## Heat Transfer And Combustion in Multiphase Systems Prof. Saptarshi Basu Department of Mechanical Engineering Indian Institute of Science-Bangalore

## Lecture 20 Heat transfer and evaporation in droplets

So, now that we have established this interfacial heat and mass transfer, we know how the interface actually behaves. Now let us look at some of the speciality problem right now okay. (Refer Slide Time: 00:33)



And one of the speciality problem is basically droplet evaporation and combustion okay why is it important as I said that droplet evaporation is fundamental to almost all applications that you can think of okay. From gas turbine to spray dryer to surface patterning whatever you can think of. This droplet may be in the sessile mode.

Sessile mode means it may be on a surface like this or it can be in a spray mode where the droplet is basically injected as a part of a spray okay, like this, there are many droplets okay. So, these droplets basically represent a multi-phase system right because there is a liquid core and then there is a gas exterior right whatever it may be it may not be water it can be anything okay.

And the liquid core can be multi-component, multi-species kind of a system as well right for example as I said water ethanol was one example okay. These days for example when we go to renewable type of energy okay people use something called bio-fuels say for example. So, bio fluid is a mixture of n number of hydrocarbons okay.

From very small hydrocarbons to very large hydrocarbons right, so these liquid phases usually have a multi-component nature or a multi-species nature to it right. Apart from that so that is the gas phase okay. This is the sessile droplet; the sessile droplet for example is used in surface patterning a lot right.

So, this sessile droplets might contain you know polymers or nano-particles okay and it can be used to create structures from the surface right. Similar things can be done in biological applications also okay like flow cytometry and things like that. Will all have this droplet based framework okay?

And they will have this droplet based thing is also used in micro-fluidics application for example where a droplet containing cells okay maybe actually use for some point of care type of diagnostics right, okay. So, there are many applications of droplet vaporization okay and drop the deformation, droplet atomization.

And it covers region from health care all the way upto combustion that means although we have to power generation. So, it affects every part of our life so to say okay. However the combustion part is a little bit more special it deals with only combustion that when these droplets actually participate in combustion what happens okay.

So, in this particular series of lectures that will follow, we will first try to anchor what is droplet vaporization. How we can actually analyze droplet vaporization? And then we will go on to have a little bit more special topic concerning combustion okay. And we are going to use a PPT as well as the journal to actually look into what we can have okay. (Refer Slide Time: 03:15)

Phenomenology	
<ul> <li>Many technological processes in which it is desired to gasify and combust a given ma- liquid in a gaseous medium at either a very fast or a controlled rate.</li> </ul>	s of
<ul> <li>Heat and mass exchange rates between the liquid mass and the gaseous medium incr with increasing interfacial area, a standard technique to increase the overall gasification is to disperse the liquid mass into an ensemble of liquid fragments [Spraying or Atomize</li> </ul>	ease 1 rate (tion]
Single droplet exchanges heat, mass, and momentum with its gaseous medium $P_{1,r,T}$	- 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
A complete analysis will involve the four interacting processes	d gas
Liquid-phase transport     Gas phase transport     Phase change at liquid-gas interface     Chemical reactions in the gas phase.     Thermal recirculation within the droplet	)

Now what is the phenomenology of droplet evaporation right that is the key thing okay. Let us consider one droplet now that is because multiple droplets have got its own problems okay. There are interference between multiple droplets okay. So, this is important droplet vaporization because most of the technological processes that we talked about just now.

It is desirable basically that you want to gasify the droplet or even combust the particular liquid in a gaseous medium okay. At a fast rate or at a controlled rate with various degrees of control right we want to do that right we basically want to control this evaporation rate okay in order to achieve certain goals, whatever those goals may be right.

So, what does this droplet actually do if you carefully look at the droplet okay? This is an isolated droplet that we have put over here this is isolated that means let us not assume that it is surrounded by any other droplet. It us like one building block that we have taken out, okay. So, there is what this droplet actually do is that there will be heat and mass exchange between the droplet as well as the gaseous medium in which it is in right, okay.

