Heat Transfer And Combustion in Multiphase Systems
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Lecture 13
Governing equations: Averaging techniques-III

Welcome! So, we have done all this volume averaging for the last few lectures. And this lecture we will try to finish this chapter.
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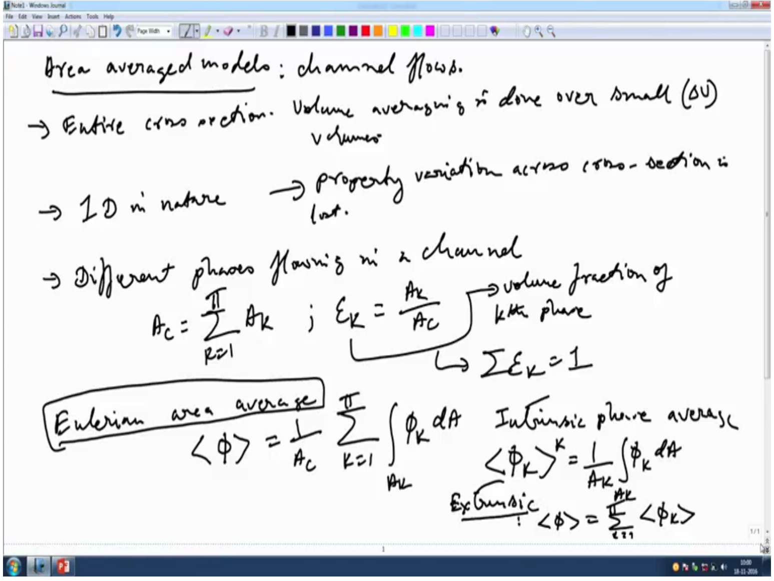

Area averaged models : channel flows.

→ Entire cross section. Volume averaging is done over small (ΔV) volumes

→ 1D in nature → property variation across cross-section is lost.

→ Different phases flowing in a channel

$A_c = \sum_{k=1}^{\pi} A_k$; $\varepsilon_k = \frac{A_k}{A_c}$ → volume fraction of kth phase → $\sum \varepsilon_k = 1$

$\boxed{\text{Eulerian area average}}$ $\langle \phi \rangle = \frac{1}{A_c} \sum_{k=1}^{\pi} \int_{A_k} \phi_k \, dA$

Intrinsic phase average $\langle \phi_k \rangle^k = \frac{1}{A_k} \int_{A_k} \phi_k \, dA$

Extrinsic $\langle \phi \rangle = \sum_{k=1}^{\pi} \langle \phi_k \rangle$

By doing area averaged models these are particularly important for channel flows. So, there is a basic difference between area averaged and the volume averaged models that we did. Now area averaging is usually done over the entire cross section right. So, the entire cross section okay and it is distinct from the volume averaging which is done over a small elemental volume.

Volume averaging is done over small that is delta V volumes okay. So, area averaging by default is kind of like a 1d in nature right because it is done over the entire cross section. So, basically you have a unidirectional nature of the whole thing. And distribution of properties that means variation of properties along the cross sectional area is also lost.

So, property variation across cross section is lost okay. So, you know keeping these things in mind let us look at how the area averaging model actually works. So, I assume that there are different phases. Phase is flowing in a channel it is like a channel flow okay. So, Ac is basically given by the summation k at the number of phases.

So, this is the total cross sectional area of that particular channel which is nothing but the sum total of all the individual areas that are occupied by the individual phases. Similarly Epsilon k which is basically the volume fraction of the kth phase is given by this. So, this is nothing but the volume fraction of kth phase okay. And it comes with the natural consequence, that if you sum total all this Epsilon k it will become one, okay.

So, if you do an Eularion area average okay, if you recall all the notation this is basically given by this. And we will have a small tutorial where we will cover some it show some example problems that how this area averaging and volume averaging works and that we will do later. So, it is basically nothing but this okay.

So, this is an Eularion area average of any particular quantity that you may be interested in, right, okay. So, if we; if you recall what was intrinsic averaging. So, intrinsic phase average given by if you recalls the notation from the earlier classes okay, so that is the intrinsic phase averaging right.

