Heat Transfer And Combustion in Multiphase Systems
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Lecture 18
Interfacial Transport Including Dynamic Behaviour

So, the next we as we said in the last class that we have done the mass we have done the momentum. Now we are going to do the energy equation, right, okay.

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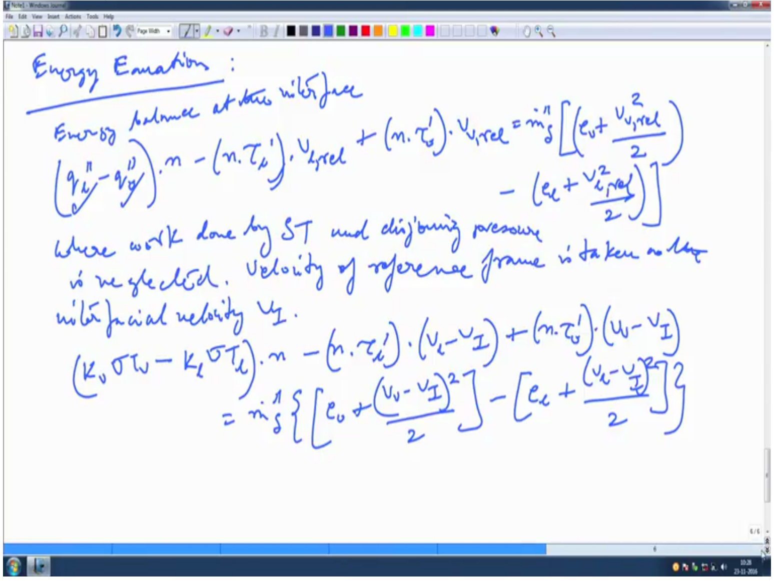

Energy Equation :

Energy balance at the interface

$$\left(q_l'' - q_v''\right)\cdot n - \left(n\cdot\tau_l'\right)\cdot v_{l,rel} + \left(n\cdot\tau_v'\right)\cdot v_{v,rel} = \dot{m}_\delta''\left[\left(e_v + \frac{v_{v,rel}^2}{2}\right) - \left(e_l + \frac{v_{l,rel}^2}{2}\right)\right]$$

where work done by ST and disjoining pressure is neglected. Velocity of reference frame is taken as the interfacial velocity v_I.

$$\left(K_v\nabla T_v - K_l\nabla T_l\right)\cdot n - \left(n\cdot\tau_l'\right)\cdot\left(v_l - v_I\right) + \left(n\cdot\tau_v'\right)\cdot\left(v_v - v_I\right) = \dot{m}_\delta''\left\{\left[e_v + \frac{\left(v_v - v_I\right)^2}{2}\right] - \left[e_l + \frac{\left(v_l - v_I\right)^2}{2}\right]\right\}$$

So, energy balance at the interface ql double prime - qv double prime dot n relative + square by 2 - el okay where if this particular form where work done by surface tension ST and disjoining pressure is neglected okay. And the velocity of the reference frame is taken as the interfacial velocity VI right.

Velocity of reference frame is taken as the interfacial velocity VI okay. So, now this can be further written in this particular form Kv, so, this is basically we are using the Fourier law of heat conduction right. To replace the q this and this right the two flux term. This you have already seen earlier okay. So, this equation can be further written in terms of the enthalpy, let us go to the next one.

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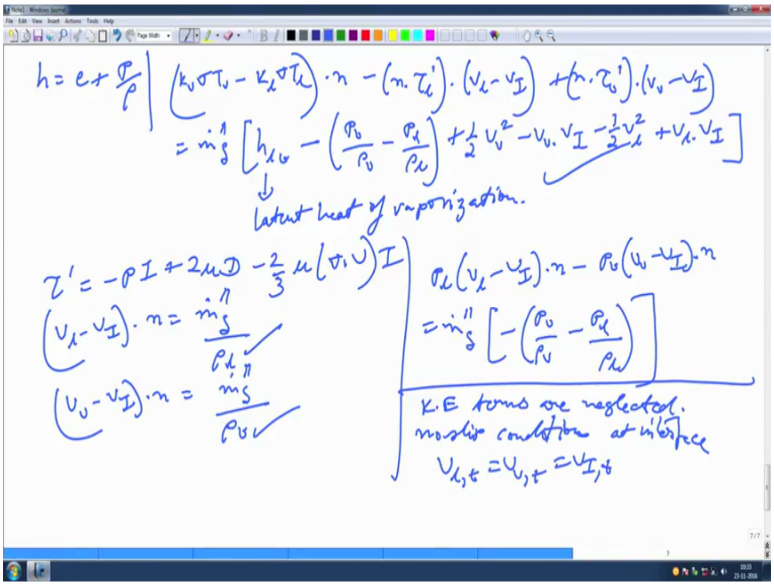