It is important so, what will happen is that there will be heat that will come from the ambient and the droplet will generate the vapour which will actually get mixed with ambient right that is vaporization. The reverse you can talk about is condensation right. So, there will be heat that will be given out by the droplet the vapour that is there around the droplet will just condense okay.

In that case the droplet size will increase. So, this single droplet actually exchanges heat mass and momentum with a gaseous medium right. If I want to analyze this droplet right okay in its complete and holistic view okay what are the things that we need to do, right. That is a pertinent question over here forget about it is it is combusting or not.

Just to do the heat and mass transfer what equations do we need to write what is the mode of analysis that we need to do right okay. So, the complete analysis will involve basically four emphasize the word for interacting processes one is a liquid phase transport that means what happens in the liquid phase that means weeding the droplet.

What happens, the gas phase transport that means what happens in the gas phase okay. The phase change at the liquid and gas interface right. And if you have combustion then what are the chemical reactions that are taking place in the gas phase right. These are the four things that we need to answer right.

So, we have to basically solve the governing equation that is mass, momentum, energy, species right in both the phases that means in the liquid phase, in your gas phase. These are all the equations that we derived right so far in the lectures right. How to write equations in the liquid phase and in the gas phase right.

So, here in this case this is a very dispersed kind of a system it has got a well defined boundary it is not a problematic boundary anymore not in this spherical configuration because the droplets we are assuming it is spherical okay. But you need to solve for the liquid phase, the gas phase, the phase change as well as the chemical reactions.

These four things need to be solved properly right. What are the important considerations for this, okay? The important consideration is that there could be relative motion between the droplet and the gas phase this is normally it is true because for example we are spraying droplets okay into a heated environment right. Let us take that example okay.

They are spraying something in an environment okay. So, the liquid phase velocity or the speed with which the droplet and injected into that particular phase may be different from whatever is the gas phase velocity right okay. So, the gas may be moving at a difference so it is say you have a pipe there is a flow which is going at a velocity v1 right you are injecting droplets into this flow field right at a velocity is say for example v2 right.

So, this v1 may not be equal to v2 in most of the cases there would not be right. So, there will be a relative motion between the gas and the liquid phase okay. There will be something called a Stefan slow due to surface gasification. As this droplet is operates right the interface is actually shrinking okay or expanding depending on which way you think it is right either it is evaporation or it is a condensation phenomena.

So, there is something called a Stefan flow which is created due to this surface gasification that means there is conversion okay of species from one phase to the other that is what is happening correct. Now there is also droplet drag if there is relative motion between the droplet and the gas phase there has to be dragged right.

Because if say for example you put a cylinder in a flow field right you develop a drag of the cylinder right that is very common okay. But that drag comes because of the relative motion pressure drag and there is viscous of drag also. Same drags will be applicable over here also not in that exact form but it will be applicable right.

Similarly there will be internal recirculation also inside the droplet where there is a flow outside why should not there be a flow inside right. The droplet is also in the liquid phase right. So, there will be a flow inside, for all these important considerations needs to be taken into account if we want to solve this full problem right okay. (Refer Slide time: 08:53)



So, given that introduction, let us look at some of the interesting features okay, of this now, okay. And some of the data that comes from our group was essentially from my group. So, I can share a few data with you, okay, like for example as I said we have assumed that the droplets are normally very spherical that is true in some of the cases right.

In some of the cases it is not okay, so, you get these catastrophic shapes right, got it. So, this is this you cannot tell that it is spherical this is non spherical right. So, this becomes a problem in which the interface is not very well-defined right. So, there are situations like hydrodynamic thermal and shear instabilities okay that leads to configurations like this right.

But we will come to that later okay, so, there may be also temperature and concentration species profile within the droplet because of internal recirculation right. So, as you can see in this particular configuration okay, the species profile is not very symmetric right. It is shows that kind of a behaviour is not that so that happens because it is a flow that is generated inside the droplet okay.

Forget about this for the time being this is if you have particles matters within the droplet how do they accumulate okay, one other interesting thing. (Refer Slide Time: 10:18)



So, these are some of the examples where you can see that the hell why the droplet is very useful okay. Now one other interesting thing that will come after this one that most of the time people assume that there is something called a spherically symmetric model right. What is a spherically symmetric model?

