Heat Transfer And Combustion in Multiphase Systems
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Lecture 17
Interfacial transport

So, last class we started to do explaining that what is the disjoining pressure, what is the value of the disjoining pressure and if certain liquid actually wet's a particular solid wall what are the different regimes.

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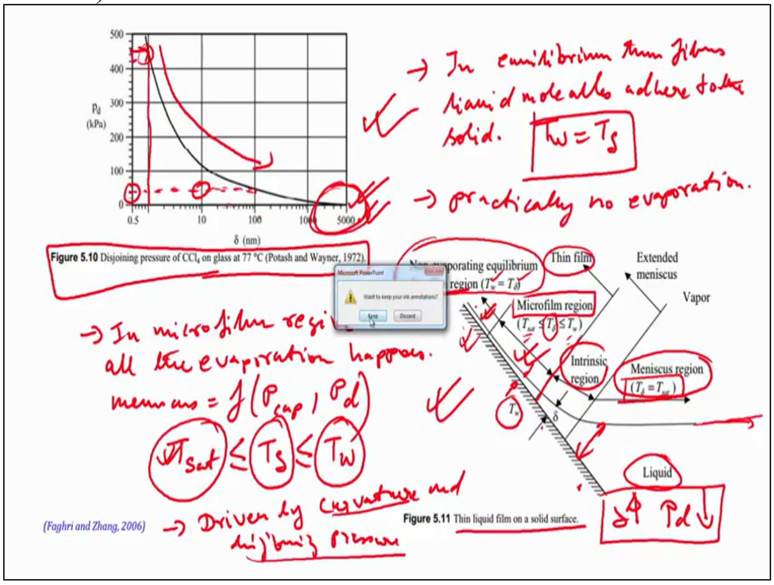

Figure 5.10 Disjoining pressure of CCl_4 on glass at 77 °C (Potash and Wayner, 1972).

Figure 5.11 Thin liquid film on a solid surface.

(Faghri and Zhang, 2006)

→ In equilibrium thin films liquid molecules adhere to the solid. $T_w = T_s$

→ practically no evaporation.

→ In microfilm regi all the evaporation happens.

meniscus = $f(P_{cap}, P_d)$

$T_{sat} \le T_s \le T_w$

→ Driven by curvature and disjoining pressure

$\delta\uparrow \; P_d\downarrow$

So, if you recall this particular part of the presentation what we had was that there was a liquid which wetted a wall over here, this is the wall okay. And this is basically after this is a liquid basically continuous. So, it is like a wetting it is like completely wets the surface. It is like an extended meniscus.

So, up to this region from this region onwards it is called the thin film region okay. So, thin film region, we already established what the thin film region is that what is the magnitude of the thickness of the film? Thickness that is needed, so, this is the thin film region. Now in the thin film region there exists, a non evaporating equilibrium thin film region.

Where the wall temperature is the same as the liquid temperature okay, then there is a micro film region okay were both the capillary pressure as well as the disjoining pressure plays a major role okay. Here the film temperature is greater than a saturation temperature but lower than the wall temperature.

So, this is called the microfilm region here the flow is basically are driven by both capillary as well as by the disjoining pressure okay. And so it is driven by the curvature as well as by the disjoining pressure that is what we wrote okay. Then there is a transition region basically and then there is a thick film region which is the meniscus region where the temperature of the film is the same as a saturation temperature okay.

And here because of the large thickness of the film okay the disjoining pressure does not play, much of a role okay. And we also establish the fact that as the thickness of the film goes up the disjoining pressure effect comes down. And if you recall this graph over here you can easily see that when the film thickness is about 5000 nanometer which is 5 microns.

The human hair is about 100 micron, so, it is basically one-fifth of the rather one by 20th of a human hair okay. So, that is the thickness that we are talking about when the thickness of the film is like that the disjoining pressure is basically practically equal to zero that means it is virtually negligible okay. So, in the microfilm region most of the evaporation actually takes place, it is ideally suited for that okay.

So, but in another case okay another interesting question that that pops up while we are doing this analysis is that what is the nature of the saturation vapour pressure over a curved interface right. So, this is a curved interface that what we see over to her right what is the saturation pressure looks like okay, how it is influenced okay. So, let us look at that in details okay.