$h = e + \frac{p}{\rho}$

$$(k_v \nabla T_v - k_l \nabla T_l)\cdot n - (n\cdot\tau_l')\cdot(v_l - v_I) + (n\cdot\tau_v')\cdot(v_v - v_I)$$
$$= \dot{m}_s'' \left[h_{lv} - \left(\frac{p_v}{\rho_v} - \frac{p_l}{\rho_l}\right) + \frac{1}{2}v_v^2 - v_v\cdot v_I - \frac{1}{2}v_l^2 + v_l\cdot v_I\right]$$

$\downarrow$ latent heat of vaporization.

$$\tau' = -pI + 2\mu D - \frac{2}{3}\mu(\nabla\cdot v)I$$

$$(v_l - v_I)\cdot n = \frac{\dot{m}_s''}{\rho_l}$$

$$(v_v - v_I)\cdot n = \frac{\dot{m}_s''}{\rho_v}$$

$$\rho_l(v_l - v_I)\cdot n - \rho_v(v_v - v_I)\cdot n = \dot{m}_s''\left[-\left(\frac{p_v}{\rho_v} - \frac{p_l}{\rho_l}\right)\right]$$

K.E terms are neglected.
noslip condition at interface
$v_{l,t} = v_{v,t} = v_{I,t}$

So, in terms of enthalpy, this become; these are basically long equation. So, just take the enthalpy that's the definition for enthalpy right, okay. Now based on this you can write Kv is basically nothing but the Fourier law of heat conduction terms that we have written over here, long equations this is the corresponding work done by the stresses VI = m hlv that is latent heat component Pv Rho v - Pl Rho l + half Vv square VI – half Vl square +Vl into VI.

So, this is basically the latent heat of vaporization, right, okay. Similarly likewise we told earlier that the stress tensor is already predefined right PI + 2 mu D - two-third mu okay to the corresponding identity. So, since the relative velocities of the interface also satisfy this condition Vl – VI into n must be equal to m dot prime divided by Rho l right.

That is the velocity of the interface think about it adjust that how much of the interface actually move that is basically what is taken up by the mass flex. The amount of mass that is going to the other phase okay and similarly we can write it be Vv - VI into n = m dot remember that this is liquid density this is the corresponding vapour density, right, okay.

So, combining all these things together if we write it now Pl Vl - VI dot n - Pv Vv - VI dot n m double dot - Pv – sorry - Pl divided Rho l okay that is the expression, that is a form that you get okay if you do this kind of substitution. Now from the relative velocities of the interfaces you get that okay.

Now if you to simplify the energy equation that you have over here this guy that you have over here. We can make several assumptions because that is a long equation, right. Normally you do not encounter equation, as large as this right. So, how to actually make it appear a little bit simpler. So, there are two ways that you can do it one is that if you neglect the kinetic energy terms, neglect it, right.

That is one way of doing it and if you assume that they are there are no slip conditions at the interface okay. By no slip condition therefore you mean Vlt must be equal to Vvt must be equal to VIt, right, okay, across the interface all this velocity should be the same in the tangential direction that is a no slip basically okay.

So, the energy equation therefore okay, if you apply all these cases okay, there that there is no kinetic energy change, there is no slip etcetera, etcetera.