So when you have taken only the phase into consideration that means the area occupied by that particular phase okay. It is averaged over that particular phase. So, extrinsic on the other hand if you look as extrinsic phase averaging that is given by nothing but the summation of the individual phase averaging of the of the quantity that we recognized earlier. So, this is the extrinsic phase averaging okay.

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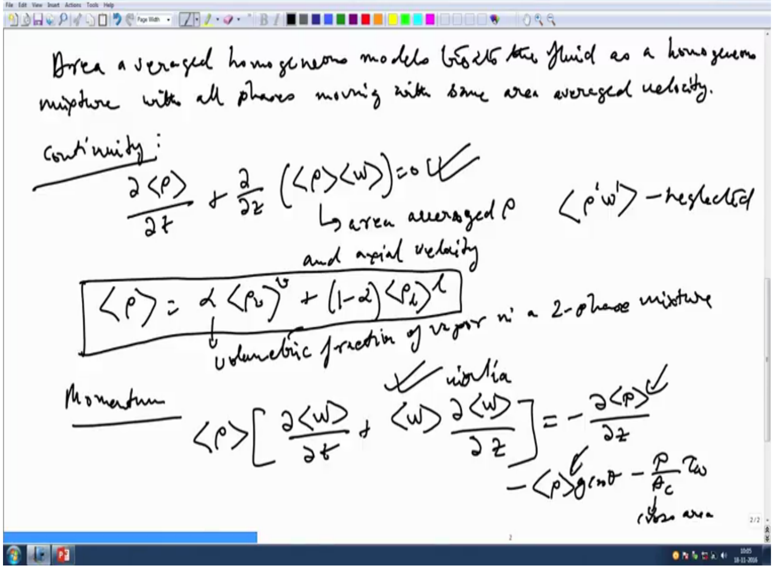

Area averaged homogeneous models treats the fluid as a homogeneous mixture with all phases moving with same area averaged velocity.

Continuity:

$$\frac{\partial \langle \rho \rangle}{\partial t} + \frac{\partial}{\partial z}\left(\langle \rho \rangle \langle w \rangle\right) = 0 \quad \langle \rho' w' \rangle - \text{neglected}$$

↳ area averaged ρ and axial velocity

$$\boxed{\langle \rho \rangle = \alpha \langle \rho_v \rangle^v + (1-\alpha)\langle \rho_l \rangle^l}$$

α ↓ volumetric fraction of vapor in a 2-phase mixture

Momentum

$$\langle \rho \rangle \left[\frac{\partial \langle w \rangle}{\partial t} + \langle w \rangle \frac{\partial \langle w \rangle}{\partial z}\right] = -\frac{\partial \langle p \rangle}{\partial z} - \langle \rho \rangle g \sin\theta - \frac{P}{A_c}\tau_w$$

inertia

A_c ↓ cross area

So, area averaged homogeneous model basically treats the flow as a homogeneous mixture with all the phases moving with the same area average velocity right. So, area averaged homogenous models and we will use all these models in different contexts okay treats the fluid as a homogeneous mixture.

With all phases moving with the same area averaged velocity which makes sense, right average velocity okay it makes perfect sense over here. So, based on this if we write the continuity and we will move fast with this because you are already done a lot of continuity equations in the two-volume averaged portion sections that we did.

So, this is basically the; remember one dimension only remains the other two dimensions are basically lost, right because you have averaged them out, okay. So, these are basically nothing but area averaged quantities area averaged, sorry averaged Rho and axial velocity, got it, okay.

So, and as usual earlier the product of quantities like the deviations of Rho and w we have to we have not; we have neglected them right. So, earlier we knew already that we usually neglect quantities like this right, okay. These are basically neglected correct this you already knew earlier okay, from the earlier exercises that we did okay.

So, this Rho is basically nothing but if there are two components, two phases, more phases is not a problem either okay. So, this is basically the volumetric fraction of the vapour in a two phase mixture that is just a special case this is the general equation okay. This is just a special case.

If there, if it is a two phase mixture this is how you should actually write it, got it, okay. So, similarly we can write the momentum okay, momentum should not be a problem either, okay. So, in the case of momentum this becomes much easier because we are dealing with only one dimension. The other dimensions are not there because of this okay.