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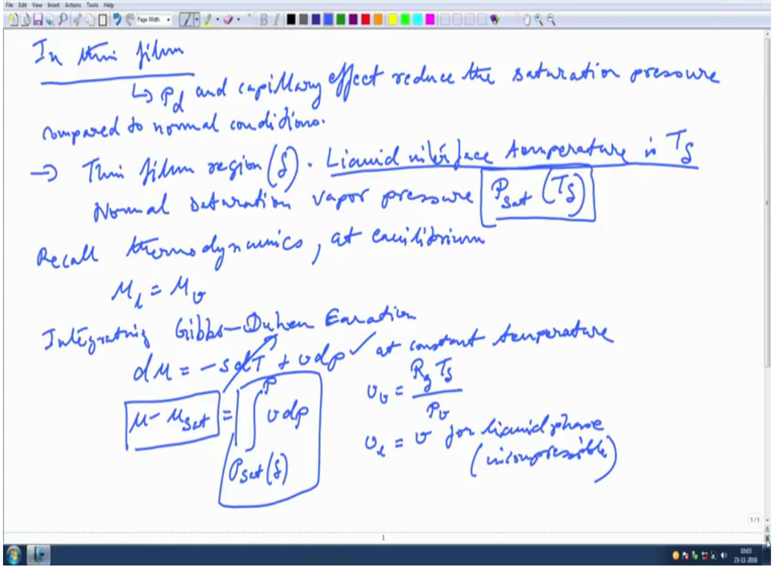

In thin film

↳ P_d and capillary effect reduce the saturation pressure compared to normal conditions.

→ Thin film region (δ) · Liquid interface temperature is T_δ

Normal saturation vapor pressure $\boxed{P_{sat}(T_\delta)}$

Recall thermodynamics, at equilibrium

$\mu_l = \mu_v$

Integrating Gibbs–Duhem Equation

$d\mu = -s\,\cancel{dT} + v\,dp$ ✓ at constant temperature

$\boxed{\mu - \mu_{sat}} = \boxed{\int_{P_{sat}(\delta)}^{P} v\,dp}$

$v_v = \frac{R_g T_\delta}{P_v}$

$v_l = v$ for liquid phase (incompressible)

So, let us look at the journal part now okay. Now in the thin film region, so, our question that we are going to answer is that what is the nature of the saturation pressure, right. How the saturation pressure is changed across the interface whether there is a separation of the saturation pressure or not okay.

So, my initial statement will be the disjoining pressure and the capillary effect okay, reduce the saturation pressure saturation pressure okay compared to two normal conditions okay. This is a statement and we are going to prove this particular statements right. So, what we are saying is that what the disjoining pressure and the capillary pressure has an effect on the saturation pressure, right, okay.

Now the thin film region we are only talking about the thin film region over here delta okay. The liquid interface temperature okay is T delta right. So, the normal saturation vapour pressure will be normal saturation okay vapour pressure will be pressure will be T sat whatever is the saturation pressure corresponds to that, corresponding to that temperature.

And you can almost look it up from the thermodynamic table right. You know the temperature you can find out what is the corresponding saturation pressure. So, we are stating that a liquid interface temperature is T delta and the corresponding saturation pressure is T sat T delta okay. And we are going to say that whether this pressure okay the actual saturation pressure whether it is different from this or not okay.

So, recall if you do a little bit of recall of your thermodynamics okay. At equilibrium remember we are only talking about equilibrium over here, we are not talking about as receding or an advancing liquid front okay. We are not talking about that mu l is the same as mu v right, that is what we proved earlier, got it, okay, at equilibrium, right.

So, the chemical potential of the liquid and the vapour phases should be the same. Now integrating the Gibbs equation okay which is basically nothing but this and we integrate it at constant temperature, got it, okay. So, this is a Gibbs equation okay once you integrate it at the constant temperature, this term basically vanishes because it is a constant temperature integration.

So, it is mu - mu sat = P sat at whatever the delta is P Vdp, correct. So, this is basically a the mu - mu sat will integrate it okay that is basically integrating this term you get P sat to P okay. That is the arbitrary limit of integration okay. Now there are two other things that we would like to mention one is that your, Vb that is the specific volume right of the vapour phase is given by Rg sorry Rg T delta divided by the corresponding Pv correct, okay.