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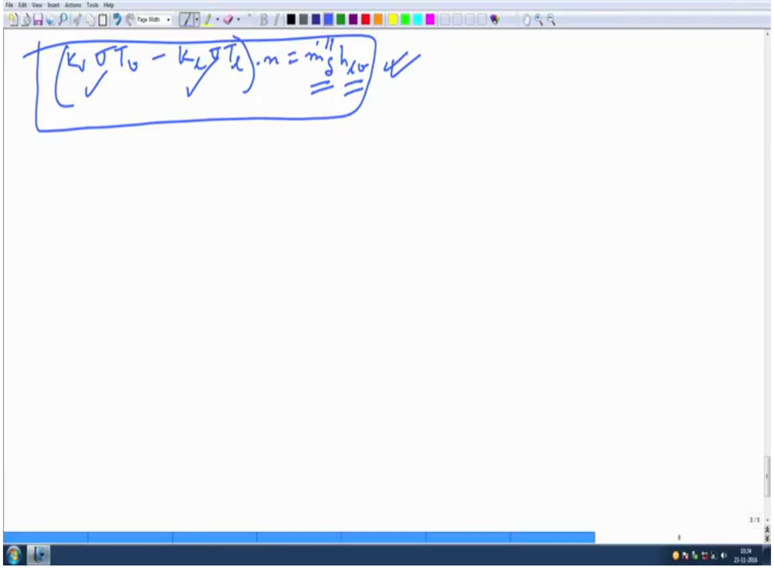

$$\boxed{(k_v \nabla T_v - k_l \nabla T_l)\cdot n = \underline{\underline{\dot{m}''_s}}\, \underline{\underline{h_{lv}}}}$$

It becomes a perfectly manageable form which is given by n, this I argue to do the math; it just requires the substitution dot n that is it, right. So, it is basically two heat conduction terms that you have right. And the corresponding mass transfer due to evaporation or whatever and the corresponding latent heat okay.

So, we can arrive at this particular form of this particular equation just by making those assumptions over there okay. So, this is also widely used in fact in most of the cases we will use this right. So, this is the conduction is equal to whatever the mass that is taken away right, by this okay.

Now you can compare, you can make, you can have analogies in which you can compare the order of these two terms. And you can make assumptions that which gradient to neglect which is gradient; to take okay and things like that okay. But other than that, that is what you are supposed to get okay.

Now before we go to the species equation okay and species equation comes with it a little bit of the, how the phase change actually happens okay. Let us look at an interesting term over here. So, we will come to this thing a little later. But before that I wanted to cover the;

because we are still in the momentum part of the equation and momentum and energy we have covered.

Let us look at some of the dynamic behaviours of these interfaces because we have done this momentum balance across the interface and things like that. Let us look at the dynamic behaviour of the interfaces before we go on to the species balance and then we talked about how evaporation actually happens, right.

How the slopes matter’s, just now we established that there is a to conduction terms basically right, that you saw is equal to whatever is the mass that is that actually evaporates right from one phase to the other, right. If we neglect all the kinetic energy terms and everything okay, which is very normal practice to neglect the kinetic energy terms?

Because other terms are so much more significant right, so but before doing that let us do the dynamic part of the interface okay which is what we are going to do in this few slides.

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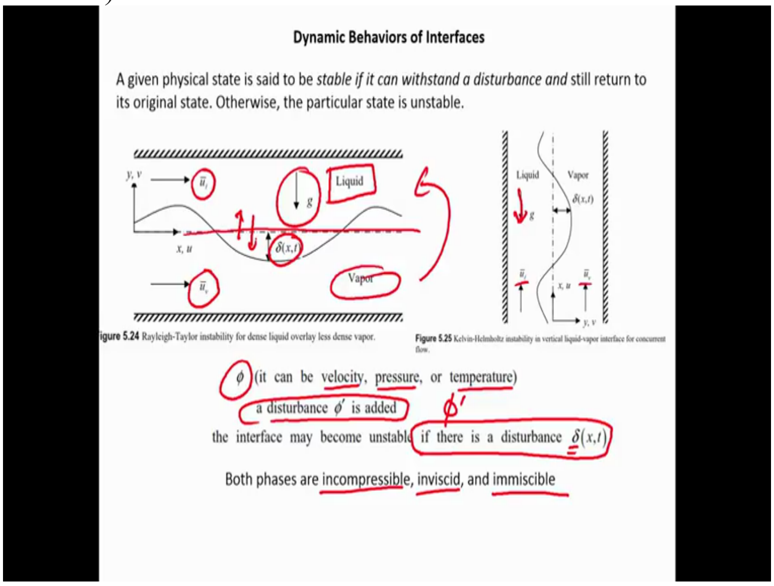

Dynamic Behaviors of Interfaces

A given physical state is said to be *stable if it can withstand a disturbance and* still return to its original state. Otherwise, the particular state is unstable.

igure 5.24 Rayleigh-Taylor instability for dense liquid overlay less dense vapor.

Figure 5.25 Kelvin-Helmholtz instability in vertical liquid-vapor interface for concurrent flow.