Spherically symmetric model is that a model of a droplet which is actually vaporizing into a quiescent ambient where there is no flow okay. So, the droplet is nice and easy sitting over there. There is no flow if you are considering flow around the drop that there are no such flow right and it is isotropically actually vaporizing inside a quiescent environment.

This particular thing is very hard to achieve because normally even your droom okay at this condition actually has a flow okay. If you put an anemometer or a putative over here, you will detect the flow and that flow is not very trivial okay. So, by spherically symmetric model we mean that the droplet is vaporizing in a spherically symmetric fashion where there are no flow effects.

When there is slow the symmetry is usually destroyed. As we will show in the next one if the; if there is something like a buoyancy or if there is or something like an external flow you can see what happens within the droplet. This is basically the outer flow stream line okay, streamlines of the outer flow.

You can see that there is a flow separation that happens at the back of the droplet this is very common if you guys have done fluid mechanics. You know that behind a cylindrical or a spherical obstacle that is always there is always a flow separation. The similar thing happens within the droplet also. In addition however you can see that there are re-circulating flows that are created within the droplet, right, okay.

Similarly, okay, so, you can have another situation in which the flow is very high probably like a little bit of a high velocity flow and you have basically two recirculation right. So, we are not considering these two at all in a spherically symmetric analysis right because there is strong flow inside the droplet, whatever the nature of that flow may be okay.

It can be buoyancy driven flow, it can be imposed flow, it can be natural internal recirculation, it can be imposed recirculation due to the gas phase velocity not it okay. So, that part is very important okay. (Refer Slide Time: 12:42)



We will look at this a little bit later. Now if we want to solve this equation along with combustion okay and this is not to basically scare people away, okay. So, basically this is the droplet this is where the combustion front is basically what we call the flame and this will basically the gas phase flow field, right, okay.

Now you can see it is a multi-component molecular diffusion when you have combustion you have complicated kinetics, the chemical kinetics is extremely complicated. So, you can have as much as 50 species okay, 250 reactions sometimes even more okay. There will be non luminous thermal radiation.

There will be UV emission, so, you have to solve for all those things okay. So, you have a mass conservation, you have multiple species conservation equation that you need to solve and you need to solve it for both phases that is by interesting part right, okay. Then you have the energy conservation equation and the droplet surface you have to take into account the surface regression, the evaporation, condensation, radiative heat addition.

In the droplet interior you have to solve all these equations right. So, it becomes a framework in and all these equations will be coupled with each other based on the interface condition. There will be all matched and the interface or solutions have to match at the interface right and they have to physically make sense also, right.

So, this is a very onerous job as you can understand right. There are a lot of reactions floating around there are a lot of things that you need to solve. There are gas phase, liquid phase and all these things okay. So, how but fortunately the spherically symmetric assumption offers a very good tool right.

It offers a solution without much of a headache right. So, before we go that go there and we will emphasize on something called a d square law and then we will try to prove that how it actually comes. Now some people some researchers actually did some experiments right. Using this kind of a in using this kind of a droplet.

And they what they did was that is suspended or droplet okay. They maintained it probably in a box or something where there is no flow got it, okay. And it is quiescent, so, the spherically symmetric assumption can be applied. When they did the measurement and when they normalize the droplet diameter by the initial droplet diameter square right.

And here you are plotting something like delta T by D naught square right. Forget about that time scale that is not important as of now okay. So, the important part is that D by D naught squared okay. So, when they divided the instantaneous droplet diameter with the initial diameter of the droplet.

So, what they basically did was the proper perhaps they took a camera okay and they basically monitored how this droplet is actually evaporating. So, they were basically calculating the droplet diameter at each time instant okay and after they were done with that okay the square the diameter and they divided it by the initial diameter square right and they plotted it.

What this form was something so, it will be always 1 to 0 right because a droplet on the shrink and this is the evaporation case. Let us not consider condensation okay and they saw that they got a straight line, a linear line all right. They got a linear no matter what fuel they did with okay they always got a linear profile.