Now that is the volume of the vapour phases like Vl on the other hand is equal to V okay for the liquid phase. As you mean whether liquid is incompressible, got it, okay.

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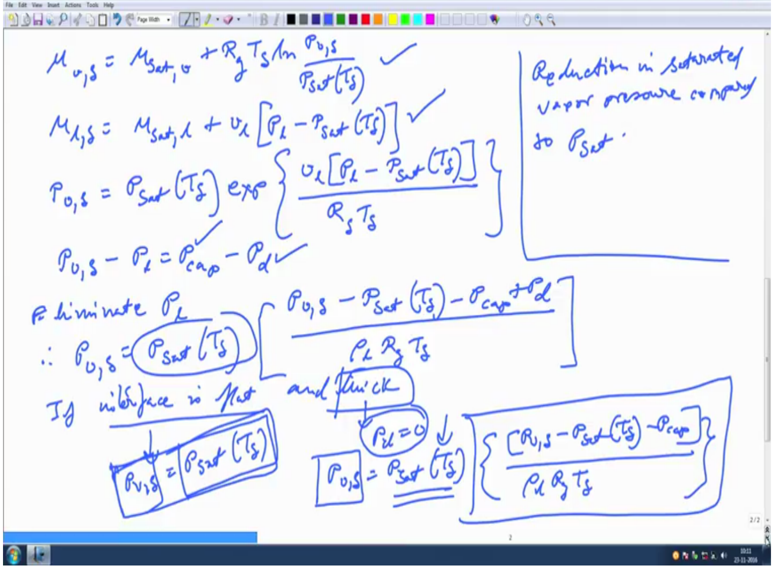

$$\mu_{v,\delta} = \mu_{sat,v} + R_g T_\delta \ln \frac{P_{v,\delta}}{P_{sat}(T_\delta)} \checkmark$$

$$\mu_{l,\delta} = \mu_{sat,l} + v_l \left[P_l - P_{sat}(T_\delta)\right] \checkmark$$

$$P_{v,\delta} = P_{sat}(T_\delta) \exp\left\{\frac{v_l\left[P_l - P_{sat}(T_\delta)\right]}{R_g T_\delta}\right\}$$

$$P_{v,\delta} - P_l = P_{cap} - P_d \checkmark$$

Eliminate P_l

$$\therefore P_{v,\delta} = \boxed{P_{sat}(T_\delta)}\left[\frac{P_{v,\delta} - P_{sat}(T_\delta) - P_{cap} + P_d}{\rho_l R_g T_\delta}\right]$$

If interface is flat and thick

$$\boxed{P_{v,\delta}} = \boxed{P_{sat}(T_\delta)}$$

$$\boxed{P_d = 0}$$

$$\boxed{P_{v,\delta}} = P_{sat}(T_\delta)\left\{\frac{\left[P_{v,\delta} - P_{sat}(T_\delta) - P_{cap}\right]}{\rho_l R_g T_\delta}\right\}$$

Reduction in saturated vapor pressure compared to P_{sat}.

So, now we can write basically two sets of equations for the liquid and the vapour. So, the mu of the vapour phase okay add delta is equal to mu sat at delta at sorry sat v + Rg T delta ln Pv delta divided by the P sat T delta, got it that the first set okay. Now the second set mu l into delta is equal to mu sat l + Vl Pl – Pl – P sat T delta, got it.

These are the two sets of equations right, in one case Vl is incompressible. So, it is constant in other case it is not, so, we are integrating it, right. If you have not understood it let us go through it once again. So, mu - mu this is a common equation, right, okay. So, this integral is a common equation now the vapour volume as you can see is a function of the vapour pressure, correct. That comes from the; so, this particular equation is well known, right.

On the other hand for the liquid phase okay. We are assuming it is incompressible. So, it is basically a constant okay. So, you integrate this equation one for the liquid phase and one for the vapour phase. Once you do that okay you get two sets one is for the vapour okay where do you have a change in that volume, right. The volume has been substituted by the corresponding pressure form.