So, in the case of this, if you know your pipe flow and other kind of equations like this, this should become very apparent that why it should look like this okay. But of course the pipe flow does not retain the convective term okay. But in the entrance length or where the flow is developing these terms would be there, okay.

This is P minus if there is any gravity dependent gradient that should be there okay this is the inclination effect essentially and then P by Ac into Tau w okay where the c is basically the cross-sectional area okay. So that is the net equation and all the terms are unknown inertia okay the pressure term, the gravity or the inclination term the shear stress term right, the wall shear stress term. So, that is pretty logical okay.

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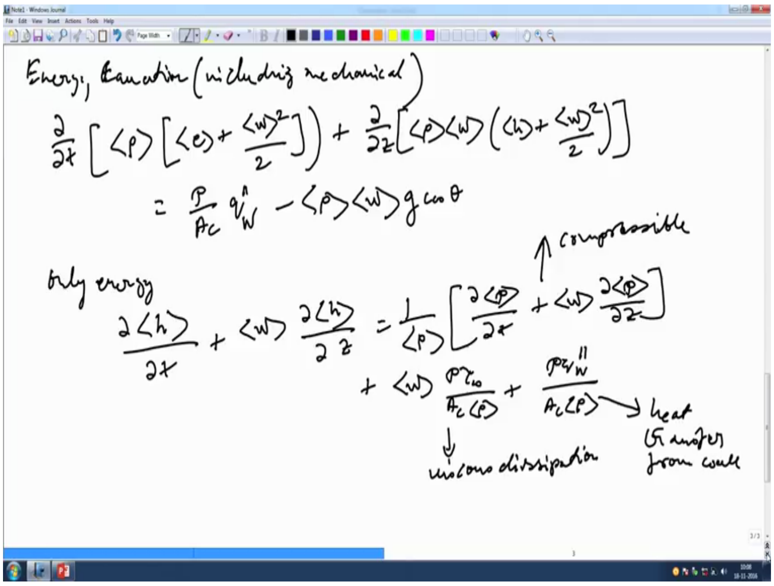

Energy Equation (including mechanical)

$$\frac{\partial}{\partial t}\left[\langle\rho\rangle\left[\langle e\rangle+\frac{\langle w\rangle^2}{2}\right]\right)+\frac{\partial}{\partial z}\left[\langle\rho\rangle\langle w\rangle\left(\langle h\rangle+\frac{\langle w\rangle^2}{2}\right)\right]$$

$$=\frac{P}{A_c}\hat{q}_W-\langle\rho\rangle\langle w\rangle g\cos\theta$$

only energy

$$\frac{\partial\langle h\rangle}{\partial t}+\langle w\rangle\frac{\partial\langle h\rangle}{\partial z}=\frac{1}{\langle\rho\rangle}\left[\frac{\partial\langle p\rangle}{\partial t}\overset{\uparrow\text{ compressible}}{+}\langle w\rangle\frac{\partial\langle p\rangle}{\partial z}\right]$$

$$+\underbrace{\langle w\rangle\frac{P\tau_w}{A_c\langle\rho\rangle}}_{\text{viscous dissipation}}+\frac{P q''_W}{A_c\langle\rho\rangle}\rightarrow\text{heat transfer from wall}$$

So, based on this we know already what the momentum equation looks like this. So, the energy equation I am writing the energy equation including the mechanical energy term okay. So, here plus okay this is g cos theta okay. So, that is the net energy equation including the mechanical energy term if we just deal with only energy that means you take out the kinetic energy and all those out of the expression.

So, this is written on enthalpy basis, so, this term is basically becomes your viscous dissipation, this equation and this term becomes your heat transfer from the wall okay. And this is basically the compressible okay fairly common. So, this is the net equation as you can see it is much simpler than what the volume averaged equations that we wrote, okay. And so, there is only one more equation that is left after this.