So, the linearity of the profile suggests basically that with time okay there is a constant slope. So, this means that D by D naught square okay will be basically some kind of a constant slope. Some kind of a slope kt right, no matter what happens with how you normalize time. So, it shows that there is a constant slope always.

The slope value can vary depending on what droplet you are dealing with but they are; they got a constant slope right. No matter which droplet they actually dealt with okay. So, this was actually proposed in the famous D square law okay D square law by Law and Srignano okay. So they actually proposed that there will be something like a D square law which the droplet has to obey okay in the case of this quiescent spherically symmetric evaporation okay. (Refer Slide Time: 17:04)



Now, what is the pollute to that d square law okay. So, first and foremost it is very curly symmetric okay forced and natural convictions are neglected not just neglected they are assumed to be negligible right. So, this reduction leads to a one dimensional analysis okay because there is no flow.

There is no spray effect that means, there is one droplet is isolated and it is all by itself in an infinite environment. There is no other droplet that is present, so it is very restrictive right diffusion is a rate controlling parameter that means the liquid does not move relative to the droplet center that means there is no internal motion of the liquid, got it.

The surface regresses into the liquid as evaporation happens that means the surface slowly shrinks right that is common right, a droplet should shrink okay. Therefore heat and mass transfer in the liquid occurs only because of diffusion with a moving boundary, but not convection, got it.

So, there are no conviction effects it is basically a diffusion problem there is no flow inside the droplet outside the droplet okay. There is heat and mass transfer only due to diffusion with a moving boundary. So, it is a slowly shrinking droplet okay where there is no flow okay and it is an isobaric process.

It is a constant pressure process we are not varying that the pressure, constant gas phase property okay. That is because the gas phase properties if for example due to evaporation on during the time of evaporation, during temperature changes if a specific heat thermal conductivity diffusion coefficient density etcetera varies.

You will have a different problem, so we are assuming that is a constant property. Now the sixth assumption is the most important one. It says gas phase quasi steadiness what is quasi steady. Steady means steady that means there is no temporal derivative if you take a temporal derivative it will come out to be zero.

Quasi steadiness actually means it is steady with respect to certain time scales right. So, because there is a significant disparity between the liquid and a gas usually that is the case it is of the order of thousands right. Liquid properties at the droplet surface okay at the droplet surface changes at a rate much lower than the gas phase transport processes okay.

This assumption actually breaks down far away from the droplet surface we will see where it breaks down. But the liquid properties at the droplet surface that is the regression rate shrinking of the droplet the temperature and the species concentration if there is a multicomponent thing. Changes at a rate which is much lower than the gas phase transport processes.

So, that means that gas phase changes so fast right. The liquid is almost seeing a steady gas phase outside. It is not able to detect what is going on in the gas phase because it is own movement is very slow right. So, that is one of the key importance, the important parameter that a gas phase with respect to the droplet is actually a quasi steady thing okay.

This will happen if there is a very fast process that is going on and if you are very slow then you would almost see like a steady thing. This happens everywhere like for example with your human eye, right. With a human eye in some cases if I change something very quickly okay you may not be able to see that the difference or subtle difference.

So, why that happens because I am actually not able to respond that fast right. But if my response time is slow and that gas phase response time is really fast then of course you will see the gas phase almost like a quasi-steady kind of a situation okay. (Refer Slide Time: 20:48)