And in the other case you get it just simple in terms of Vl because Vl is constant you can take it out okay. So, it is a very simple integration. So, now, so, you have now two sets of equations over here okay. Now let us try to do a combination of the two now. It is P sat T delta into exponential Vl Pl – P sat T delta divided R delta T delta, got it.

When you combine the two forms that is what you get okay. So, the pressure difference now in the liquid and the vapour phase okay can also be given as right because this is the capillary pressure, this is the disjoining pressure. Capillary pressure comes from the curvature effect thus as we said for a flat interface this should have been the same right whereas the disjoining pressure comes because of the thinness of the liquid film, got it.

So, using these equations we eliminate Pl okay. So, once you eliminate Pl what you get is P sat T delta and you can work out the math, the details, the algebra part that is Pd, got it, okay. So, that is the net expression that you get okay. Now if the interface is flat okay and thick, so, the interface can be both flat and thick right.

So, it is like you have a bucket full of water that is like a typical interface which is very thick, right, okay. So, in that particular case when the interface is thick you get Pd = 0, right. If Pd = 0 only if it is a thick interface but it is not necessarily flat you get Pv delta = P sat T delta and you have the expression which is basically Pv delta – P sat T delta - P cap okay divided by Rho l Rg T delta, got it, okay.

So, there is a reduction and when the interface is flat you know that all the terms basically vanish okay. When the interface is flat and thick you basically have your P delta is a same as your saturation pressure okay, when both of these things happens, right. So, in general what you see is that the vapour pressure, saturation vapour pressure at the interface is actually lowered compared to the P sat, right.

So, what we can say over here is that there is a reduction in saturated vapour pressure compared to P sat, right. So, compared to P sat okay there is a reduction this is P sat okay compared to P sat there is a reduction in the saturation vapour pressure, got it, okay because of this factor that is it seemed right.

So, what does the curvature do the curvature if there is a disjoining pressure and the capillary pressure there is a definite reduction of the saturation pressure near the interface. Even if there is no disjoining pressure there is a reduction in saturation pressure at the interface because of the normal curvature.

And if the interface is however flat you however recover the old form that means your saturation pressure and your normal saturation pressure which is given by this is the same as the saturation pressure across the; on the other side of the interface, right, okay. So, here therefore in this particular part of this particular lecture.

What we have done is that we have covered that how the interface actually what will be the pressure difference across the interface right and we also know now which part of the interface should evaporate, which part of the interface should not evaporate, which will become what kind of a curvature will actually govern the flow.

And we have defined and very interesting quantity which is the disjoining pressure over here. So, take away from this particular section is Pd and P cap okay. And these are very, very important in many, many applications okay, especially for thin film dynamics okay. When you do materials processing also like for example polymer, thin polymer films okay.

You would have a lot of role of the disjoining pressure over there okay. We have not gone into the intermolecular perspective except giving an idea that this is governed by the Lenard Jones potential 612 or you can adopt a form like that. But apart from that we have not gone into the detail for working out okay.

But we have said it arises out of the dispersed phase as well as the electro static forces right. But that is not needed also because this is not a full-fledged statistical mechanics class okay but what we have done over here is to provide you with an overview but how to attack okay a problem like this where you can have an effect of the disjoining pressure okay.

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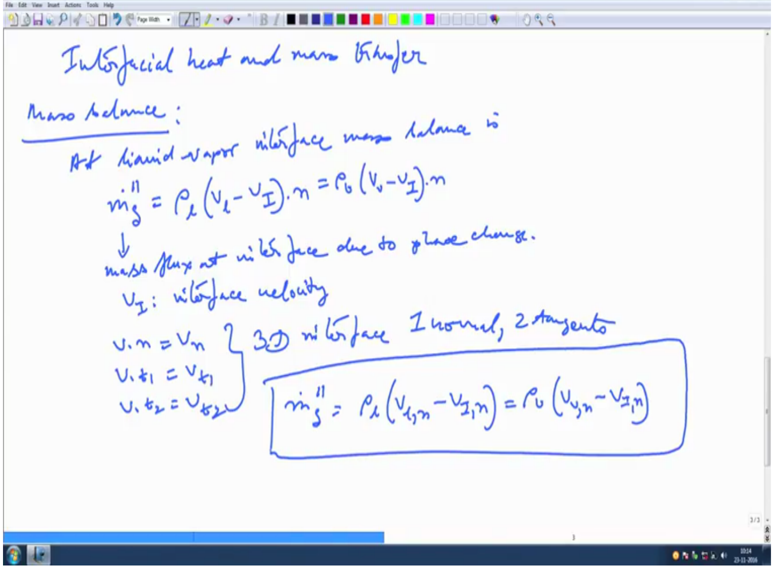