ϕ (it can be velocity, pressure, or temperature)

a disturbance ϕ' is added

ϕ'

the interface may become unstable if there is a disturbance $\delta(x,t)$

Both phases are incompressible, inviscid, and immiscible

So, these kinds of examples are seen everywhere, right. Say for example you have a vapour right and you have a liquid right okay. So, basically by vapour and liquid means is a there is a very large variation in density okay. Vapour liquid density is almost like thousand times right. So, what happens is that there is a layer of liquid which is sitting over a layer of vapour, right.

Let us look at the problem in this particular fashion right. There is a layer of liquid which is sitting over a layer of vapour and both of these two are actually moving right. So, here there is a velocity which is uv which is the vapour phase velocity. There is a velocity which is ul okay this average velocity which is in a liquid phase okay.

And there is an interface which basically separates the two, got it, okay. Let us not talk about evaporation and all those things for the time being okay. Let us keep this problem very simple now what happens is that our physical state of this interface okay is said to be stable if it can withstand a disturbance and return to its original state okay.

So, if the statement is like that means if this is the interface and I give it up give it a spanking let me if I just perturb up the interface and I create this perturbation which is delta, right. It is basically you can consider it to be like a string right and I am basically just clipping a part of the string, right.

So, it naturally will oscillate, now if this oscillation dies down and it returns to its original flat configuration which is this configuration right, then we will call that this is a stable situation, right. So, nothing happens you impose a disturbance that disturbance decays away and so it is a stable configuration okay.

So, in this particular case gravity is appearing this way, the horizontal pipe and you have a liquid and vapour both of which are actually moving right very common problem, okay, need not be always liquid vapour, it can be disparate density fluids also. On the other side of it, if we, if I just take this and if I just rotate the whole thing okay.

So, what will happen is that gravity is now fabering appearing like this correct. And you have a liquid velocity and you have of vapour velocity same thing interface is now vertical instead of horizontal. And again you perturb it and see how the perturbation actually evolves, got it, okay. So, here the parameter that we are interested in can be anything.

This is Phi it can be velocity, it can be pressure, it can be temperature, it can be species also whatever okay. And disturbance of this magnitude which is given by Phi prime, Phi with a prime, right. That is actually added that means when there is a velocity u we are adding a perturbation which is u prime, right.

So, and the interface gets disturbed by this magnitude which is given by delta okay. So, the both phases we are assuming they are incompressible. They are invicid so that we get rid of all those all those viscous stress terms. And it is not it is immiscible that means they do not mix with each other.

That means there is no diffusion that happens okay in terms of one species going into another, right. So, you can you can think about it in this particular way right okay.
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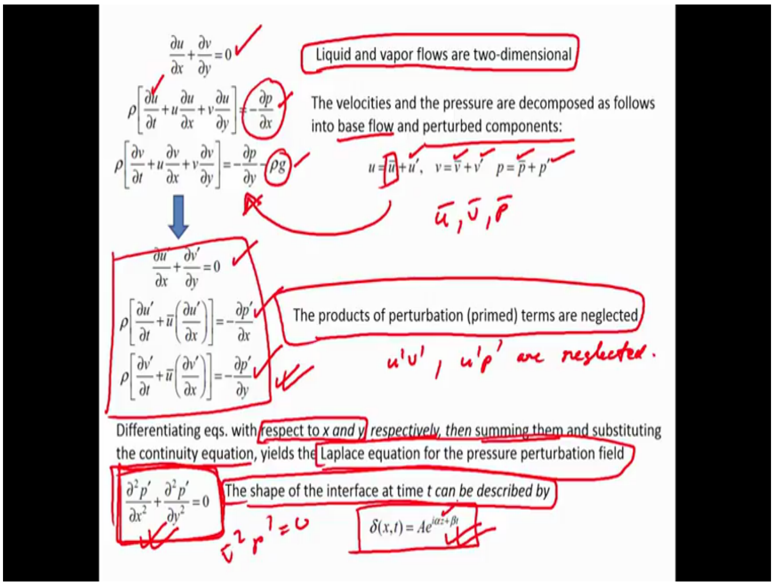

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Liquid and vapor flows are two-dimensional

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}$$

$$\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial p}{\partial y}-\rho g$$

The velocities and the pressure are decomposed as follows into base flow and perturbed components:

$$u=\bar{u}+u', \quad v=\bar{v}+v' \quad p=\bar{p}+p'$$

$\bar{u}, \bar{v}, \bar{p}$

$$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}=0$$

$$\rho\left[\frac{\partial u'}{\partial t}+\bar{u}\left(\frac{\partial u'}{\partial x}\right)\right]=-\frac{\partial p'}{\partial x}$$

$$\rho\left[\frac{\partial v'}{\partial t}+\bar{u}\left(\frac{\partial v'}{\partial x}\right)\right]=-\frac{\partial p'}{\partial y}$$

The products of perturbation (primed) terms are neglected

$u'v'$, $u'p'$ are neglected.