Gas-phase QUASI-steadiness - characteristic times analysis rd environment the gas-phase heat and mass diffusivities,  $g_{
m s}$  and  $\delta_{
m s}$  are of the same order on rate,  $K = -d(D_0^2)/dt$  is of the order of tional hydrocarbon droplet va d atmosphere. Thus, there ratio is of the same order as the ratio of the liquid-to-gas densities,  $\frac{1}{K}$ . It means that gas mass and heat ρ. diffusion occurs much faster than droplet surface regression time. If we further assume that ies of the environment also change very slowly, then during the characteristic gas-pha on time the boundary locations and conditions can be considered to be constant. Thus th gas-phase processes can be treated as steady (time independent because the surface "freezes"), with the boundary variations occurring at longer time scales When (at which value of  $D_{\infty}$ ) this assumption breaks down, i.e. when the diffu the surface regression time? When surface regression characteristic time becomes equal to gas mass/heat diffusion time. i.e. when  $D_{2}^{2}/\delta_{x} = D_{x}^{2}/K_{x}^{2}$  emembering that  $\frac{1}{2} = \sqrt{\frac{p_{x}}{2}}$  must still be valid mass/heat diffusion time, i.e. when  $D_g^2/\delta_g = D_g^2/K_i^2$  temembering that  $\frac{\delta_g}{K} \approx \frac{\rho_{ig}}{\rho_i}$ n the droplet), we can conclude the (it doesn't depend on the distance breaks down when  $\frac{D_{\infty}}{D_{\phi}} \approx \sqrt{\frac{\rho_{ig}}{\rho_{z}}}$ ard atmospheric conditions it breaks down at  $D_{a} \approx D_{g} \cdot \sqrt{\frac{\rho_{ag}}{\rho_{g}}} \approx D_{g} \cdot \sqrt{10^{3}} \approx 32D_{g}$ It means that our model will be valid for the distances less than this o

Now the gas phase quasi-steady. Let us look at some of the characteristic time scale analysis right. Now in a standard environment where there is a gas phase heat and mass diffusivity like for example so the gas phase heat diffusivity is alpha g mass diffusivity is say delta g okay. They are off usually of the same order right. They are usually of the same order which is roughly given as 10 to the power of 0 centimetre square second inverse right, okay.

A typical example is Louis number equal to 1 kind of a situation right okay. Now the droplet regression rate is of the order of this 10 to the power of minus 3 centimetres square per second inversion okay. That the droplet from the experimental data if you if you consider that how it vaporises, is this, is the rate of regression surface regression right.

How the droplet diameter is shrinking right that happens in the order of 10 to the power of -3 centimetre inverse second okay. This happens actually in a time scale of 10 to the power of 0, right. This is for conventional hydrocarbon fuels evaporating in atmosphere right. So, that the ratio of the same order of liquid to gas densities okay.

That is, this is liquid, this is gas okay if you divide these two things like the mass diffusivity divided the regression rate it means essentially that the gas mass and heat diffusion occurs much faster than the droplet regression time, got it. So, the mass diffusion, the mass transfer that is happening from the droplet surface okay is happening at a much, much faster time scale compared to the regression.

And this ratios will actually show you one is 10 to the power of -3, one is 10 to the power of minus; 10 to the power of zero right. And alpha and delta are basically the diffusive time skills right. So, it essentially means that when you have a disparate density difference like

this, this thing Rho liquid by Rho g you can readily see over here okay because this is of the order of thousand.

There is, this is also the order of thousand, right which is very clear from over here. This is on the order of houses is not that so okay? So, it essentially means that the diffusion process is very fast compared to the regression or the surface regression okay. So, that is the gas phase processes can be assumed to be steady okay time independent because of surface almost freezes during that period.

The surface does not see anything else okay with the boundary variations occurring at much longer scales. Now the question then actually remains that what value of this D that means the separation distance right. This assumption actually breaks down that means when the diffusion time is equal to the surface regression time right.

So, when the surface regression time becomes equal to the gas phase heat and mass diffusion time that means this particular relationship when it is actually valid right. So D naught square by k will be equal to what value of D okay. What value of the diffusion length right for which these two time scales will be the same remembering that these two is of the order of thousand always right.

So, once you do that you will find that this D infinity is actually 32 times the droplet diameter okay. So, that means if your distance is less than that okay you are okay. You can assume that this gas phase quasi steadiness but if you are far, far away right then of course this will not be valid, got it.

So, it is 32 times the droplet diameter at that particular distance you can actually have this thing breaking down. That means you cannot apply gas quasi steadiness at that kind of a distance right which is 32 diameters away from the droplet okay which makes kind of sense right because this is basically the scale at which the droplet surface actually regresses.

This is the scale at which the vapour will travel to that kind of a distance. This D infinity is basically a distance right. So, long the distance is small comparable with the droplet diameter okay this assumption is still valid right that means your quasi steadiness is valid. However if the distance becomes very large that means you are trying to compare whether a point which is 32 diameters away from the droplet is actually quasi steady or not.