Interfacial heat and mass transfer

Mass balance:

At liquid-vapor interface mass balance is

$\dot{m}_s'' = \rho_l (v_l - v_I) \cdot n = \rho_v (v_v - v_I) \cdot n$

↓
mass flux at interface due to phase change.

v_I: interface velocity

$$\left.\begin{array}{l} v \cdot n = v_n \\ v \cdot t_1 = v_{t_1} \\ v \cdot t_2 = v_{t_2} \end{array}\right\}$$ 3D interface 1 normal, 2 tangents

$$\boxed{\dot{m}_s'' = \rho_l (v_{l,n} - v_{I,n}) = \rho_v (v_{v,n} - v_{I,n})}$$

Now having done this, let us look at some of the interesting conservation laws that we wrote earlier, right. We wrote this is conservation laws in terms of; if you recall the interfacial heat and mass transfer lecture, right. What did we do over there in the interfacial heat and mass transfer lecture? We did not include the effect of surface tension and disjoining pressure effect, right.

When we actually did it when we wrote the interfacial equations right, so, let us now include those things over here okay because once you include those things. Now you can have a holistic view right, of the interface. So, let us look at the mass balance first and I would not go through the details of the math, I will just write down the basic equations right at the liquid vapour interface right.

Once I write it you will recall that what I wrote earlier the mass balance is given by okay do you recall this particular form Vl – Vi, right, dot n right, okay, that is the interface that the reference speed of the interface to Rho V, Vv - VI dot n okay this is the mass flux at interface due to phase change recall VI this is just to recap VI was an interface velocity, right, okay.

Now this is the normal component of the velocity, this is the first tangent; this is basically the second tangent because you know it is a 3d interface. Interfaces are normally 3d right it is a 3d interface. So, basically you need one normal and two tangents right to define it, got it, okay. So, now the mass balance therefore write it Rho l Vln - VI n = Rho v VIn –VIn okay.

So, that is what we have for the in terms of the mass balance and we have included the 3d nature of the interface over here, right, okay, by combining all these terms that is the mass term.

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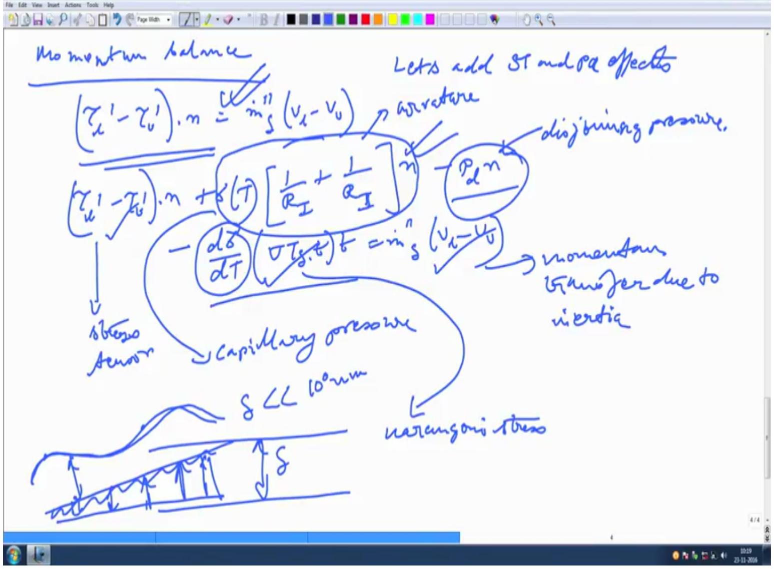

Momentum balance

$$(\tau_l' - \tau_v')\cdot n = \dot{m}''_s (v_l - v_v)$$

Lets add ST and PD effects

$$(\tau_l' - \tau_v')\cdot n + \sigma(T)\left[\frac{1}{R_I} + \frac{1}{R_{II}}\right] n - P_d n - \frac{d\sigma}{dT}(\nabla T_s \cdot t)\, t = \dot{m}''_s (v_l - v_v)$$

So, now let us look at the momentum balance, so, the momentum balance okay if we start writing out the momentum balance. Now the momentum balance that actually includes all the surface tension forces right because it should come from the momentum balance over here, right, okay. Now normal momentum balance will be something like this it is very similar to the mass balance right.