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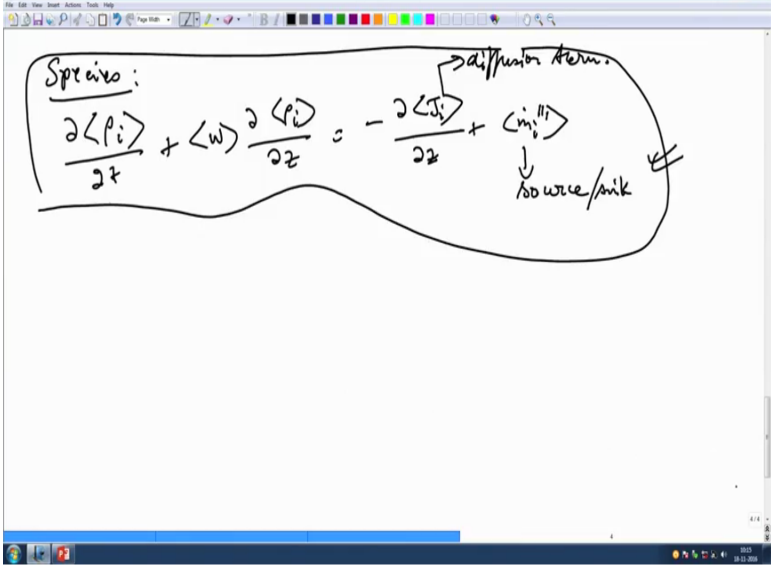

Species:

$$\frac{\partial\langle\rho_i\rangle}{\partial t}+\langle w\rangle\frac{\partial\langle\rho_i\rangle}{\partial z}=-\frac{\partial\overset{\rightarrow\text{ diffusion term.}}{\langle J_i\rangle}}{\partial z}+\underbrace{\langle\dot{m}_i'''\rangle}_{\text{source/sink}}$$

Which is basically the species balance okay. The species equation is almost exactly like the continuity equation, well almost like the heat the energy equation sorry okay. This is Ji plus the corresponding mi okay. So, this is basically the source and sink term. Now what is the source and the sink term.

Because some species may be actually transferred that means if you have a transfer from the liquid to the vapour phase for example that is a species transport. If a chemical reaction is going on that is species source and sink. This is basically nothing but the diffusion right, okay. So, this basically shows this entire band of equation that we have this if you go to the previous one this one energy.

And if we go to the one before that this is some continuity and this is the corresponding momentum. So, all these things are written now for the area averaged one okay. And once we do a tutorial or a sample problem in which we will solve that sample problem okay by using a combination of this area average as well as this volume averaged.

But that will be done later okay and that will be supplied as a study material where you can actually take a look which basically uses this equation. And see's how that can be applied to a real life problem okay. So, we can, I can give you the problem definition that how is it actually solved.

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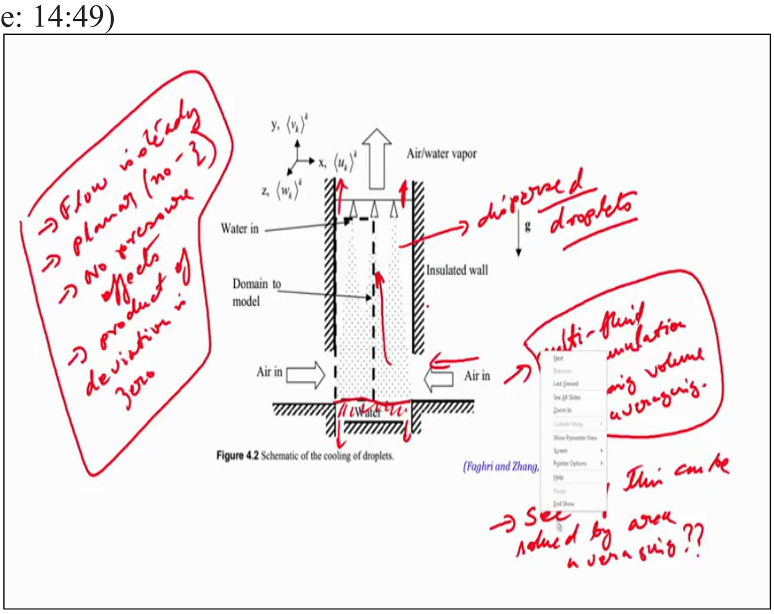

Figure 4.2 Schematic of the cooling of droplets.