Then that particular assumption is not valid got it, okay. So, that is roughly a rough figure 32 diameters, so that means our model will be valid for less than that right. So this d square law

is very powerful but it comes with its own caveats that these are the assumptions that we have stated and we have assumed that it is a liquid vapour.

Of course if this difference between the densities of the liquid and the gas if they are very small okay, very, very small okay. Like for example if you go to supercritical type of cases okay. So, there that could be very small okay so in these that particular cases you cannot assume actually quasi steadiness, okay.

So, be prepared to understand what assumptions and why they are applied and why it is valid that is an important part of this argument okay. (Refer Slide Time: 26:21)



So, as some of the additional assumptions are that you have to assume a single fuel species or whatever species that you are assuming is a constant and uniform temperature, droplet surface temperature. That means there is no spatial variation of the droplet surface temperature. There is no temporal variation of the droplet surface temperature as well okay.

And it is basically a gas phase model we would not do anything to the liquid phase okay, it is basically a gas phase model. So, as you will find when I do them the modelling it will be just a gas phase modem. And we will do something to the liquid phase but not that much. The saturation vapour pressure exists at the droplet surface okay that means the vaporization happens to that obeying the Clausius-Clapeyron type of equation okay.

So, the evaporation at the surface is always at a thermodynamic equilibrium okay. So, it produces the vapour which is at a saturation pressure corresponding to the droplet surface temperature okay. So, it is not of not that faster process also that the evaporation is always in a nice thermodynamic equilibrium state okay.

It is not away from equilibrium okay and then there is no effect like Soret and Dofour these effects basically comes due to the second order effect. These are all second-order effects. So, these are neglected usually they are and the radiation effects also are neglected, got it. So, these are the ten assumptions basically okay that goes into the making of the d square law which is experimentally people are validated right.

But however I did not do the math as of now. So, the math will be done and done in the next few slides or the next few journals, entries. But I want you to kind of understand before we go to the next lecture okay. To state that the d square law is valid experimental people have seen it right.

And in order to apply it mathematically there are a set of assumptions that we have said. Two principal assumptions are that there is a gas phase quasi steadiness that is very, very important right the liquid phase does not feel any flow, does not feel anything essentially, it just vaporizes. The saturation vapour pressure exists so it is a thermodynamically equilibrium process always the, that is how the liquid is going into the vapour phase.

There are no second-order effects okay forget about second order effects that may be important in some specific applications not for this okay. And you have a constant and uniform surface temperature right. So, whatever heat that you are supplying to the droplet basically goes towards a latent heat okay.

And of course there is no spatial change in temperature that is because it is a spherically symmetric case right. So, there is no temperature gradient also okay within the droplet as such okay. So, when there is no gradient within droplet can you imagine what that is called, so in you are dumping heat to the droplet.

The droplet there is no internal temperature variation within the droplet not with time but with space right, why is that? That happens because the droplet has got a very high value of thermal conductivity right. So, when is the droplet have got a very high value of thermal conductivity it is called basically any infinite conductivity approximation.

That means whatever heat that you are supplying is only, the in during the initial phase the droplet surface, the droplet core versus the droplet does not see any temperature gradient okay. So, that heat is instantaneously conducted right. Now if this temperature does not vary with time also then that means that whatever heat now you are supplying to the droplet is basically going towards heating or evaporating the solvent, right.

So, these are the few assumptions that will come this is an important assumption and we once we do the math we will see that what is the implication of this particular assumption. Take a look so you remember the constraints that what are the time scale analysis is that we did okay. And remember another important thing is that the diffusion is the rate controlling step okay. There is no conviction effective at all.

And constant properties and other things are fairly obvious. So, we should not be bothered about that okay. So, what we do is that we finish this lecture over here we will then start and derive this is D square law from first principles right and show that why it should come constant basically match the experimental okay.

And there once we write the equations you will see that we will write only gas phase equations mostly gas phase equations with some limiting boundary conditions okay. It will be better if we do it in the next class and complete the droplet spherically symmetric then we will move to more complicated cases okay, thank you.