Except now that is the change of momentum is equal to the forces okay. Now let us include this we knew already. Let us add the 2 effects surface tension and Pd effects okay. So, this gets added to the left hand side of this equation, to this side of this equation. So, what do we have Tou l - Tou V prime dot n + Sigma T if you recall already that Sigma T was responsible for that Morangonie flow and related things.

So, this is like RI + 1 by R2 n - Pd into n okay Pd being disjoint; I will go across to each and every term over here d sigma by dt, this is where the Morangonie stress term comes into the picture Vl -Vv okay. Now let us look term by term this is basically nothing but the stress tensor right in the stress tensor we all know this okay.

This is basically nothing but the capillary pressures which we covered right a capillary pressure right. This is basically the disjoining pressure only happens for thin film, correct okay. This is basically what we call the Morangonie stress because you have this gradient right d sigma by dt okay, so you have a Morangonie stress okay if there is a temperature variation right.

And this is basically the momentum transfer due to inertia right. But as you can see when there was no; if the interface is flat say for example absolutely flat interface this term will go off to 0, correct. The second term will go of, this will only come when there is a curvature right.

If there is no curvature why should the term actually be retained? If the film is very thick the disjoining pressure will go. If there is no temperature variation, right, okay. This term will also go. So, we would go back to our original formulation across the interface right. So, if it is a straight flat interface and thick one right, okay.

If this is thick okay then of course your equation boils down to basically that right. Moment you have a curvy kind of an interface right you actually are including this term into the picture if there is a temperature variation from here to here okay say that can happen because they you have a heater okay.

You have a differential heat flux, say the temperature is increasing or you are supplying heat to this substrate in this particular fashion right. What will happen this surface temperature will also start to vary right because of that you will have actually have a variation of the Marangoni stress right across the interface that means to be accommodated over here, right, okay.

And if this film is very thin, let me see if this delta now becomes much, much less than say 10 nanometers or 100 nanometer. Then you actually bring this come into the picture right, okay. So, the momentum equation now basically boils down to this particular, these three additional terms all of which comes from the curvature, temperature, gradient right.

As well as the disjoining pressure all of which we covered in the previous lectures to give you guys an idea of right, okay.

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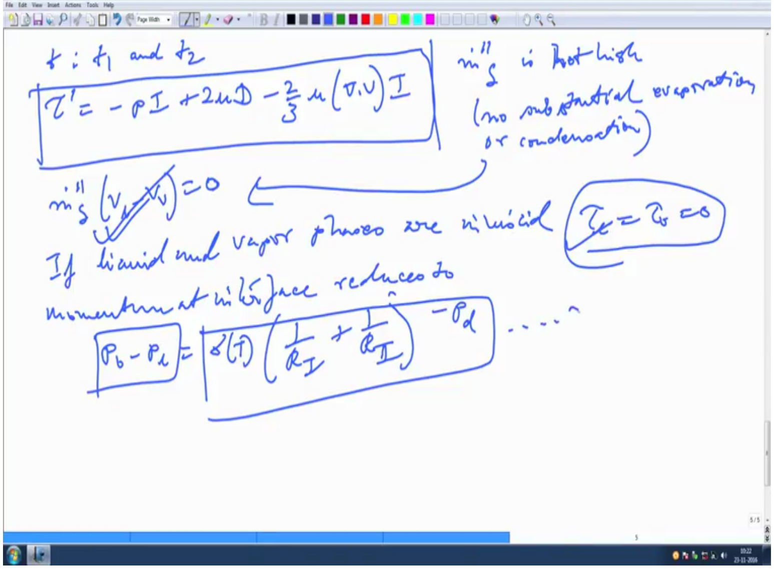

t : t_1 and t_2

$$\boxed{\tau' = -pI + 2\mu D - \frac{2}{3}\mu(\nabla \cdot V)I}$$

$\dot{m}''_S$ is not high (no substantial evaporation or condensation)