Differentiating eqs. with respect to *x and y respectively, then* summing them and substituting the continuity equation, yields the Laplace equation for the pressure perturbation field

$$\frac{\partial^2 p'}{\partial x^2}+\frac{\partial^2 p'}{\partial y^2}=0$$

$\nabla^2 p' = 0$

The shape of the interface at time *t can be described by*

$$\delta(x,t)=Ae^{i\alpha z+\beta t}$$

Now the liquid and the vapour flows are essentially two dimensional in nature that is another set of assumptions okay. So, based on that what we can do is that we can write three equations this is the mass and the two momentum right. Why the viscous term is not there, this is because we have neglected viscosity.

We have assume that it is invicid in nature, right. So, what we have, you have the you variation of you this is the convective derivative which is basically the inertial term. And this is the corresponding pressure we have added gravity in the first case, okay. That is because of; in one case in a y momentum equation gravity enters into the picture.

So, if you look at this particular equation this is a y-axis so gravity is acting along the y-axis correct, okay. So, that is why the gravity term is already there, right. Now we are going to decompose the velocity. So, there are basically three variables right, u, v, p right. So, we are going to decompose the velocities and pressures into a base flow and some perturbed components, right.

So, it is almost like a team to the Reynolds decomposition that we do okay in your turbulent flows, right. So, what happens is that you have a u bar which is a mean or what is called the base flow and then there is a perturbation component, v is given by v bar and a perturbation component. P is given by p bar and a perturbation component.

So, everywhere there is that perturbation component, right. So, you substitute all of these things over here in this set of equations, right. So, what you get that is the set of equation and you recall that your u bar and your v bar and your p bar all satisfied the Navier stokes and the mass and the continuity equation, right, okay.

So, based on this substitution you now arrived at three additional sets of equations, right, okay. The first one is basically nothing but du prime dx v prime dy that is the continuity version of the perturbed component, okay. You can think about it like that okay. The next component is this, the next component is that.

Now the products of the perturbation and why does your g actually drops out that is because it does not have a perturbation, right, okay. And we have assumed that the densities of course fixed okay. So, there is no perturbation on the property right that can also happen. But here we are assuming that the perturbation is not happening on the density.

And the product of the perturbation that means the prime terms are neglected that is u prime, v prime is neglected. Similarly if there are quantities like u prime, p prime these are all neglected okay, so, they are all neglected right, okay. Now we take this set of equation and we differentiate it with respect to x and y right, respectively, okay.

And then we sum them and we substitute the continuity equation. This is yields what we call the Laplace equation for the pressure perturbation filed. If you do this you get this particular equation which is basically nothing but the pressure perturbation or this is Laplace you can see that this is basically this, right, okay.

It is the Laplace of the pressure perturbation okay. It is not the pressure it is the pressure perturbation, so, that is what we have done over here. So, basically what we have done is that we have basically taken the derivative with respect to x and y okay and then we have substituted the continuity we have sum them and we get the Laplace pressure field, okay.

So, this is the pressure field or the perturbed pressure field that you get right and the shape of the interface that we showed at any particular point of time is assumed to follow an expression like this. This is very typical linear stability type analysis that people are normally familiar with, right.