(Faghri and Zhang,

→ Flow is steady
→ planar (no-z)
→ No pressure effects
→ product of deviation is zero

→ dispersed droplets

→ …lti-fluid …mulation …ing volume averaging.

→ See This can be solved by area averaging??

So, if you look at for example this expression here okay it is from okay, so if you look at the; this particular problem okay and which we are going to solve okay. This is basically nothing but a cooling tower, so to say, okay. So, what happens is that air enters here and it goes up okay so this is basically nothing but hot air, okay.

And water is sprayed from the top like a carton it is like a shower curtain okay almost like a shower curtain. Water is sprayed in the form of this dispersed phase. These are basically dispersed like droplets basically, got it. So, they are spread like droplets okay and what do they do? They interact with this airflow okay.

So, naturally there is a transfer of heat that takes place. So, and so there is a phase change of water droplets to water vapour okay and the air then it escapes out, okay and the water afterwards collects at the bottom. But we are not concerned about this or this you can say to instantaneously drained out, okay.

So, if we have to analyze a problem like this we can use both the multi fluid formulation using the volume average. So, this can be done by multi fluid formulation and we will give you as a write-up multi fluid formulation using volume average, volume averaging right, that can be done.

And we will try to see whether it can be done through area averaging also because there are it is like a constant area section this thing's are occupied by the two phases okay. I mean this problem does not have a numerical answer but it is basically how we can cast our equations okay. As you can understand most of the multiphase fluid problem do not have a numerical answer because it is a complicated physics right.

So, we are going to only be able to cast it in certain forms right but how we are able to cast it okay. So, we can write some of the assumptions and this is just to kick-start the problem not mean necessarily do it. We can assume that the flow is steady because that will get rid of the transient term okay.

It is planar; it is a planar flow okay. So, that means there is no Z variation okay. So, Z variation can be neglected there are no pressure effects, I mean these are all assumptions you can solve the problem including that, okay just will make life a little bit simpler. And product of the deviations as we already did in the; in our derivation is zero, okay.

So, using these assumptions can we write down what is the continuity, momentum, the corresponding x momentum, y momentum right, okay and the corresponding the forces that are acting on the system, right. Can we actually write a complete; can we cast this problem basically in a nice theoretical framework with all the governing.

And with all the governing equations write down all the boundary conditions recall these walls are insulated so you can have a actually a real heat flux condition at these walls. So, all these things we need to cast okay and show that this can be solved by the multi fluid formulation using volume averaging technique that we talked about okay.

So, this will be given to you as a write up okay. So, you can study that write up you can also read it from the textbook for this course which is transport phenomena in multiphase systems okay. It is an example problem there so but we will also give you a write up in which we will explain the details right that how this is actually done.

So, based on this what we have, we have we can also solve this problem as I said. Let us see okay as a student you guys can also try see if this can be solved by this area averaging. So, that is the question okay of course we will give you some answers to that but at this point this is a question if you are following the course in a sequential manner.

Can you also think about casting it in some other way okay. So, we will do a couple of example problems like this okay and we will give it to you in the form of write ups okay. So, let us move on to our journal and let us move to the next page.

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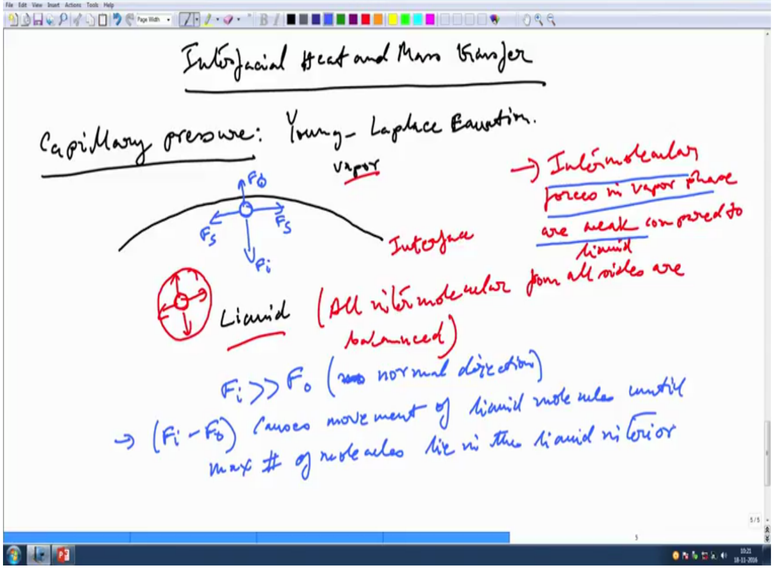

Interfacial Heat and Mass Transfer

Capillary pressure: Young- Laplace Equation.