$\dot{m}''_S(\cancel{V_l - V_v}) = 0$

If liquid and vapor phases are inviscid $\boxed{\tau_l = \tau_v = 0}$

momentum at interface reduces to

$$\boxed{P_v - P_l} = \boxed{\sigma(T)\left(\frac{1}{R_I} + \frac{1}{R_{II}}\right) - P_d} \dots$$

Now this t of course can be t1 okay and t2, so that is the two tangents okay. Now this Tau prime any Tau prime can be written as the pressure plus the 2D minus two third not all of these terms are reaching into the corresponding I okay. In many of the cases so, this is the general form of the stress tensor right.

This we already know from your Navier-Stokes equation and whatnot right. So, in many cases in triple dot prime this one okay it is not high. It is not high right that means the other way of saying is that there is no evaporation or condensation or whatever it is very because you are talking about okay.

So, no substantial I would say this evaporation never stops right. No substantial evaporation or condensation, got it, okay. No normal evaporation or condensation in that case what will happen is that your m triple dot Vl - Vv this particular term will be equal to 0 right. If that is the case corollary of this right, if the liquid and the vapour phases are invicid say for example right.

So, if liquid and vapour phases are in invicid, it can be invicid right. In that particular case what will happen is there Tau l = Tau v = 0 right, there is no viscous shear right, okay. So, in that particular case the momentum at the interface reduces to okay Pv - Pl sigma T 1 by RI + 1 by RII - Pd right. This is we are very familiar with this is that capillary pressure form right, okay.

So, in most of the cases you get away using this because in many of the books you will find that this is what they write that across the interface this is the momentum balance right. Then not exactly always true. If there is evaporation okay, you have to include the evaporation term let us mandatory, right. If there is viscous stress you have to include the viscous stress at the interface.

But in many cases this may be considered negligible okay compared to the curvature effect that we have here okay. So, this is basically what we call the revised momentum equation, got it, okay. In most of the cases this would be kind of valid okay. Now thus the follow-up will be in the next we will start with the energy equation.

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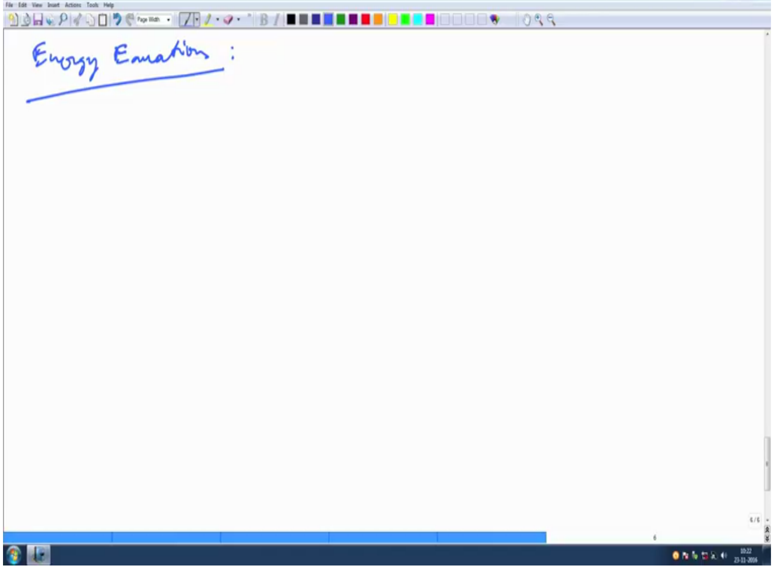

Energy Equation :

So, the energy equation and we will see how the energy equation actually varies over the interface because we have done mass, than momentum. We have shown how surface tension and the disjoining pressure enter into the picture how Marangoni stress enters into the picture. Now we will do the energy and then correspondingly do the species right.

Because these are the other two snippets right where we actually have to include these effects over there okay. So, that we will do in the next lecture okay, which follows okay thank you.