So, what this had, it has basically as z component that means the interface fluctuates in the z direction. If you look at it okay so that is the interfacial fluctuation and there is that there is a time component okay.
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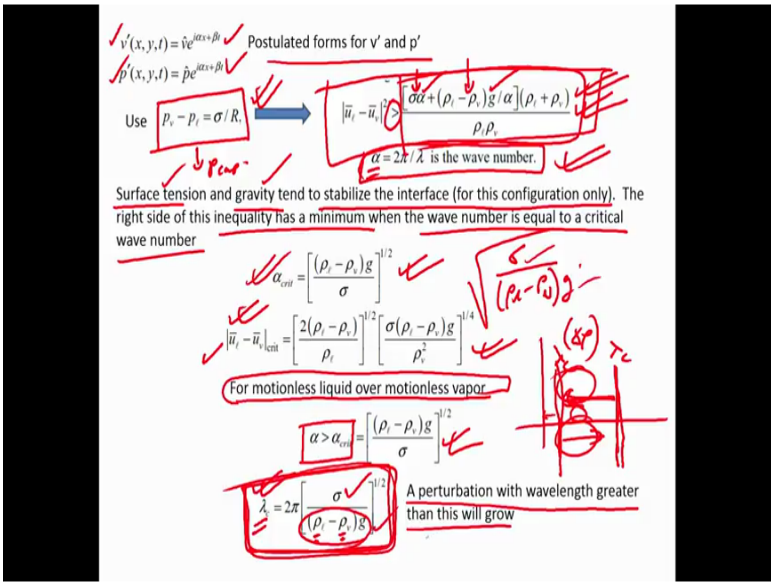

$v'(x,y,t) = \hat{v}e^{i\alpha x+\beta t}$

$p'(x,y,t) = \hat{p}e^{i\alpha x+\beta t}$

Postulated forms for v′ and p′

Use $p_v - p_\ell = \sigma / R_c$

p_{cap}

$$|\bar{u}_\ell - \bar{u}_v|^2 > \frac{[\sigma\alpha + (\rho_\ell - \rho_v)g/\alpha](\rho_\ell + \rho_v)}{\rho_\ell \rho_v}$$

$\alpha = 2\pi / \lambda$ is the wave number.

Surface tension and gravity tend to stabilize the interface (for this configuration only). The right side of this inequality has a minimum when the wave number is equal to a critical wave number

$$\alpha_{crit} = \left[\frac{(\rho_\ell - \rho_v)g}{\sigma}\right]^{1/2}$$

$$\sqrt{\frac{\sigma}{(\rho_\ell - \rho_v)g}}$$

$$|\bar{u}_\ell - \bar{u}_v|_{crit} = \left[\frac{2(\rho_\ell - \rho_v)}{\rho_\ell}\right]^{1/2}\left[\frac{\sigma(\rho_\ell - \rho_v)g}{\rho_v^2}\right]^{1/4}$$

$(\Delta\rho)$ T_c

For motionless liquid over motionless vapor

$$\boxed{\alpha > \alpha_{crit}} = \left[\frac{(\rho_\ell - \rho_v)g}{\sigma}\right]^{1/2}$$

$$\lambda_c = 2\pi\left[\frac{\sigma}{(\rho_\ell - \rho_v)g}\right]^{1/2}$$

A perturbation with wavelength greater than this will grow

Similarly the postulated forms of v prime and p prime is also taken as a similar form, right. Like we took in the case of delta okay we took it in the case of delta we have done a very similar thing for v prime and p prime okay. These are postulated forms okay we are postulating that this will be the form.

And then we use the corresponding Laplace this is a two-dimensional filed okay only one curvature is needed right. So, the p capillary pressure is what we are using over here, okay. So, based on these postulated form one can show and this is not a linear stability analysis class. We are not going through the detailed math of how we are arriving it, arriving at this okay.

What we get is that an expression like this that means the square of the velocity difference between the two phases not the perturb but the mean velocity square. And you have taken a modulus of that that means you have taken a minus taken absolute value of that and squared it okay. So, it does not matter which one is positive or negative right.

Which one is slow or fast correct, okay, so that is given by this particular expression like? Let us look at it term by term there is a term involving Sigma which is the surface tension. There is a term involving the gravity right, okay. And this alpha that we have written over here is called the wave number okay.

It is 2 pi by lambda, lambda is the wavelength. So, alpha is the wave number. Now the surface tension and gravity so, this is such a tension term this is gravity term right that is what we told. So, first of tension and gravity tends to stabilize the interface for this configuration okay. So, if you look at the configuration for their; for this particular configuration right.

Both gravity as other surface tension tries to stabilize the interface that means it tries to bring the interface back to its original configuration which is flat right, okay. So, both gravity and this tries to do it okay. So, the right side of this and remember this is also an inequality form right so this must be greater than that okay.

