit, okay. Now, what can you say about this quantity is easy? This is well known, right.

But if you have to evaluate this, then, okay, then, if the temperature gradient needs to be evaluated in the liquid phase, okay. Then you need to solve the full liquid phase equations, right. Otherwise how would you know what is the temperature gradient at the surface, is not that so? This is the liquid, right. You know what is that you have an expression where it requires the temperature gradient at the surface, right.

Now, unless you know the temperature gradient throughout the temperature profile throughout, right. How are you going to know what is the temperature gradient at the surface? So, it requires the full solution, right. It requires a full solution so that full solution is needed in order to evaluate this.

Now, here is where we are going to make an assumption, okay. Now, this temperature gradient if you think about originally what we did a- is almost equal to four third PI a cube Rho l Cl okay. That means that if this exactly means that all the heat that is conducted goes into heating of the droplet.

This is the heat that is conducted, right. It goes into the heating of the droplet where this is the temporal signature of the same, understood. What does this mean? This means is essentially the slope, right, of the temperature curve here. And what does this mean? This essentially means the heat that is conducted inside the liquid phase is not that so?

You have always seen k dt by dr or lambda dt by dr whatever you call. It is basically nothing but the heat flux, right. That you know from your conduction studies, right. So, essentially this means how much heat is conducted within the droplet. And the amount of heat that is conducted within the droplet is basically used up in heating the droplet, basically increasing the temperature of the droplet, in a temporal fashion, but not in a special manner, okay.

So, all the heat actually goes over there. So, therefore, therefore lambda dT by dr. This is the slope in the vapour phase right that is equal to Rho v Qv, Qv is a latent heat component plus for third pi a cube Rho l Cl it is actually Cpl. So, you can write it in whatever way that you want okay, in the liquid, okay. This is basically Cl is basically Cpl, okay. Now, this portion is usually considered to be small, okay.

This is usually considered to be small not always but it is usually considered to be small. And there is a limit called the wet-bulb limit where this is basically equal to 0, okay. But that happens not immediately. But after a certain span in the droplet lifetime, right. So, if this is equal to zero. If you take this particular thing out, then, this is basically dT by dr = Rho sorry Rho v into Qv. Qv is a latent heat, okay.

Or if this if you are going to include this also then this Qv is actually written as Qv effective, okay. So, you sum the two, okay. In some cases, for example, Sirignano write the same thing as L effective, okay. So, QV is written as L and QV effective is written as L effective, okay. It is a notation thing, basically, means the same thing essentially, right. So, in the wet bulb limit this can be 0.

In some cases the magnitude of this can be 0 as well, okay. Under whatever is the consideration, so what we have done is that we have established a relationship in which the slope, okay is actually matched with the corresponding latent heat, okay of vaporization. Similarly if you recall what we did in the case of the other species balance, sorry.

What we did in the species balance also we have established this particular for once again that is the slope of the species in the vapour phase, okay. That is also given by these two terms, got it, okay. This should be clear, okay. Now, let us look at it. So, we are coming close to finishing this particular thing, okay.

Now, we already know that the phase changer, the interface happens very fast, right. That we established already, right. It happens very fast. So, therefore, we assume that equilibrium conditions actually prevail right.

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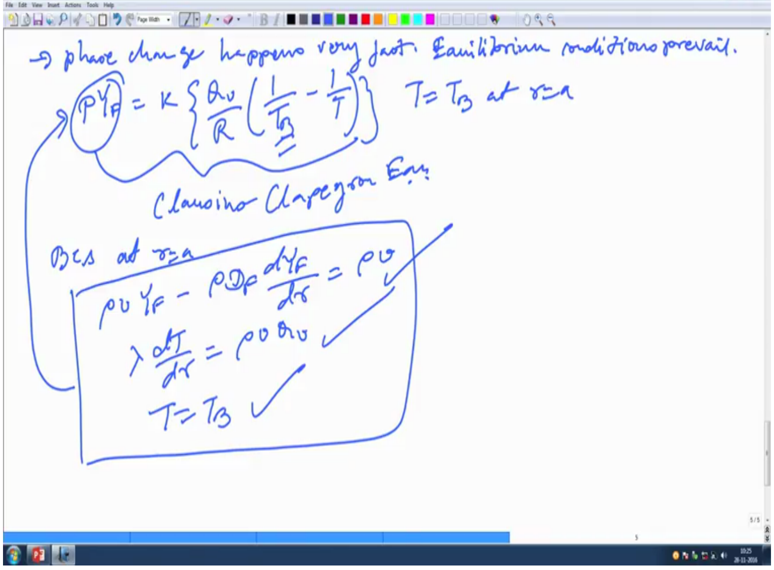

→ phase change happens very fast. Equilibrium conditions prevail.

$$\underbrace{PY_F = K\left\{\frac{Q_v}{R}\left(\frac{1}{T_B} - \frac{1}{T}\right)\right\}}_{\text{Clausius Clapeyron Eqn}} \quad T = T_B \text{ at } r = a$$

BCs at r = a

$$\boxed{\begin{aligned} &\rho v Y_F - \rho \mathcal{D}_F \frac{dY_F}{dr} = \rho v \ \checkmark \\ &\lambda \frac{dT}{dr} = \rho v Q_v \ \checkmark \\ &T = T_B \ \checkmark \end{aligned}}$$

So, the phase change happens very fast, okay. So, we can assume that equilibrium conditions prevail, got it. Since equilibrium conditions prevail, we can write the Clausius-Clapeyron equation okay to link the mass flux to the YF basically. The species mass fraction to the corresponding temperature at the interface, right.

So, PYF = k Qv by r QV is l the latent heat. TB is basically the surface temperature T = TB at r = a. So, this is nothing but the Clausius-Clapeyron equation, okay. So, the boundary conditions at r = a is written as Rho v YF minus clear, okay. So, that is the species, this is the energy and this is the corresponding temperature, okay.

And there is a connecting relationship, okay, between the temperature and the species using this, right, the Clausius-Clapeyron equation which we already covered in the last class, okay.

So, still now, we are not in the d square law format okay in the, in this lecture. In the next lecture, we will see how to go from here to the d square law, okay.