→ Intermolecular forces in vapor phase are weak compared to liquid

Liquid (All intermolecular from all sides are balanced)

$F_i >> F_o$ (normal direction)

→ $(F_i - F_o)$ causes movement of liquid molecules until max # of molecules lie in the liquid interior

Now that we have covered we know more or less what these phenomena’s are. We are, we know what are the governing equations. We know the thermodynamics of the systems; we have covered quite a bit of materials in terms of getting you a feel that how multi-phase systems actually work.

Now let us look at some of the important things when this is interfacial heat and mass transfer and from here we will go on to some specialized topics after this okay which will involve mainly droplets okay. So, in interfacial heat and mass transfer okay the primary thing that you guys already know to a certain extent there is something called capillary pressure.

These are utility things that we need to know okay because this will help you to understand whatever we are going to do after this. So, this is capillary pressure is basically Young

Laplace equation. And how do you actually derive it, how do you what is the physical explanation of the same. So, that is what we are going to do over here.

Now if you assume that there is an interface, now look at it very carefully okay. This is the interface, so say this is liquid, the vapour okay. Let us use, so, if there is a fluid there is a liquid molecule. Let us look at the state of the two liquid molecules right. One liquid molecule, so, this is the interface that separates the two.

This is liquid; this is vapour, right, okay. So, assume that there is a liquid molecule which is sitting inside the liquid pool, far inside okay. What happens is that if you look at the intermolecular forces that are acting on this is very isotropic in nature, okay. So, that molecule is basically balanced from all sides okay.

So, the intermolecular forces from all sides okay are balanced okay. So, that is what happens when the molecule is actually inside okay. So, when the liquid molecule is inside the liquid pool, far inside the liquid pool away from the interface okay. It is basically in a balanced condition right.

All the intermolecular forces balance from all sides okay are balanced, got it, okay. And you should also note one very important observation the intermolecular forces in the vapour phase is weak compared to the liquid phase okay. That means the binding forces intermolecular forces in vapour phase is are weak compared to liquid okay.

So, that is a good; so, what happens to a molecule now which is at close to the surface like this right. So, what will happen is that this molecule on the two sides that is Fs on the two tangential sides okay. Most likely it is in a balanced condition because it is within the liquid right, whereas this force if we look at this as O and because Fi okay.

This Fi which is a pull that is exerted by the liquid phase on the molecule is sufficiently large compared to the pull that is exerted by the vapour because of this reason; intermolecular forces in the vapour phases are weak, okay. So, what happens is that this is basically nothing but the force in the normal direction right tangential direction is still balanced okay.

So, what happens is that this pull, this Fi this excess force rather tries to pull the molecule at the liquid vapour interface towards the liquid phase. So, if the molecule is here the liquid tries to pull it inside okay because of the dominance of this particular force over there okay. So, in other words Fi – F naught this differential right causes movement of liquid molecules, more molecules okay until maximum number of molecules lies in the liquid interior, got it.

You can understand the phenomena right let me explain it one more time. There is a dis-balance of the force right the vapour that is in the vapour phase the pull that is coming from the vapour phase is weak and a pull that is coming from the liquid phase is strong. So, naturally the molecule is pulled back into the liquid interior till there is a certain amount of packing, right.

So, that the maximum number of molecules lie in the liquid interior okay and one ready result of this is that this leads to the interface having minimum area, got it. So, this leads to the minimum area of the interface okay, one of the reason why you can see; when whenever you have all those droplets right.

You have always you do not have a sheet of water okay that is formed correct unless it is lying on the substrate that is a different issue, okay. But normally it will lead to this sphere's right. So, because it leads to a interface having the minimum area, right. So, you understood with these phenomena that so that is what governs.