So, the right side of this inequality has a minimum value, right side means this site has got a minimum value when the wave number is equal to the critical wave number right. So, if you minimize this part the right hand side of this equation right okay. What will happen is that this has a minimum and that minimum okay that minimum alpha because all other quantities okay, so, are fixed right. So, only alpha is the variable that you have here.

So, basically you minimize this quantity in terms of alpha right okay. So, that critical alpha is given by this particular parameter which is nothing but Rho l - Rho v that is the difference in density of the two phases's right and multiplied by g divided by Sigma and the root over of that right, okay. This sounds familiar to a certain audience; you can think about it and give the answer in the next class.

That what does this actually resemble right, this sigma divided this you can write it like this Rho l – Rho v into g and root over of that okay. This is an interesting quantity okay that is associated with this okay. So, what is this basically means that it is basically stigma and the corresponding gravity essentially right, okay.

Similarly when you substitute this alpha critical over there okay in the expression you get a critical difference of a velocity differential, right, okay. And that is given by this particular expression okay. For motionless liquid over motionless vapour that means when the both the phases are actually static right, okay.

So, this was it was it is actually moving right. Now if the vapour and the liquid phases are all motionless in nature then alpha must be greater than alpha critical that means it must be greater than this particular quantity. Lambda c that is the critical wavelength okay is given by this particular expression which is nothing but this particular length scale that we have here.

And this perturbation, a perturbation which has a wavelength greater than this lambda c will actually grow. That means when you have a motionless liquid sitting over motionless vapour what will happen is that if you perturb it okay with up with an amplitude okay which is greater than this particular critical wavelength okay. This will actually grow right.

So, look at one particular thing a perturbation greater than this wavelength that is the key clause over here right. So, when this Rho l and Rho v becomes very close to each other right that means you are dealing with water and something which is a little bit more denser than water right. What will happen in that particular case, right?

This lambda c will be a very large quantity right. So, in order to perturb that particular interface you need an exceedingly high amplitude right, because that amplitude space is very large lower than this would not do anything right. So, the perturbation needs to have a huge amplitude okay, it is a long amplitude okay, a long with wavelength essentially, right.

Similarly on the other hand if this density difference is huge typically what happens between liquid and vapour right, it is a huge density difference right. This quantity is very large right. So, the wavelength space if you look at it, so, previously if this was the critical lambda c right. The critical lambda c is pushed back right to the short-wavelength regime correct.

When you actually dick have a large density differential which is delta Rho correct. When it gets pushed over here to this side okay any wavelength which is greater than this will make the interface unstable. Previously what happened when the interface was here right when the critical wavelength was here any disturbance in this region will not be able to create de-stabilize the interface.

On the other hand either if the density difference is too large okay essentially what we will have, you will have the disturbances okay for of very short wavelengths also can actually grow right. So, that is a very key takeaway point, right. Even if you do not remember anything if you just remember this okay. This actually shows that if the perturbation for a motionless configuration.

If the perturbation is greater than this amplitude it will grow and you should always remember that surface tension and gravity here tries to stabilize the interface. So, we have to fight against to them essentially, right, and that is what exactly what you are doing over here by making your wavelength greater than that right.

So, this particular thing is called it is a got a celebrated name it is called the Rayleigh-Taylor instability okay. It is called the Rayleigh-Taylor instability, so you can have a recap we actually have a flat interface okay with two fluids moving on both sides. We add a perturbation component to the whole thing right we decompose it into two components I mean in into a mean flow or a base flow.

And a perturbation part and then if we differentiate the equations and do some math basically it is just a tool and it is very standard these days okay. You get a Laplace of the pressure perturb of pressure field. And then you assume forms of delta, the pressure field and the velocity field and then you add the interfacial conditions which is basically nothing but the capillary pressure right.

And then you can show that what the velocity difference should be right, okay, for flow to get unstable right. And for motionless configuration okay the perturbation needs to have an amplitude which is greater than the greater than this, got it, okay. So, Rayleigh-Taylor instability is seen in many cases also not just in one case it is seen in multiple fields okay.

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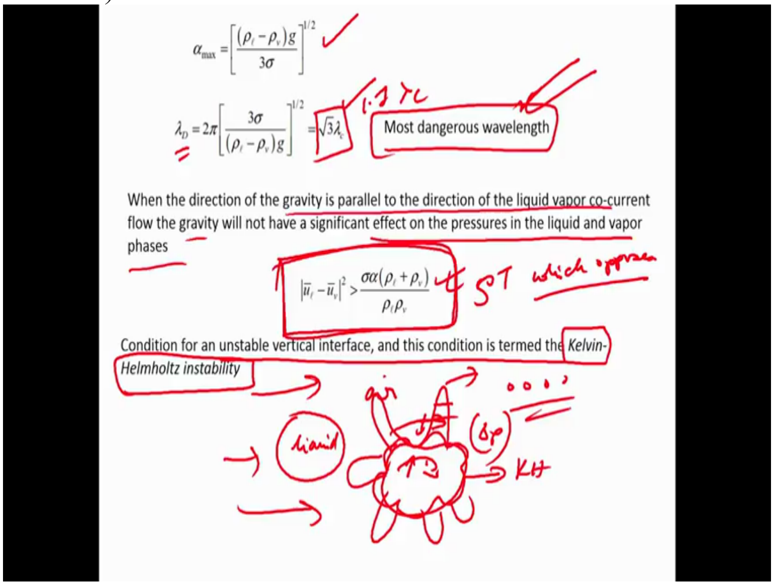

$$\alpha_{\max} = \left[\frac{(\rho_l - \rho_v)g}{3\sigma}\right]^{1/2}$$

$$\lambda_D = 2\pi\left[\frac{3\sigma}{(\rho_l - \rho_v)g}\right]^{1/2} = \boxed{\sqrt{3}\lambda_c}$$

Most dangerous wavelength

When the direction of the gravity is parallel to the direction of the liquid vapor co-current flow the gravity will not have a significant effect on the pressures in the liquid and vapor phases

$$\boxed{|\bar{u}_l - \bar{u}_v|^2 > \frac{\sigma\alpha(\rho_l + \rho_v)}{\rho_l \rho_v}}$$

Condition for an unstable vertical interface, and this condition is termed the *Kelvin-Helmholtz instability*

Now this is Rayleigh Taylor okay also we can define something called see that the right-hand side you can also maximize it right. So, you get a maximum alpha and then you get an lambda c which is called the most dangerous wavelengths, basically the wavelength that grows the fastest right.

So, that is actually is root over 3 that is approximately 1.7 of lambda c okay of the critical. So, It is about 1.7 times if you are perturbing it at that amplitude the perturbations will grow okay very rapidly. Now in this case was the gravity was the gravity was up well it was perpendicular to the flow direction right.

In the case of the other configuration the vertical slow you actually have gravity which is in the direction of the flow. In that particular case you can show this okay and there it gravity does not have a significant effect on the pressure of the phases. So, there you have this particular expression is satisfied okay.

So, it is basically ul - uv square on the on this side is only surface tension now which plays a role right okay which opposes okay. So, a condition for an unstable interface is there for this particular type of interface which is vertical and gravity is actually in the direction of the flow, it is basically called Kelvin Helmholtz instability okay.

So, one is Rayleigh Taylor instability, this is the Kelvin Helmholtz instability both are very similar. It is just a configuration difference which way the gravity is pointing okay that is what actually leads to this kind of configurations right. So, you get an idea that when the; you have an interface okay it is basically the force balance across the interface.

And not only just the static force balance right it is also the dynamic behaviour of the interface that is very important. Because when you do heat and mass transfer analysis specially say you take the configuration when a droplet is a liquid droplet right. Now, when it is put it to a flow of air right what happens?

The droplet actually should get perturbed this is an interface between droplet and the liquid and the air right okay and there is a flow there is a recirculation inside the droplet there is a flow outside right. And there is a huge difference in density this delta Rho is large right. So, naturally you will develop waves on the droplet surface right.

And these waves can actually lead to this caged type instabilities that is in fact what you see in your atomization type of studies that there are surface waves that are created and this actually leads to ultimately leads to break up. Because when these waves actually grow right they will grow right because that is the most dangerous wavelengths right.

So, they will grow and then they can actually break okay. So, they can actually atomize okay and give rise to small satellite droplets, got it, okay. So, this is an important exercise that is because we showed the dynamic nature of the interface now. We showed the static nature of the interface okay.

And we covered all the equations the relevant equations that goes with it okay and after so in the next class what we will try to do is that we are going to try to see that how the species valve now can be entered into the picture and we look at the mass transfer. The mass and heat transfer side of things this we have looked from the force balance point of view that how the;

So, these interfaces actually do not actually evaporate this example that I showed over here okay. So, in the next class we will try to do the species balance and move from there, thank you.